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Abstract

This paper reviews recent works related to optimal control of energy storage systems. Based on a contextual analysis of more than 250 recent papers we attempt to better understand why certain optimization methods are suitable for different applications, what are the currently open theoretical and numerical challenges in each of the leading applications, and which control strategies will rise in the following years. The reviewed research works are divided to "classic" methods and "advanced" methods, in order to highlight the current developments and trends within each of these two groups. The classic methods include linear programming, dynamic programming, stochastic control methods, and Pontryagin's minimum principle, and the advanced methods are further divided into metaheuristic and machine learning techniques.

Keywords: Energy storage; Linear programming; Dynamic programming; Stochastic optimization; Pontryagin's minimum principle; Machine learning

1. Introduction

Over the past few years energy storage technologies are slowly emerging as an essential component of modern power systems \cite{1,2}. Batteries in
particular are being used in increasing numbers both in electric vehicles and in conjunction with renewable energy systems due to their reduced costs [12]. One primary driver for the growing interest in storage systems is the increasing use of renewable energy sources [13, 14]. A common claim is that renewable sources such as wind and solar are intermittent and unreliable, and thus require storage devices to be properly integrated in a utility system. To address this challenge one idea is to use storage devices for energy balancing: surplus energy is stored when the power demand is low, and used later when “the wind is not blowing, or the sun is not shining” [5, 15, 16]. Another common claim is that storage systems are crucial if the penetration level of renewable sources exceeds a certain threshold [11, 17]. This threshold however depends on many factors, varies from one system to another, and is currently not sufficiently well understood. On shorter time scales, fast reacting storage devices are crucial for frequency and angle stability. Since renewable energy sources and other power electronics based devices have little inertia, they may jeopardize the grid stability and the overall dynamic behavior [18–22]. This problem may be mitigated by fast-reacting storage systems that are installed alongside low-inertia sources.

A well-known challenge is how to optimally control storage devices to maximize the efficiency or reliability of a power system. As an example, for grid-connected storage devices the objective is usually to minimize the total cost, the total fuel consumption, or the peak of the generated power, while operating the device within its limits [23, 24]. The problem in this case is to decide how much energy should be stored, and when to store it. Such optimal control problems are generally hard to solve due to their high numeric complexity, and since finding an optimal solution requires computing the stored energy at each and every point in time. In addition, such optimization prob-
lems are usually not convex, either since the objective function is not convex, or since it is not defined over a convex set [17]. As a result, gradient-based optimization methods are usually inefficient, and tend to converge to local minima.

In light of these practical and theoretical problems, this paper reviews the state-of-the-art optimal control strategies related to energy storage systems, focusing on the latest challenges and trends. While several other works in the recent literature review storage technologies [1, 4, 6] and their various applications [25, 37], in this paper we perform a systematic review of the mostly used control strategies, and attempt to better understand why certain methods are the most efficient in different scenarios, what are the currently open theoretical and numerical problems in each application, and which applications and control strategies will rise in the following years. Specifically, one of our goals is to highlight correlations between certain control methods and certain applications, which is accomplished by a contextual analysis of more than 250 recent papers. These are divided to “classic” methods and “advanced” methods, in order to highlight the current developments and trends within each of these two groups.

2. Controlling storage systems: From models to optimization problems

Storage devices come in various sizes and serve different needs [11, 17]. For instance, the term grid-scale energy storage encompasses a number of technologies such as pumped hydroelectric storage, compressed air storage, batteries, flywheels, superconducting magnetic energy storage, and super-capacitors [1, 4, 6]. These technologies are also characterized by many parameters such as energy density, capacity, power density, efficiency, lifetime, cost, and many others. Obviously different devices have different dynamics and constraints, and are therefore represented by various models. For instance, a dynamic model describing the device at seconds or sub-seconds time frames must take into account the accurate physical behavior of the device. In addition, such a model must describe the specific inverter connecting the storage device to the grid, and the specific control algorithm being used [38, 43].

For the purpose of designing optimal control strategies fast dynamic phenomena are usually negligible, so storage systems are often modeled based
on the following simple differential equation:

\[
\frac{d}{dt}E(t) = \begin{cases} 
\eta_c(E(t), P(t))P(t), & P(t) \geq 0, \\
\eta_d^{-1}(E(t), P(t))P(t), & P(t) < 0,
\end{cases}
\]  

(1)

where \( E(t) \) is the stored energy, \( P(t) \) is the total power flowing into the device, \( \eta_c(E, P) \) is the charging efficiency, \( \eta_d(E, P) \) is the discharging efficiency, and \( \eta_c(E, P) \) and \( \eta_d(E, P) \) have values in the range \((0, 1]\). This model focuses on the relations between power and energy, and is useful mainly at time scales of seconds to days. On shorter time scales other dynamic effects have to be considered, and the above model becomes inaccurate. Likewise, on longer time scales factors such as degradation, losses, and leakages are becoming increasingly important, and again the above model becomes inaccurate.

One of our objectives in this paper is to compare different optimal control strategies using a common dynamic model, and based on specific examples. To this end, consider an energy storage device which is used for energy trading in a typical power network which consists of loads, conventional, and renewable power plants as shown in Fig. The device is assumed to be lossless, the power flowing into the device is \( P(t) \), the price of energy is \( C(t) \), and the device capacity is \( E_{\text{max}} \). The objective is to minimize the total cost of energy:

\[
\text{minimize} \quad \int_0^T C(t)P(t)\,dt
\]

subject to

\[
E(t) = \int_0^t P(\tau)\,d\tau, \quad 0 \leq E(t) \leq E_{\text{max}},
\]

(2)

where \( C(t) > 0 \). Here the optimal total cost is always zero or negative, since at the worst case one can simply choose \( P(t) = 0 \). In order to use numeric algorithms such problems are usually converted to discrete time. In this case define \( \Delta = T/N, \ t = i\Delta, \ C_i = C(i\Delta) \), and \( P_i = P(i\Delta) \) for \( i = 0, 1, \ldots, N \) to obtain

\[
\text{minimize} \quad \Delta \sum_{i=1}^N C_i P_i
\]

subject to

\[
E_k = \Delta \sum_{i=1}^k P_i, \quad k = 1, \ldots, N, \quad 0 \leq E_k \leq E_{\text{max}}.
\]

(3)
This problem will be revisited in the following sections, where we will show how to solve it using different techniques.

![Diagram of a grid-connected storage device](image)

**Figure 1:** Representation of a typical grid-connected storage device.

### 3. Classical optimization techniques

#### 3.1. Linear programming strategies

A straight-forward approach for solving problems such as (3) is linear programming \[45\]. This method can be used only if the objective function is linear, and the constraints are either linear equalities or linear inequalities. Such problems can be efficiently solved by several classical methods such as simplex algorithms, or interior point algorithms. Linear programming techniques are often used in energy trading problems. As an example, problem (3) can be written in the canonical form of linear programs as

\[
\begin{align*}
\text{maximize} & \quad c^T x \\
\text{subject to} & \quad Ax \leq b, \\
& \quad x \geq 0,
\end{align*}
\]

where \( x = (E_1, \ldots, E_N)^T \) is the vector of variables, \( A = I_{N \times N} \) is the matrix of coefficients, and \( c = (C_1 - C_2, C_2 - C_3, \ldots, C_{N-1} - C_N, C_N)^T \), \( b = E_{\text{max}} I_{N \times 1} \) are vectors of known coefficients. This simple problem has a trivial analytic solution, for instance

\[
E_i = \begin{cases} 
E_{\text{max}}, & \text{if } C_{i+1} > C_i, \\
0, & \text{if } C_{i+1} < C_i, \\
E_{i-1}, & \text{if } C_{i+1} = C_i 
\end{cases}
\]
for \( i = 1, \ldots, N - 1 \) with \( E_0 = 0 \) and \( E_N = 0 \). This solution is globally optimal but unrealistic since the resulting powers are unbounded, and may become very large. A more realistic solution is obtained if the detailed operating limits of the storage device are taken into account.

Major advantages of linear programming techniques are their simplicity, low numeric complexity, and the fact that most solvers will converge to a globally optimal solution. Naturally a main disadvantage is that both the objective function and constraints must be linear. Another disadvantage is that uncertainties are not trivially handled. For instance, in the simple example above the problem can only be formulated in the canonical form if all the costs are assumed to be known. In practical problems this is not always the case. These ideas are demonstrated in several works from the recent literature, as shown next. A considerable number of works focus on optimal scheduling of storage systems. For example, work [46] studies a campus central chiller plant containing a bank of electrical chillers and a thermal energy storage device. The best operation strategy for the chillers is found by solving a mixed-integer linear programming problem, while the thermal energy storage is managed based on dynamic programming. The main objective is to minimize the daily electricity cost. This method yields a low-complexity solution, since the dynamic programming algorithm allows to divide the original problem to simpler sub-problems, which are then solved based on mixed-integer linear programming. In [47] two optimization methods are applied to an islanded microgrid that includes renewable energy sources, diesel generators and battery energy storage systems. The first method uses linear programming, and its objective is to minimize the energy produced by the diesel generators. The second method uses mixed-integer programming with an objective to minimize the total cost. Both models control the battery state of charge. The mixed-integer programming method is more accurate but more complex than the linear programming method.

