Model based control of a water tank system

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Overview of the talk

- Preliminaries: theoretical background
- Problem statement
- Problem solution / Practical results
- Comparison

NARX vs. ANARX

Nonlinear AutoRegressive eXogenous model:

$$y^{[n]} = \varphi\left(y, y^{[1]}, \ldots, y^{[n-1]}, u, u^{[1]}, \ldots, u^{[n-1]}\right)$$

Additive NARX model:

$$y^{[n]} = f_1\left(y^{[n-1]}, u^{[n-1]}\right) + \cdots + f_n(y, u).$$

Criteria	**NARX**	**ANARX**
Accuracy	high	acceptable
IO to state-space	sometimes	always
Linearizability	sometimes	always

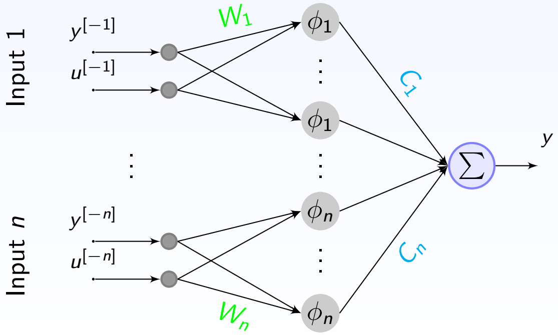
In the above equations $u : \mathbb{Z} \to \mathcal{U} \subset \mathbb{R}$ is the input signal and $y : \mathbb{Z} \to \mathcal{Y} \subset \mathbb{R}$ is the output signal.

To simplify expressions we use abridged notation. In particular, for $k \in \mathbb{N}$ ($k \in \mathbb{Z}^-$) the notation $\xi^{[k]} := \xi(t+k)$ stands for kth-step forward (backward) time shift of $\xi : \mathbb{Z} \to \mathbb{R}$.

ANARX and Neural Networks

$$y = \sum_{i=1}^{n} C_i \phi_i \left(W_i \begin{bmatrix} y^{[-i]} & u^{[-i]} \end{bmatrix}^{\mathrm{T}} \right),$$

i indicates the number of sublayer, ϕ_i is an activation function, C_i and W_i are matrices of the output and input synaptic weights, l_i is the number of hidden neurons.



Close-loop, reference model and controller

Reference (etalon) model:

$$y + a_1 y^{[-1]} + \cdots + a_n y^{[-n]} = b_1 v^{[-1]} + \cdots + b_n v^{[-n]},$$

where $a_1, \ldots, a_n \in \mathbb{R}$ and $b_1, \ldots, b_n \in \mathbb{R}$.

Controller:

$$\boxed{\eta_1 = a_1 y - b_1 v + C_1 \phi_1 \left(W_1 \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}} \right)}$$

$$\begin{aligned}
\eta_1^{[1]} &= \eta_2 + b_2 v - a_2 y - C_2 \phi_2 \left(W_2 \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}} \right) \\
&\vdots \\
\eta_{n-2}^{[1]} &= \eta_{n-1} + b_{n-1} v - a_{n-1} y - C_{n-1} \phi_{n-1} \left(W_{n-1} \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}} \right) \\
\eta_{n-1}^{[1]} &= b_n v - a_n y - C_n \phi_n \left(W_n \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}} \right).
\end{aligned}$$

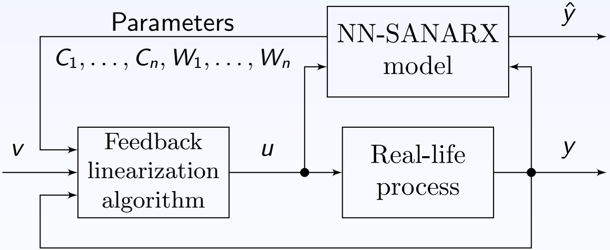
SANARX

Simplified ANARX structure can be obtained by assuming that ϕ_1 is a linear function. The latter yields

$$u = T_2^{-1}(\eta_1 - (T_1 + a_1)y + b_1 v),$$

where T_1 and T_2 are the first and second elements of the vector $C_1 W_1$, respectively. Note that $T_2 \neq 0$.