As mentioned above, one advantage of linear programming is that it has known convergence properties and can quickly solve problems with a large number of variables. These properties make linear programming a suitable method for solving large-scale energy storage optimization problems. Work [48] proposes an optimal strategy for managing a microgrid. The objective in this problem is to supply the load at the lowest marginal cost. A linear programming based solution is proposed in order to achieve low computational time. Work [49] uses forecasts and historical data of wind to plan the day-ahead and week-ahead targets for a pumped-hydro energy storage device. In this paper the storage is operated based on the wind power production during the day, under the constraint of a fixed stored energy at
the end of each day. Several of the latest works are summarized in Table 1.

3.2. Dynamic programming strategies

Dynamic programming is an optimization method that solves complex problems by dividing them into a series of simpler sub-problems, using a multi-stage decision process [63–68]. Unlike several other optimization methods, dynamic programming algorithms scan all the feasible solutions of a problem to locate the globally optimal one. A direct scan of the entire solution space is numerically impossible, so the problem is solved in a number of steps, based on a recursive formula. As defined by Richard Bellman: “An optimal control policy has the property that no matter what the previous decisions (i.e., controls) have been, the remaining decisions must constitute an optimal policy with regard to the state resulting from those previous decisions” [25].

Dynamic programming algorithms are very general, and apply to both linear and nonlinear objective functions and constraints. In addition, the objective function may be convex or non-convex, and the algorithm is guaranteed to converge to the globally optimal solution, if it exists. On the other hand, dynamic programming methods can only be used if the cost function can indeed be computed using a recursive formula. Another disadvantage is that prior knowledge of all past and future signals is required, which is not always realistic. However, this difficulty can be resolved by using stochastic dynamic programming methods, as described in Section 3.3. The complexity of dynamic programming algorithms increases linearly with the number of stages, but exponentially with the number of state variables. This phenomenon is often called the “curse of dimensionality” [69]. The practical meaning for energy storage related problems is that the complexity increases linearly with the number of time samples, but exponentially with the number of storage devices, and with the number of state variables describing each device.

To demonstrate a basic dynamic-programming solution, recall the energy trading problem (3) presented in Section 2. This cost function may be computed using the recursive formula

$$V_k(E_k) = \min_{0 \leq E_{k-1} \leq E_{\text{max}}} \{V_{k-1}(E_{k-1}) + (E_k - E_{k-1})C_k\}, \quad (6)$$

where $E_k$ is defined in the range $[0, E_{\text{max}}]$. The process starts with $k = 1$, for which by definition $V_1(E_1) = E_1C_1$. Then for every $k \geq 2$ the recursive formula (6) is used to compute $V_k(E_k)$. The next step is to compute the
Table 1: Summary of linear programming methods.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Category</th>
<th>Application</th>
<th>Objective</th>
<th>Decision variables</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>EB</td>
<td>Optimal scheduling and operation of a virtual power plant</td>
<td>Minimize generation and (dis)charging costs</td>
<td>Generated power by renewable sources and fuel cell and battery (dis)charging rates</td>
<td>Power and storage limits</td>
</tr>
<tr>
<td>51</td>
<td>EB</td>
<td>Grid-connected PV array</td>
<td>Minimal peak of generated power</td>
<td>Generated power from PV and storage (dis)charging rates</td>
<td>Battery capacity, (dis)charging rates</td>
</tr>
<tr>
<td>52</td>
<td>EB</td>
<td>Optimal scheduling in MGs</td>
<td>Minimal total cost and power generation</td>
<td>Generated power from wind, PV and storage (dis)charging rates</td>
<td>Storage capacity, generation power</td>
</tr>
<tr>
<td>53</td>
<td>EMS</td>
<td>Self-scheduling of compressed air ES</td>
<td>Flatten the load curve and minimize the operation cost</td>
<td>Generated power from wind, PV and storage</td>
<td>Physical limitations</td>
</tr>
<tr>
<td>54</td>
<td>EMS</td>
<td>Isolated power system</td>
<td>Minimal cost of daily operation</td>
<td>Energy capacity of the Storage station</td>
<td>Battery capacity, frequency regulation</td>
</tr>
<tr>
<td>55</td>
<td>EMS</td>
<td>Regenerative braking in electric railway systems</td>
<td>Minimal total investment and operating costs</td>
<td>ES size and cost, battery (dis)charging rates</td>
<td>Storage capacity, economic constrains</td>
</tr>
<tr>
<td>56</td>
<td>EB</td>
<td>Power-flow in grid-connected MGs</td>
<td>Minimal total cost and load smoothing</td>
<td>Buying/selling prices, (dis)charging rates</td>
<td>Battery energy, power rate change</td>
</tr>
<tr>
<td>57</td>
<td>EB</td>
<td>OPF</td>
<td>Minimal cost of generation’s operation</td>
<td>Generation factors, (dis)charging rates</td>
<td>Storage capacity, generator</td>
</tr>
<tr>
<td>58</td>
<td>EMS</td>
<td>Unit commitment with distributed energy source</td>
<td>Maximum market benefit, minimum cost of ES</td>
<td>ESS size</td>
<td>Capacity</td>
</tr>
<tr>
<td>59</td>
<td>EMS</td>
<td>Unit commitment with renewable sources</td>
<td>Minimal total cost</td>
<td>Number of renewable sources, generated power from generators and storage (dis)charging rates, on/off state of thermal units</td>
<td>N – 1 contingency analysis, storage capacity</td>
</tr>
<tr>
<td>60</td>
<td>EMS</td>
<td>Unit commitment with wind generators</td>
<td>Minimal total cost</td>
<td>Transmission network limitations, (dis)charging rates boundaries</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>ET</td>
<td>Energy trading</td>
<td>Maximum profit</td>
<td>Storage (dis)charging prices, SoC, generated power by conventional and wind generators</td>
<td>Profit regularization, SoC, (dis)charging rates</td>
</tr>
<tr>
<td>62</td>
<td>EV</td>
<td>EV charging</td>
<td>Maximum revenue</td>
<td>EV and storage (dis)charging rates</td>
<td>Day-ahead prices, charging, power capacity, storage SoC</td>
</tr>
</tbody>
</table>
optimal stored energy. The process starts with $k = N$ for which $E_N^* = \arg \min_{0 \leq E_N \leq E_{\text{max}}} V_N(E_N)$, and then for every $k \leq N$,

$$E_{k-1}^* = \arg \min_{0 \leq E_{k-1} \leq E_{\text{max}}} \{V_{k-1}(E_{k-1}) + (E_k^* - E_{k-1})C_k\}. \quad (7)$$

Finally, the optimal powers $P_i^*$ are

$$P_1^* = \frac{E_1^*}{\Delta},$$

$$P_i^* = \frac{E_i^* - E_{i-1}^*}{\Delta} \quad \text{for } i = 2, \ldots, N. \quad (8)$$

This is the globally optimal solution of the original problem.

Due to various advantages, dynamic programming based algorithms are used extensively for solving energy storage optimization problems. Several studies use dynamic programming to control storage in residential energy systems, with the goal of lowering the cost of electricity \[70–72\]. For example, work \[72\] uses dynamic programming to optimally control a residential energy storage system, considering scenarios with and without local electricity generation, and under different weather conditions. It is demonstrated that real-time predictions of the load and generated power improve the results under most tariffs, especially when the energy produced locally is low. Work \[71\] proposes a dynamic programming based control strategy to minimize electricity costs with different combinations of PV panel sizes and storage capacities. The results are then used to determine the optimal PV panel size and storage capacity combination considering the investment costs. Similarly, work \[70\] uses dynamic programming to evaluate the profits of a PV and storage system under different price structures.

Several studies propose dynamic programming solutions to optimize the power flow within a grid. For instance, in \[73\] an energy management strategy is formulated for a microgrid that includes solar panels, a wind turbine, a diesel generator, and a battery energy storage system. The goal is to find the optimal energy balance that meets the power demand and minimizes the total fuel consumption. Here the computation time is reduced by using the Pontryagin’s maximum principle to narrow down the possible minima to several options, which reduces computation time by several orders of magnitude, and improves the precision of the solution. Another constraint affecting the optimal power flow is the rate at which power is supplied, as discussed in \[74\]. This study presents a new control algorithm for a grid-connected system containing loads, renewable energy sources, and a storage device. The aim is to optimize the revenue from energy trading, under several constraints.
including a limited grid power ramp-rate. Dynamic programming is used in this work to obtain a reference for the storage device control system.

Dynamic programming methods often rely on prediction algorithms to improve the results under uncertainty conditions \cite{2, 4, 6}. For instance, paper \cite{6} presents an algorithm for managing the energy stored in a battery in a residential PV system. Two different strategies are applied with the aim of minimizing the total cost of electricity. The first one is based on the current state of charge, and the latter uses weather predictions, which are based on daily weather forecasts and demand profiles. The results indicate that the prediction algorithm achieves significant improvement when compared to the first method, particularly in cases of cloudy sky and during certain seasons. Another example of using predictions can be seen in \cite{5}, which minimizes the power variance in a system that includes a generator, load, and a storage device. The paper proposes a real-time control strategy for the generator which is based on dynamic programming and a linear regression analysis to forecast the load curve. Several of the mentioned works are summarized in Table 2.

### 3.3. Stochastic control strategies

Stochastic control methods model and solve optimization problems involving uncertainties. For instance, when controlling storage devices one leading approach is Model Predictive Control (MPC), while another is to formulate the problem as a Markov decision process, and to solve it based on stochastic dynamic programming. In contrast to classical dynamic programming, which optimizes the control policy for predetermined signals, stochastic dynamic programming optimizes the control policy over a family of possible signals \cite{25}. Likewise, the output of the algorithm is not a single optimal solution, but an optimal control strategy which is implemented in real-time based on actual measurements.

As an example, consider again an energy storage device which is used for energy trading, as in Section 2. The objective is to find the optimal trading strategy considering uncertainties in the future cost of energy which can be related to power variation from renewable sources. At every instance $k$ the power flowing into the device is $p_k$, and the cost of energy is $C_k$. The cost function is not known in advance, and is modeled as a Markov chain with conditional density functions $g_k(C_k|C_{k-1})$ which are assumed to be known. It is also assumed that $C_k > 0$ and the initial cost $C_0$ is given. At every instance $k$ the objective is to choose the present and future powers.
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Category</th>
<th>Application</th>
<th>Method variation</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>[77]</td>
<td>EMS</td>
<td>Dynamic pricing</td>
<td>Stochastic</td>
<td>Minimize the average cost of electricity used and investment in storage while satisfying demand</td>
</tr>
<tr>
<td>[72]</td>
<td>EB</td>
<td>Real-time pricing</td>
<td>Backward step is modified to extend the battery life-time</td>
<td>Optimal control of a residential ESS without and with local generation</td>
</tr>
<tr>
<td>[78]</td>
<td>EMS</td>
<td>Energy pricing</td>
<td>Basic</td>
<td>Minimize the sum of energy cost and demand charge</td>
</tr>
<tr>
<td>[70]</td>
<td>EMS</td>
<td>Energy balancing</td>
<td>Stochastic</td>
<td>Reduce the daily energy expenditure of the network operator while serving the network subscribers traffic load</td>
</tr>
<tr>
<td>[80]</td>
<td>EMS</td>
<td>Grids</td>
<td>Stochastic</td>
<td>Compute the maximum value of ESS in grid applications under uncertain energy prices</td>
</tr>
<tr>
<td>[81]</td>
<td>EB</td>
<td>Smart MGs</td>
<td>Evolutionary adaptive with reinforcement learning framework</td>
<td>Maximize reliability, self-sustainability, environmental friendliness, customer satisfaction, and extend battery life</td>
</tr>
<tr>
<td>[82]</td>
<td>EB</td>
<td>MGs</td>
<td>Stochastic dual</td>
<td>Minimize the expected total energy costs subject to storage capacity, line capacity, and other physical constraints</td>
</tr>
<tr>
<td>[79]</td>
<td>EMS</td>
<td>MGs</td>
<td>Combined with PMP</td>
<td>Develop optimal and efficient energy management strategy</td>
</tr>
<tr>
<td>[83]</td>
<td>EMS</td>
<td>MGs</td>
<td>Deep learning adaptive</td>
<td>Real-time management and control decision making for the energy management of micro-grid</td>
</tr>
<tr>
<td>[74]</td>
<td>EB</td>
<td>MGs</td>
<td>Basic</td>
<td>Minimize the operation cost and trade the energy</td>
</tr>
<tr>
<td>[84]</td>
<td>EMS</td>
<td>Households</td>
<td>Combined with decision tree</td>
<td>Real-time energy management in a multi-source system</td>
</tr>
<tr>
<td>[85]</td>
<td>EV</td>
<td>Urban trains</td>
<td>Heuristic</td>
<td>Optimize train running time and speed profiles</td>
</tr>
<tr>
<td>[86]</td>
<td>EV</td>
<td>Plug-in HEVs</td>
<td>Approximated cost-to-go part</td>
<td>Reduce computational demand and the memory requirements</td>
</tr>
<tr>
<td>[87]</td>
<td>EV</td>
<td>HEVs</td>
<td>Multi-dimensional</td>
<td>Determine the maximum potential of hybrid storage for reducing fuel consumption</td>
</tr>
</tbody>
</table>
\{p_k, \ldots, p_N\} that maximize the profit, or equivalently minimize the loss:

\[
\minimize \quad \Delta \sum_{i=k}^{N} p_i E[C_i | C_k = c_k]
\]

subject to \( e_j = \Delta \sum_{i=1}^{j} p_i, \; j = 1, \ldots, N, \)

\[0 \leq e_j \leq e_{\text{max}}, \quad |p_i| \leq p_{\text{max}}.\]  

Here \( \Delta \) is the sample time, \( N \) is the number of samples, \( e_{\text{max}} \) is the device capacity, and \( p_{\text{max}} \) is its maximum power. The present cost \( C_k \) is assumed to be known, so the objective function is defined with respect to the expected costs, given that \( C_k = c_k \). It is also assumed that the past powers \( \{p_1, \ldots, p_{k-1}\} \) are known.

Following is a solution based on stochastic dynamic programming. First, note that a necessary condition for an optimal solution is \( e_N = 0 \), since otherwise one can always decrease \( p_N \) to obtain a more optimal solution. An equivalent problem is therefore

\[
\minimize \quad \Delta \sum_{i=k}^{N} p_i E[C_i | C_k = c_k]
\]

subject to \( e_j = \Delta \sum_{i=1}^{j} p_i, \; j = 1, \ldots, N, \)

\[0 \leq e_j \leq e_{\text{max}}, \quad |p_i| \leq p_{\text{max}}, \; e_N = 0.\]  

To solve this problem define the value function

\[
V_k(e_{k-1}, c_k) = \Delta \min_{p_k, \ldots, p_N} \sum_{i=k}^{N} p_i E[C_i | C_k = c_k]
\]

subject to \( 0 \leq e_j \leq e_{\text{max}}, \; |p_i| \leq p_{\text{max}}, \; \Delta \sum_{i=k}^{N} p_i = -e_{k-1}, \)

where the energy values \( e_j \) are defined in [9]. This function describes the optimal cost at instance \( k \) assuming that \( e_{k-1} \) and \( C_k = c_k \) are known. Note that the last constraint is equivalent to the constraint \( e_N = 0 \), since \( \Delta \sum_{i=k}^{N} p_i = \Delta \sum_{i=1}^{N} p_i - \Delta \sum_{i=1}^{k-1} p_i = e_N - e_{k-1} = -e_{k-1}. \)

The value function may be calculated step-by-step based on recursion. This may be done offline, before any information on the actual costs is available. The process starts with \( k = N \) for which \( V_N(e_{N-1}, c_N) = h(-e_{N-1})c_N, \)
where the function $h(x)$ is defined as follows

$$h(x) = \begin{cases} x, & \text{if } |x| \leq p_{\text{max}} \Delta, \\ \infty, & \text{otherwise}. \end{cases} \tag{12}$$

Then for $k = N - 1, \ldots, 2$

$$V_k(e_{k-1}, c_k) = \min_{0 \leq e_k \leq e_{\text{max}}} \{ E[V_{k+1}(e_k, C_{k+1}) + h(e_k - e_{k-1})C_k | C_k = c_k] \}$$

$$= \min_{0 \leq e_k \leq e_{\text{max}}} \{ E[V_{k+1}(e_k, C_{k+1}) | C_k = c_k] + h(e_k - e_{k-1})c_k \}$$

$$= \min_{0 \leq e_k \leq e_{\text{max}}} \{ r_k(e_k, c_k) + h(e_k - e_{k-1})c_k \}, \tag{13}$$

where $e_{k-1}$ is defined in the range $[0, e_{\text{max}}]$, and $r_k(e_k, c_k)$ is given by

$$r_k(e_k, c_k) = E[V_{k+1}(e_k, C_{k+1}) | C_k = c_k] = \int_\theta V_{k+1}(e_k, \theta) g_{k+1}(\theta | c_k) d\theta, \tag{14}$$

and $g_{k+1}(\theta | c_k)$ are the known conditional density functions at each instance.

After the value function has been computed the optimal trading strategy may be found in real-time, during the trading process. At every instance $k$ the actual cost $C_k = c_k$ is already available. In addition, the optimal energy $e_{k-1}^*$ is known, since it was calculated at the previous step. Based on this data, the optimal energy and power at instance $k$ may be calculated by

$$e_k^* = \arg\min_{0 \leq e_k \leq e_{\text{max}}} \{ E[V_{k+1}(e_k, C_{k+1}) | C_k = c_k] + h(e_k - e_{k-1}^*)c_k \}$$

$$= \arg\min_{0 \leq e_k \leq e_{\text{max}}} \{ r_k(e_k, c_k) + h(e_k - e_{k-1}^*)c_k \}, \tag{15}$$

$$p_k^* = \frac{1}{\Delta} \left( e_k^* - e_{k-1}^* \right)$$

with $e_0^* = 0$. These energies and powers are the globally optimal solution of the original problem.

Many problems have different elements of uncertainty, such as varying load curves, varying energy production of renewable sources, or time-varying price signals. In many energy storage optimization problems these uncertainties are crucial, and substantially affect the optimal energy management and overall system cost [77, 88–90]. Such uncertainties are often handled by stochastic dynamic programming methods. For instance, work [77] explores an energy storage management problem in a system that includes renewable energy sources, and considers a time-varying price signal. The goal is to
minimize the total cost of electricity and investment in storage, while meeting the load demand. The model considers losses which are described based on the device round-trip efficiency and dissipation mechanisms.

In addition, many studies use stochastic control methods to minimize the effects of forecast errors related to power demands or renewable energy production [91–93]. For instance, paper [91] proposes a power management strategy for a Li-ion battery and flywheel energy storage system coupled to a PV array, within a microgrid. The applied algorithm considers changes in PV production by applying a stochastic framework, which is used to produce a smoother battery power profile, and to limit the power flow between the units. This method is shown to be robust, and is not sensitive to the initial state-of-charge of the storage device. Work [92] applies stochastic optimization to increase the system flexibility by compensating unavoidable real-time mismatches between the production and consumption of electricity. Uncertainty in renewable energy generation is considered by modeling the upper and lower bounds of the forecast error. The work considers a pumped-hydro storage system, and takes into account several mechanical constraints for increased accuracy. Moreover, work [93] proposes a stochastic dynamic programming framework for optimal energy management of a smart home with a plug-in electric vehicle energy storage. The goal is to minimize electricity costs while supplying the consumer demand and EV charging requirements. This paper analyzes various scenarios such as vehicle-to-grid, vehicle-to-home and grid-to-vehicle, and uses probabilistic models to describe the drive time and route length.

Markov decision processes, a common decision making tool, are often used with stochastic dynamic programming [78, 79, 89, 94]. Paper [94] explores a system with a renewable source and a storage device, and proposes to model bidding processes as continuous-state Markov decision processes. The problem is then solved based on stochastic dynamic programming. To avoid the “curse of dimensionality”, the value function is approximated by a set of linearly independent functions, and the results are compared to the ones obtained by linear programming. Works [78, 89] study the optimal operation of a residential energy storage device. The system includes a renewable source, and is subjected to random electricity prices. The model includes a single state, which evolves as a Markov chain. This state describes both deterministic and random factors, such as the current time, consumer demand, and weather conditions, all of which might affect pricing and generation of renewable energy. In [89] this formulation is extended to include the effects of power electronics converters interfacing the sources and loads. These have different conversion efficiencies, and thus complicate the problem.
Several studies address the “curse of dimensionality” in the context of stochastic dynamic programming [82, 95–97]. For instance, in [82] the objective is to minimize the expected total energy costs within a grid-connected microgrid over a finite horizon. The main constraints are the storage capacity and the line capacity. The model considers uncertainty in demand, in renewable generation, and in electricity prices, and takes into account the battery state of charge, losses, and efficiency. The algorithm is designed to be computationally efficient, and uses an elimination method that considerably reduces the number of possible solutions. Work [97] discusses the arbitrage value of storage devices. The ramp-rate constraint of the storage system is relaxed in order to obtain an analytical solution for the energy management problem, thus lowering the computational complexity. Numeric simulations support the suggested method, and provide additional information such as the expected optimal profit, the payout of the storage and the optimal storage sizing. Several of the above works are summarized in Table 3.

3.4. Strategies based on Pontryagin’s minimum principle

Pontryagin’s minimum principle is used in optimal control theory to find guidelines for the best possible control [101–108]. Unlike other methods, the minimum principle often does not provide a single well-defined solution, but only states necessary conditions which may lead to optimal control strategies. This is both an advantage and a disadvantage. While the minimum principle is not straightforward to use, it may yield simple solutions with relatively low complexity, which can be implemented in real-time [109]. This simplicity is often critical when the optimal solution should be applied in practice. A typical application is secondary storage management in electric vehicles, in which the future load is not precisely known, nor can it be easily modeled as a stochastic process.

To demonstrate this method, consider a simple power system consisting of a fueled primary source, a load, and a storage device as presented in Fig. 2. The challenge is to determine the optimal power generated by the primary source at every point in time \( t \), such that the total cost is minimized:

\[
\text{minimize} \quad F_{\text{tot}} = \int_0^T F(u(t) + P_L(t))dt \\
\text{subject to} \quad \frac{dx(t)}{dt} = u(t), \quad x(0) = 0, \quad x(T) = 0, \\
0 \leq x(t) \leq E_{\text{max}},
\]

where \( u(t) \) is the generated power, \( P_L(t) \) is the load profile, \( x(t) \) is the stored energy, and \( F(\cdot) \) is a cost function representing fuel consumption or any
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Category</th>
<th>Application</th>
<th>Method details</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>[77]</td>
<td>EMS</td>
<td>Grid-connected storage device</td>
<td>Stochastic programming</td>
<td>Minimize the average cost of electricity used and investment in storage while satisfying demand</td>
</tr>
<tr>
<td>[95]</td>
<td>EB</td>
<td>Grid-connected storage device</td>
<td>Online energy procurement algorithm based on Lyapunov optimization technique</td>
<td>Mitigate mismatches between the forecasted and actual renewable energy generation</td>
</tr>
<tr>
<td>[96]</td>
<td>EB</td>
<td>Grid-connected wind farm</td>
<td>Adaptive optimal policy with approximated objective function</td>
<td>Maximize expected daily profit by time-shifting wind energy, considering uncertainties in generation and prices</td>
</tr>
<tr>
<td>[90]</td>
<td>EMS</td>
<td>Generic grid-connected storage device</td>
<td>Stochastic dynamic programming</td>
<td>Co-optimize the use of a storage device that is put to multiple uses</td>
</tr>
<tr>
<td>[79]</td>
<td>EMS</td>
<td>EMS for a telecommunication operator</td>
<td>Markov decision process with dynamic programming</td>
<td>Minimize the energy bill while serving customers requests</td>
</tr>
<tr>
<td>[78]</td>
<td>EMS</td>
<td>Households</td>
<td>Markov decision process</td>
<td>Minimize the total cost by charging storage from purchased power and discharging it for personal consumption</td>
</tr>
<tr>
<td>[88]</td>
<td>EMS</td>
<td>Households</td>
<td>Model predictive control</td>
<td>Maximize battery lifetime, including forecasts of generation and demand</td>
</tr>
<tr>
<td>[91]</td>
<td>EMS</td>
<td>MGs</td>
<td>Simultaneous perturbation stochastic approximation</td>
<td>Limit power flow and to produce smooth storage profile</td>
</tr>
<tr>
<td>[92]</td>
<td>EB</td>
<td>Grid-connected PHES</td>
<td>Modelling forecasts of wind generation</td>
<td>Increasing system flexibility by compensating for real-time mismatches between production and consumption</td>
</tr>
<tr>
<td>[93]</td>
<td>EV</td>
<td>Smart home</td>
<td>Stochastic dynamic programming</td>
<td>Minimize electricity cost while supplying consumer demand and EV charging requirements</td>
</tr>
<tr>
<td>[98]</td>
<td>EB</td>
<td>Grid-connected storage device</td>
<td>Monte Carlo simulation</td>
<td>Determine the operational schedule of the controllable storage device in conjunction with varying renewable generation and demand</td>
</tr>
<tr>
<td>[99]</td>
<td>EB</td>
<td>MGs</td>
<td>Model predictive control</td>
<td>Optimal scheduling the ESS to a desired charging profile to optimize the power flow between the MG and the utility grid</td>
</tr>
<tr>
<td>[100]</td>
<td>EMS</td>
<td>EV</td>
<td>Stochastic dynamic programming</td>
<td>Minimizing charging costs while meeting consumer demand, minimizing the impact of charging on the grid</td>
</tr>
</tbody>
</table>
other cost. It is assumed that this function is twice differentiable and strictly convex.

To use Pontryagin’s minimum principle it is convenient to define a modified running cost function, $J$, that incorporates the constraint on $x(t)$:

$$ J = \int_0^T [F(u(t) + P_L(t)) + \phi(x(t))] \, dt, $$

(17)

where the function $\phi(x)$ is defined as

$$ \phi(x) = \begin{cases} 
\frac{\alpha}{2E_{\text{max}}} (x - E_{\text{max}})^2, & x > E_{\text{max}}, \\
0, & 0 \leq x \leq E_{\text{max}}, \\
\frac{\alpha}{2E_{\text{max}}} x^2, & x < 0 
\end{cases} $$

(18)

with $\alpha > 0$. If the constant $\alpha$ is very large then the energy is practically limited to the range $0 \leq x \leq E_{\text{max}}$, so the bounds on $x(t)$ in the original problem may be ignored. Based on Pontryagin’s minimum principle, an optimal solution must obey the following conditions:

- $\frac{d}{dt} x^* = (F')^{-1}(\lambda^*) - P_L$;
- $\frac{d}{dt} \lambda^* = \phi'(x^*)$
with the initial condition \( x^*(0) = 0 \) and the final condition \( x^*(T) = 0 \). These two differential equations can be easily solved for every \( \lambda^*(0) \), so a low-complexity method to compute the optimal solution is to scan possible values of \( \lambda^*(0) \) until the final condition \( x^*(T) = 0 \) holds. This may be done efficiently using simple line-search algorithms. A simple example demonstrating this approach is the “shortest path” optimal control method \([24, 110]\), which yields optimal peak shaving strategies in power systems and electric vehicles. The main idea is that the optimal generated energy must follow the shortest path within two bounds set by the load profile, and the device capacity.

Optimal control methods that are based on the minimum principle can be divided into two classes: methods that use storage devices as part of a grid-connected power system \([111, 116]\), and methods that target autonomous systems such as electric vehicles, hybrid vehicles, and locomotives \([109, 117–121]\). The former mostly consider batteries and super-capacitors; however, several works such as \([111]\) also consider thermal storage devices. Typical objectives are minimal fuel consumption, minimal input power \([109, 118, 121]\), or maximal profit from energy trading \([113]\). This last work demonstrates that a bang-off-bang policy may be optimal for a capacitor type storage device. The model considers natural constraints on voltage and current, and describes a usage-dependent aging rate of the storage device. In several studies the system is modeled based on the battery state of charge. In this case, the main constraints are often the state of charge upper and lower bounds, as done in \([109, 112–114, 119, 121]\). In work \([121]\), the minimum principle is applied to a problem with two state variables, and a trade-off between two objective functions is identified: fuel consumption and the battery RMS current. The utilized model implements look-up tables to simulate the electric machine losses and fuel consumption, and proposes a method to find the minimum. As for the additional cost associated with using both the battery and super-capacitor, work \([112]\) demonstrates that the dual storage system offers some advantage in fuel saving compared to single storage system. Other constraints may include limits on power losses, and in vehicles, also the torque and angular velocity, as shown in \([109, 117, 120]\). The issue of losses is also addressed in \([116]\), where the optimal control problem relates to lossy storage devices connected in parallel to a conventional fueled generator and a varying load. The optimal solution, which is based on the minimum principle, is compared to a dynamic programming algorithm. Numeric experiments reveal that for lossless storage devices dynamic programming is beneficial, however for lossy storage devices the minimum principle provides faster and more accurate solutions. Work \([120]\) studies the optimal charging profile of
an ultra-capacitor energy storage system during a regenerative braking event. The Pontryagin’s principle is applied to understand the necessary conditions the solution must satisfy in order to maximize the achievable regeneration efficiency. In addition, the combination of batteries and super-capacitors is widely explored, partly since it may enhance the overall efficiency and delay battery degradation [117, 118, 121, 122]. In work [113] this combination is used to provide multiple grid services—the batteries provide peak shaving, while the super-capacitors are employed in order to compensate for short term forecasting errors, and for damping high-frequency oscillations. In [117] the battery degradation model is used to minimize life cycle operating cost in a serial-parallel plug-in hybrid electric buses. The minimum principle is applied to a system with two state variables in order to manage the energy sources, which results in satisfactory computational time. Work [115] shows a geometric interpretation of the Pontryagin’s minimum principle by demonstrating its equivalence to the shortest path method. Based on this result a low-complexity algorithm is developed to find the optimal solution in a system comprised of a controllable generator, a load, a power conditioning hardware, and a storage device. The resulting solution is shown to be the shortest path within two bounds set by the load profile and the device capacity.

Several works propose to combine the minimum principle with other optimal control techniques to improve the overall performance. For instance, in [123] the minimum principle is combined with particle swarm optimization to minimize fuel consumption in a military electric vehicle. Simulation results show that such strategy allows to reduce the fuel consumption by 13%. Paper [124] proposes an energy management strategy for plug-in hybrid electric vehicles. In order to improve the computational efficiency, the paper proposes a stochastic predictive controller which uses the minimum principle. Several of the mentioned works are summarized in Table 4.

4. Advanced control strategies

Advanced optimal control methods may be informally divided into meta-heuristic and machine learning techniques.

4.1. Metaheuristic techniques

Metaheuristic techniques often excel when the objective function is non-linear or non-convex, and the solution space is of high dimension. In such
Table 4: Summary of studies that use Pontryagin’s minimum principle.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Category</th>
<th>Technology</th>
<th>State Variables</th>
<th>Cost Function</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>Grid-connected device</td>
<td>EMS</td>
<td>Concentrated solar power plant</td>
<td>Thermal energy</td>
<td>Revenue from generated electricity</td>
</tr>
<tr>
<td>125</td>
<td>EMS</td>
<td>Concentrated solar power plant</td>
<td>Thermal energy</td>
<td>Total revenue from generated electricity</td>
<td>Thermal storage capacity, constraints on the power flows</td>
</tr>
<tr>
<td>113</td>
<td>EMS</td>
<td>Capacitor-type ES</td>
<td>Capacitor SoC</td>
<td>Total expenses of system</td>
<td>ES SoC, discharge current</td>
</tr>
<tr>
<td>115</td>
<td>EMS</td>
<td>Grid-connected storage systems</td>
<td>Stored energy</td>
<td>Cost over the generated power</td>
<td>Storage capacity</td>
</tr>
<tr>
<td>116</td>
<td>EMS</td>
<td>Grid-connected storage system</td>
<td>Stored energy</td>
<td>Conventional generators fuel consumption</td>
<td>Storage capacity and losses</td>
</tr>
<tr>
<td>111</td>
<td>EB</td>
<td>Centralized hybrid ESS with batteries and super-capacitors</td>
<td>Energy stored in battery and super-capacitor</td>
<td>Minimum power flow between MG and main grid</td>
<td>Energy and power</td>
</tr>
<tr>
<td>112</td>
<td>EB</td>
<td>Cooperative network of MGs</td>
<td>Energy stored in ESS for each MG</td>
<td>Minimal power flow between MGs</td>
<td>max&amp;min values of energy stored in ESS</td>
</tr>
<tr>
<td>109</td>
<td>EV</td>
<td>Hybrid ESS with battery and SC</td>
<td>Voltage across super-capacitor and battery SoC</td>
<td>Fuel consumption</td>
<td>Battery SoC and current, super-capacitor voltage and short-circuit current, DC-DC output current, engine torque and speed</td>
</tr>
<tr>
<td>117</td>
<td>EV</td>
<td>Battery SoC and energy stored in SC</td>
<td>Cost of fuel, electricity, battery degradation</td>
<td>Torque and angular velocity</td>
<td></td>
</tr>
<tr>
<td>118</td>
<td>EV</td>
<td>Energy stored in super-capacitor</td>
<td>Battery input current</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>121</td>
<td>EV</td>
<td>Hybrid ESS with battery and capacitor</td>
<td>Battery input current and lifetime</td>
<td>Battery SoC and current</td>
<td></td>
</tr>
<tr>
<td>119</td>
<td>EV</td>
<td>Hybrid power locomotive</td>
<td>Energy stored in battery and capacitor, SoC of super-capacitor</td>
<td>Fuel consumption and battery current, Total energy stored, battery SoC, capacitor voltage, Fuel cell output power, SoC</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>EV</td>
<td>Ultra-capacitor ESS</td>
<td>SoC of super-capacitor, vehicle velocity and motor current</td>
<td>Charge stored in super-capacitor at the end of braking event</td>
<td>Charging current and power losses</td>
</tr>
</tbody>
</table>
applications metaheuristic techniques may operate with relatively low computational complexity, and in many cases successfully converge to an optimal solution. However, the performance of metaheuristic algorithms is often not guaranteed. Most metaheuristic algorithms have the same fundamental structure: First a number of random solutions are generated. Then, at each iteration, each of these solutions is ranked based on the objective function, and is updated based on a specific mechanism, which forms the heart of the algorithm. For instance, this mechanism can be inspired by natural selection, social behavior, the dynamics of ant colonies, and so forth.

The use of metaheuristic techniques for energy management and optimal control of storage units is demonstrated in several works from the recent literature. Works [126–130] are based on genetic algorithms, which are inspired by the process of natural selection. In work [126], a strategy for matching renewable energy generation with heating, ventilation, and air-conditioning loads using a hybrid energy storage system is suggested. A genetic algorithm is used in this work to minimize the cost of renewable energy sources alongside storage capacity. Paper [127] proposes an energy management strategy for a microgrid system. A genetic algorithm is used for optimally allocating power among several distributed energy sources, an energy storage system, and the main grid. The objective is to minimize the cost of energy and carbon dioxide emissions, while maximizing the output power of the available renewable sources. Work [128] proposes a real-time energy management strategy for energy storage systems in electric vehicles, which is based on a genetic algorithm. The proposed strategies are analyzed and compared to rule-based solutions, demonstrating improvement in overall battery utilization. Work [129] also presents an energy management strategy for a microgrid system. Here the problem is formulated as a multi-objective optimization problem, in which the cost of power generation is minimized while preserving the life-time of the storage units. A solution is achieved by means of a non-dominated sorting genetic algorithm, which is a genetic algorithm with more than one objective. Work [130] uses NSGA-II for optimal scheduling of energy storage systems in a microgrid. The solution space reflects the charging/discharging schedule of the storage devices, and the objectives are minimal operation costs and energy curtailment.

Another common algorithm for optimal control of storage units is particle swarm optimization (PSO), which is inspired by social behavior and specifically swarm behavior. For example, [131] uses PSO for optimal operation of an ice-storage air-conditioning system, considering minimal life-cycle cost as the objective. This work explores a case study based on a typical air-conditioning system in an office building. In [132] adaptive modified
PSO is suggested for optimal operation of microgrids with renewable energy sources, fuel-cells and batteries. The problem here is formulated as a non-linear multi-objective optimization problem, in which the total operating cost and emissions are minimized simultaneously. In addition, [133] uses PSO for optimal energy management of microgrids under uncertainty in the load demand and renewable power generation. A storage device is used in this work to reduce the total cost of operation.

More works using metaheuristic techniques can be found in [134–137]. Work [134] uses a firefly algorithm which is inspired by the behavior of firefly insects. The objective is to minimize the total cost, which consists of fuel cost and start-up/shut-down costs. In [135] an artificial bee colony optimization algorithm is used for optimal economic dispatch under a demand response program. Here the objective is to minimize the total production cost, while operating the generation and storage resources within their limits. Paper [136] presents an algorithm for energy management system based on ant colony optimization, which goal is to find an optimal operation strategy for decreasing the electricity cost under hourly day-ahead and real-time scheduling. In addition, [137] proposes an energy management algorithm for a multi-microgrid systems that incorporates distributed generators, energy storage units, electric vehicles, and a demand response mechanism. Recently, work [138] developed a control strategy which is used for synchronous microgrid operation, based on an evolutionary algorithm. The analysis focused specifically on low-voltage microgrid configurations, showing that in the considered scenarios the optimal solution is determined by the energy storage capacity.

The mentioned works are summarized in Table 5. The Table clearly shows that metaheuristic techniques are mainly used for energy management applications and are rarely used for energy trading and electric vehicle applications.

4.2. Machine learning techniques

Machine learning techniques may be classified into three broad categories: supervised, unsupervised, and reinforcement learning. In supervised learning the mathematical model is designed to best predict the relations between given inputs $x = [x_1, x_2, \ldots, x_n]$ and outputs $y = [y_1, y_2, \ldots, y_n]$. The main goal is to estimate a function $f(x)$, which constitutes a mapping from $x$ to $y$, such that the expected value of a specific loss function $L(f(x), y)$ is minimized. In unsupervised learning methods the output data is unavailable, and the objective is to find patterns in the input data.
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Category</th>
<th>Technique</th>
<th>Application</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>126</td>
<td>EMS</td>
<td>Genetic algorithm</td>
<td>Energy management for ventilation and air conditioning loads</td>
<td>Minimize cost of renewable energy sources installation and storage capacity</td>
</tr>
<tr>
<td>127</td>
<td>EMS</td>
<td>Genetic algorithm</td>
<td>Energy management for MGs</td>
<td>Minimize energy cost and emissions</td>
</tr>
<tr>
<td>128</td>
<td>EV</td>
<td>Non-dominated sorting genetic algorithm</td>
<td>Optimal energy storage system in EV</td>
<td>Minimize energy consumption in a driving cycle</td>
</tr>
<tr>
<td>129</td>
<td>EMS</td>
<td>Non-dominated sorting genetic algorithm</td>
<td>Energy management for MGs</td>
<td>Minimize power generation cost and maximize useful life of storage units</td>
</tr>
<tr>
<td>130</td>
<td>EB</td>
<td>Non-dominated sorting genetic algorithm</td>
<td>Optimal scheduling of storage units in MGs</td>
<td>Minimize operation cost and energy curtailment</td>
</tr>
<tr>
<td>132</td>
<td>EMS</td>
<td>Particle swarm optimization</td>
<td>Optimization for ice-storage air-conditioning system</td>
<td>Minimal life cycle cost</td>
</tr>
<tr>
<td>133</td>
<td>EMS</td>
<td>Particle swarm optimization</td>
<td>Energy management under uncertainty</td>
<td>Minimize operating cost</td>
</tr>
<tr>
<td>134</td>
<td>EMS</td>
<td>Multi-objective particle swarm optimization</td>
<td>Multi-operation management in MG</td>
<td>Minimize operating cost and emissions</td>
</tr>
<tr>
<td>135</td>
<td>EMS</td>
<td>Firefly algorithm</td>
<td>Energy management under uncertainty and storage limitations</td>
<td>Minimize the total cost</td>
</tr>
<tr>
<td>136</td>
<td>EB</td>
<td>Artificial bee colony optimization</td>
<td>Economic dispatch under demand response program</td>
<td>Minimize the total production cost</td>
</tr>
<tr>
<td>137</td>
<td>EMS</td>
<td>Ant colony optimization</td>
<td>Energy management for MGs</td>
<td>Minimize the total electricity production cost</td>
</tr>
<tr>
<td>138</td>
<td>EMS</td>
<td>Tabu search</td>
<td>Energy management in multi-MG systems</td>
<td>Minimize the total electricity operational cost</td>
</tr>
<tr>
<td>139</td>
<td>EMS</td>
<td>Evolutionary algorithm</td>
<td>Energy management in low voltage for synchronous MG operation</td>
<td>Minimize the active power and energy losses</td>
</tr>
</tbody>
</table>

Table 5: Metaheuristic techniques.
The third category, reinforcement learning, is most commonly used for optimal control of storage systems. Reinforcement learning methods attempt to predict how software agents should operate in a dynamic environment in order to maximize a cumulative reward \cite{139}. In energy storage control problems the controller is considered as the agent, and the energy storage system is represented as the environment. At each step of the interaction the controller receives an input that indicates the current state of the storage system. The controller then chooses an action, which affects the next state of the storage system, and the value of this new state is communicated to the controller through a scalar signal. The reinforcement learning algorithm chooses these actions based on trial-and-error, in order to increase the value of the reward, which may be proportional to the peak of generated power, to the system predictability or reliability, and so forth. In such models, the environment is typically described as a Markov decision process.

Following are several works showing how reinforcement learning techniques may be used to control storage systems. Work \cite{140} uses an environment that includes a microgrid with a consumer, wind turbine, storage, and a connection to the external grid. The aim of this work is to schedule the charging times of a battery in order to maximize the consumer objectives, which may be to decrease the cost of electricity, or to increase the battery lifetime. The environment is described in this work by two Markov chain models, which describe the power output of the renewable sources. In \cite{141} a three-level hierarchical optimization under dynamic demand response is suggested for micro-grid management. The first level uses distributed demand response agents in each house to reduce the cost of electricity. The second level uses a centralized agent, which obtains and processes load demand data from all the houses. Finally, the third level optimizes the battery charging and discharging times using a reinforcement learning algorithm. In addition, \cite{142} studies the energy trading patterns of prosumers that operate wind turbines and energy storage systems. The objective of each individual prosumer is to maximize his own profit, and the trading process is modeled as Markov decision process. A deep reinforcement learning method is suggested for optimizing the prosumers’ decisions without complete information on the market model. Paper \cite{143} suggests an energy management strategy for a super-capacitor energy storage system in an urban rail transit, which is based on deep reinforcement learning. The management system is modeled as an agent that iteratively improves its behavior, and finally converges to a nearly-optimal policy. The considered simulation is based on the Beijing subway system, and shows improvement in energy saving and voltage stability when compared to a fixed-threshold strategy.
During the last few years reinforcement learning techniques are also increasingly used for energy management in electric vehicles [144–147]. In [144] an adaptive energy management method for hybrid electric tracked vehicles is proposed. The state variables in this work are the battery state of charge and the generator’s frequency. At each time step the algorithm decides how to distribute power between the battery and engine-generator, in order to minimize the fuel consumption for a given driving pattern. This method is shown to improve both the fuel economy and computation time when compared to a stochastic dynamic programming method. Paper [145] suggests an energy management algorithm for a hybrid electric vehicle with a parallel system design. The algorithm uses velocity predictions to form a Markov chain model. Then, reinforcement learning is used to determine the optimal control and optimal power distribution between the two energy sources. Work [146] also uses reinforcement learning for energy management in an electric vehicle. One of the goals considered in this work is to reduce the numerical complexity of the controller, in order to achieve real-time computations. This cannot be achieved by traditional optimization methods, so a reinforcement learning algorithm is used. The state variables are the power, voltage, and the battery state of charge. In addition, the current of the battery is chosen as the action variable, and the reward function is the total energy loss. In [147] an energy management strategy for hybrid electric vehicles is developed based on deep reinforcement learning. The suggested method selects the battery actions under different driving conditions based on an online learning architecture. Simulation results verify the effectiveness of the suggested approach when compared to a rule-based system, specifically showing an improved fuel economy.

Additional works that use machine learning techniques can be found in [148–150]. Work [148] proposes a multi-objective energy management system, which goal is to minimize the operation costs and emissions considering forecasts of the renewable energy output and the load. These forecasts are achieved by an artificial neural network, and the battery scheduling process is modeled as a fuzzy logic expert system. Paper [149] uses a recurrent neural network for short-term generation scheduling in a microgrid that contains batteries, electric vehicles, and renewable sources. The proposed method estimates the optimal amount of generated power over a time horizon of one week. Another example of efficient energy management in a storage system is shown in [150], which predicts the load using a support vector machine. These and other related works are summarized in Table 6.
Table 6: Machine learning techniques.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Category</th>
<th>Technique</th>
<th>Application</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>EB</td>
<td>Reinforcement learning</td>
<td>Battery scheduling plan</td>
<td>Minimize cost of electricity purchase and maximize the consumer independence from the grid</td>
</tr>
<tr>
<td>131</td>
<td>EMS</td>
<td>Reinforcement learning</td>
<td>Smart MG management under dynamic demand response</td>
<td>Minimize energy consumption cost</td>
</tr>
<tr>
<td>134</td>
<td>EV</td>
<td>Reinforcement learning</td>
<td>Adaptive energy management for hybrid electric tracked vehicle</td>
<td>Minimize fuel consumption over different driving schedules</td>
</tr>
<tr>
<td>135</td>
<td>EV</td>
<td>Energy management for parallel hybrid EV</td>
<td>Energy management for parallel hybrid EV</td>
<td>Minimize fuel economy and maintaining battery charge sustenance</td>
</tr>
<tr>
<td>136</td>
<td>EV</td>
<td>Energy management for parallel hybrid EV</td>
<td>Real-time energy management method for ESS in EV</td>
<td>Minimize the electricity consumption and save battery health</td>
</tr>
<tr>
<td>142</td>
<td>ET</td>
<td>Multi-agent learning system</td>
<td>Trading behavior in local energy market</td>
<td>Maximize benefit cost for each prosumer</td>
</tr>
<tr>
<td>143</td>
<td>EV</td>
<td>Energy management strategy for hybrid EV based on online learning architecture</td>
<td>Energy management strategy for hybrid EV based on online learning architecture</td>
<td>Maximize vehicle fuel economy</td>
</tr>
<tr>
<td>145</td>
<td>EMS</td>
<td>Deep reinforcement learning</td>
<td>Supercapacitor energy storage systems in urban rail transit</td>
<td>Maximize energy-saving and minimize voltage-stabilizing effects</td>
</tr>
<tr>
<td>146</td>
<td>EB</td>
<td>Operation of distributed ESSs in MG considering uncertainties</td>
<td>Operation of distributed ESSs in MG considering uncertainties</td>
<td>Minimize the amount of load shedding in the MG system and the total cost</td>
</tr>
<tr>
<td>147</td>
<td>ET</td>
<td>Multi-agent learning system</td>
<td>Micro storage management for energy market</td>
<td>Maximize benefit cost for all storage units</td>
</tr>
<tr>
<td>148</td>
<td>EMS</td>
<td>Fuzzy logic</td>
<td>Design of a fuzzy logic controller for a MG with PV plant and ESS</td>
<td>Minimize power output error</td>
</tr>
<tr>
<td>149</td>
<td>EMS</td>
<td>Artificial neural network</td>
<td>Optimal control strategy of district cooling system with ice storage</td>
<td>Minimize operating costs</td>
</tr>
<tr>
<td>150</td>
<td>EB</td>
<td>Fuzzy logic and artificial neural network</td>
<td>Multi-objective energy management using battery scheduling process for MG</td>
<td>Minimize operation cost and emissions</td>
</tr>
<tr>
<td>151</td>
<td>EB</td>
<td>Recurrent neural network</td>
<td>Short-term generation scheduling problem in MG</td>
<td>Maximize power supplied by renewable energy sources and minimize energy cost of the utility grid</td>
</tr>
<tr>
<td>152</td>
<td>EB</td>
<td>Support vector machine</td>
<td>EMS for SC-battery hybrid ESS</td>
<td>Minimize system cost and reduce the number of batteries</td>
</tr>
</tbody>
</table>
5. Recent directions and trends

One goal of this paper is to highlight correlations between certain control methods and certain applications, as they are reflected in the recent literature. Such trends are explored in this section based on content analysis of a few hundred recent papers. Based on this analysis we explain the motivation for the choice of several optimal control methods, and try to better understand why and when the reviewed methods are the most efficient.

The content analysis was based on the Scopus and IEEE Xplore databases. Analyzing the search results and several books on optimal control [155–160], it was identified that the mostly used methods for solving energy storage problems are linear programming, Pontryagin’s minimum principle, dynamic programming, and stochastic optimization [161], and the most relevant application areas are energy trading, electric vehicles, and energy balancing. These application areas are timely and can be linked to the rising concept of smart grids. In addition, there exists a vast literature on machine learning and heuristic optimal control techniques such as neural networks, fuzzy logic, genetic algorithms, etc., sometimes combined with the above mentioned methods. These are briefly outlined in Section 4, and also in a recent review papers [25, 28, 30, 33, 34, 37].

Special attention has been given to the analysis of relevant keywords describing each method and application area. The linear programming and Pontryagin’s minimum/maximum principle methods are both well-defined by their terms. However, we have noticed that dynamic programming and stochastic optimization often go in conjunction. Therefore, to make the search independent we used cross-limitations by excluding dynamic programming from the query of stochastic control and vice versa. For this reason, the search results only reflect works that focus on a specific method, while the tables in the previous sections may present several works that use combined methods. The keywords we used for the application areas are summarized in Table 7.

The search results reveal interesting trends in the current research. Figure 3 shows the results for each method, limited to a 10-year-period, and independent from the application area. The figure shows that stochastic optimization methods are widely used, probably since many energy storage applications include uncertainties. Note that stochastic optimization is usually used in combination with dynamic programming techniques, as explained in Section 3.3. Such a mixture has several known benefits including the ability to handle non-convex objective functions and nonlinear constraints. In addition, linear programming is also a popular technique, and is a natural
Table 7: Application areas: Search keywords.

<table>
<thead>
<tr>
<th>Energy trading</th>
<th>Electric vehicles</th>
<th>Energy balancing</th>
</tr>
</thead>
<tbody>
<tr>
<td>“energy trading” OR</td>
<td>“electric vehicle*” OR</td>
<td>“energy balanc*” OR</td>
</tr>
<tr>
<td>“pric*” OR “market*”</td>
<td>“HEV” OR “EV” OR</td>
<td>“peak shav*” OR</td>
</tr>
<tr>
<td>OR “bid*” OR “arbitrage”</td>
<td>“PEV” OR “PHEV” OR</td>
<td>“scheduling” OR “load</td>
</tr>
<tr>
<td></td>
<td>OR “V2G” OR “transport*”</td>
<td>“leveling” OR “load</td>
</tr>
<tr>
<td></td>
<td>OR “bus” OR</td>
<td>“load following” OR</td>
</tr>
<tr>
<td></td>
<td>“car” OR “airplane”</td>
<td></td>
</tr>
</tbody>
</table>

(a) The asterisk sign * denotes a wildcard, which can be interpreted as a number of literal characters or an empty string. For example, both price or pricing will fall into the same search pattern.
(b) The abbreviations are: EV stands for electric vehicle, H–hybrid, P–plug-in, V2G–vehicle-to-grid.

choice when the objective and constraints are linear. The popularity of linear programming is also linked to the limitations of dynamic programming techniques resulting from the “curse of dimensionality”, as discussed in Section 3.2. Thus, when dynamic programming is impractical due to numerical constraints, linear programming based methods may be a suitable alternative. The results also show that Pontryagin’s minimum principle is the least popular method. Unlike other methods, the minimum principle often only states necessary conditions, which may (or may not) lead to an optimal solution. However, while this method is not straightforward to use due to its complex mathematical formulation, our opinion is that it may yield simple and highly accurate solutions with relatively low complexity.

Figure 4 shows the search results for the selected application areas. Here we combine search strings for all methods with a particular column from Table 7. The figure shows that the number of publications related to electric vehicles and energy balancing are almost the same. In addition, energy trading started with a low number of publications in 2010, and is now at the focus of the research community. However, in our opinion this may change in the near future due to various global initiatives targeting the development of low-carbon and energy efficient transportation systems [162–165], which may shift the research focus more strongly toward electric vehicles. In addition, due to more active involvement of the end-consumer and advancements in beyond-the-meter technologies [166], it is possible that grid balancing by energy storage devices may become a major focus area.

Figure 5(a) depicts the search results for specific methods and application areas. To visualize the data we used the so-called Sankey diagram, which
shows how much each method contributes to every application area, and how much it is used. Here, we combined keywords from Table 7 with a particular method for the period 2000–2019. The longer range is chosen here to better reflect historical relations between the selected methods and the application areas. The ratio is further summarized in Table 8. It can be seen that while stochastic methods are the most popular in general (see Fig. 3), for energy balancing applications both stochastic and linear programming methods are almost equally used. At the same time, dynamic programming is used evenly in all applications, and Pontryagin’s minimum principle is mostly used in electric vehicles. The percentages in Table 8 indicate that stochastic methods closely follow the EB-ET-EV popularity pattern of dynamic programming, which again emphasizes that these methods may be combined effectively. However, this is not true for linear programming and Pontryagin’s minimum principle, since these are based on entirely different principles. Thus, while linear programming is mostly used for energy trading, the minimum principle is almost solely used in electric vehicle applications. This trend becomes even more prominent considering the keywords and index terms used in papers. For example, papers that discuss electric vehicles prefer to use the term “battery” instead of the more general term “energy storage”. If “battery” is used as an alternative to “energy storage” in the search string, then the usage of the minimum principle increases from a total of 47 to a total of 252 in the bar chart in Fig. 3 and from 29 to 188 in Fig. 5(a). This observation is visualized in Fig. 4 for all the methods used in electric vehicle applications. The other areas are not that strongly affected by specific keywords.

Similar to the above we now present relations between optimal control methods and selected storage technologies, see Fig. 5(b) and Table 9 for the period 2010–2019. We selected these shorter time-scales since in earlier years the research was mostly devoted to battery technologies, so other solutions were under-represented. Again we see that stochastic strategies dominate, and that Pontryagin’s minimum principle is a niche method. As for technologies, batteries serve as the most common energy storage solution. This can be explained by their relative simplicity and wide availability compared to other technologies. Another technology that is almost as popular is pumped-hydro, which has been almost equally analyzed by all control methods. Also observe that the minimum principle is mostly used in conjunction with battery technologies (≈86.4%), due to their active employment in hybrid electric vehicles as a secondary storage device.
Figure 3: Yearly number of publications in the period from 2010 to 2019 for various optimal control techniques.

Table 8: Relations [in %] between optimal control methods and application areas, as shown Fig. 5(a).

<table>
<thead>
<tr>
<th>From LP</th>
<th>From DP</th>
<th>From SC</th>
<th>From PMP</th>
<th>↓To EB</th>
<th>↓To ET</th>
<th>↓To EV</th>
<th>↓From LP</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.7</td>
<td>26.1</td>
<td>35.2</td>
<td>40.9</td>
<td>15.8</td>
<td>76.3</td>
<td>29.5</td>
<td>49.7</td>
</tr>
<tr>
<td>38.7</td>
<td>32.8</td>
<td>28.4</td>
<td>40.9</td>
<td>15.8</td>
<td>76.3</td>
<td>29.5</td>
<td>49.7</td>
</tr>
<tr>
<td>40.9</td>
<td>29.5</td>
<td>29.6</td>
<td>40.9</td>
<td>15.8</td>
<td>76.3</td>
<td>29.5</td>
<td>49.7</td>
</tr>
<tr>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
</tr>
<tr>
<td>45.7</td>
<td>10.4</td>
<td>43.7</td>
<td>40.1</td>
<td>13.1</td>
<td>47.6</td>
<td>37.1</td>
<td>13.1</td>
</tr>
<tr>
<td>547</td>
<td>286</td>
<td>251</td>
<td>191</td>
<td>166</td>
<td>113</td>
<td>70</td>
<td>30</td>
</tr>
</tbody>
</table>

Stochastic control
Linear programming
Dynamic programming
Pontryagin’s minimum principle

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Figure 4: Analysis of documents for various application areas, limited to the period from 2010 to 2019.

Figure 5: Visualization of two types of relations between the selected optimal control methods, application areas, and storage technologies. The abbreviations are: Ba–battery, PH–pumped hydro, CA–compressed air, and Fl–flywheel.
Table 9: Relations [in %] between optimal control methods and storage technologies, as shown Fig. 5(b).

<table>
<thead>
<tr>
<th>From</th>
<th>LP</th>
<th>DP</th>
<th>SC</th>
<th>PMP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>71.3</td>
<td>81.1</td>
<td>67.8</td>
<td>86.4</td>
</tr>
<tr>
<td></td>
<td>2.1</td>
<td>6.3</td>
<td>6.7</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>5.8</td>
<td>2.8</td>
<td>6.2</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>20.8</td>
<td>9.8</td>
<td>19.3</td>
<td>13.6</td>
</tr>
</tbody>
</table>

Table 10: Optimal control methods: advantages, challenges and application areas.

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Challenges</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>- Low numeric complexity</td>
<td>- Objective function and constraints must be linear</td>
<td>ET, EV, EB</td>
</tr>
<tr>
<td></td>
<td>- Easy to implement</td>
<td>- Prior knowledge is needed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Converges to global optimum</td>
<td>- “Curse of dimensionality”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Easy to implement</td>
<td>- Cost function must be modeled as a recursive formula</td>
<td></td>
</tr>
<tr>
<td>DP</td>
<td>- Solve problems involving uncertainties in real time</td>
<td>- Signals must be modeled as stochastic processes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Converges to global optimum</td>
<td>- “Curse of dimensionality”</td>
<td>EV, EB</td>
</tr>
<tr>
<td>SC</td>
<td>- Low numeric complexity</td>
<td>- Does not provide a single well-defined solution.</td>
<td>EV</td>
</tr>
<tr>
<td></td>
<td>- Simple solutions</td>
<td>- Convergence to optimal solution is not guaranteed</td>
<td>EV, EB</td>
</tr>
<tr>
<td>PMP</td>
<td>- Low numeric complexity</td>
<td>- Available sample data is needed</td>
<td>ET, EV, EB</td>
</tr>
<tr>
<td>Meta-heuristic</td>
<td>Low numeric complexity</td>
<td>- Time and resources</td>
<td></td>
</tr>
</tbody>
</table>
6. Conclusion

This paper reviews recent works related to optimal control of storage systems, with an attempt to better understand the unique characteristics, common uses, and mathematical foundations of the most popular optimization methods. The optimization strategies we chose to focus on have been selected following an in-depth content analysis of various sources from the main databases, as detailed in Section 5. This analysis reveals interesting trends in the current research, and may help to understand under which conditions the reviewed optimization methods are the most efficient, and why certain methods are mostly used in the context of specific applications (see Table 10). For instance, the data shows that stochastic optimization methods are widely used in combination with dynamic programming techniques, probably since this enables analysis of non-convex objective functions and nonlinear constraints. However, dynamic programming and its stochastic variants are not suitable for every application due to the “curse of dimensionality”, as discussed in Section 3.2. In such applications linear programming seems to be a natural choice, given that the objective function and constraints are linear. Moreover, in case of nonlinear objective or constrains, metaheuristic methods are used extensively, although these mainly apply to energy balancing applications. The Pontryagin’s minimum principle seems to be the least popular method, maybe due to its complex mathematical formulation, and since it only states necessary conditions which do not immediately lead to an optimal solution. This method is currently mostly used in electric vehicle applications, probably since it often yields simple solutions with relatively low complexity, which can be implemented in real-time. Machine learning techniques are also becoming an important trend due to increasing amounts of available data, and specifically reinforcement learning methods are commonly used for energy management in electric vehicles. The analysis also shows that the number of publications related to electric vehicles and energy balancing are currently almost the same, and that energy trading applications gained considerable momentum in the last several years. In our opinion these trends may change in the near future due to global initiatives related to electric vehicles, and due to the continuing integration of renewable sources and beyond-the-meter technologies, that may lead to renewed interest in grid balancing and management applications.

Concerning future research, since modern electric grids are increasingly decentralized and less regulated by governments, it is often impractical to study them from the perspective of one single entity with unlimited information and control span. This development may lead to a variety of future
research avenues regarding optimal control of storage systems, since game-
theory based descriptions may become more suitable than classic optimiza-
tion techniques. In addition, considering the incomplete and time-varying
information structure of future grid data, data-driven and online optimization
paradigms may become significantly more useful. Likewise, if we con-
sider scenarios where more information is available, online-learning may be a
possible research direction. Another topic of interest may be energy storage
management problems with many objectives, and solution techniques which
include many-objective evolutionary algorithms. Furthermore, since storage
systems are sparsely placed in a modern power grid, classical optimal control
methods may be hard to implement in several scenarios. To overcome this
challenge, sparse optimal control methods can be considered as a possible
venue for future research.

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