Control system: general scheme



Problem description

From the scheme above it follows:

$$\hat{y} + a_1 y^{[-1]} + \cdots + a_n y^{[-n]} = b_1 v^{[-1]} + \cdots + b_n v^{[-n]}$$

and

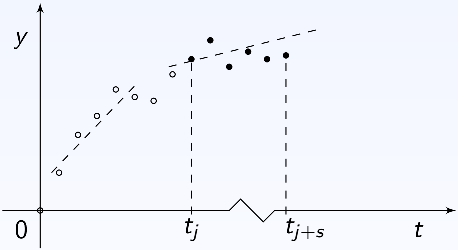
$$y + a_1 y^{[-1]} + \cdots + a_n y^{[-n]} = b_1 v^{[-1]} + \cdots + b_n v^{[-n]} + \varepsilon.$$

$\rightarrow y = \hat{y} + \varepsilon$, where ε is an error caused by imperfectness of an NN-based model describing the process.
$\rightarrow y = \hat{y} + e_{ss}$, where e_{ss} is a steady-state error after the transient process is complete.

Goal: $e_{ss} \rightarrow 0$ as $t \rightarrow \infty$.

Problem solution

Detection of a steady state:



Calibration:

Steady-state is detected $\Rightarrow e_{ss} := v - y$.

Use this value in the algorithm to *calibrate* the input signal by adding e_{ss} to the last equation in the controller.

Alpha Control Laboratory

Brief overview

- Department of Computer Control, Tallinn University of Technology
- Established in the middle of 2013
- `http://a-lab.ee/`
- Education and Research
- Research focus: computational/artificial intelligence based methods, fractional calculus

Multi-Tank System

Equations of the pump-controlled version:

$$\dot{x}_1 = \frac{1}{aw}(u - C_1 x_1^{\alpha_1})$$

$$\dot{x}_2 = \frac{h}{cwh + bwx_2}(C_1 x_1^{\alpha_1} - C_2 x_2^{\alpha_2})$$

$$\dot{x}_3 = \frac{1}{w\sqrt{R^2 - (R - x_3)^2}}(C_2 x_2^{\alpha_2} - C_3 x_3^{\alpha_3}).$$

Identification and controller design

- Data was collected from the real plant with sampling time $0.5\mathrm{s}$.
- The input signal was normalized as $u \in [0, 1]$.
- The NN-SANARX structure was used with two sublayers and 3 neurons on each sublayer. The linear activation function was chosen on the first and output sublayers as well as hyperbolic tangent sigmoid activation function (tansig) on the second sublayer.

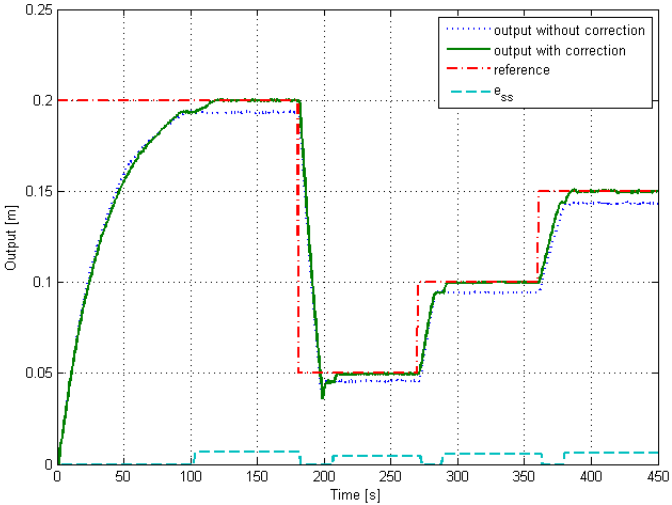
Identified model:

$$\hat{y} = T_1 y^{[-1]} + T_2 u^{[-1]} + C_2 \mathrm{tansig}\left(W_2 \begin{bmatrix} y^{[-2]} & u^{[-2]} \end{bmatrix}^{\mathrm{T}} \right).$$

Controller:

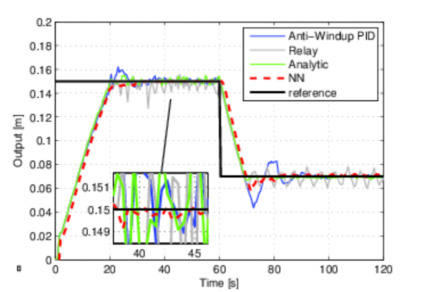
$$\begin{aligned} u &= T_2^{-1}(\eta_1 - T_1 y) \\ \eta_1^{[1]} &= v - C_2 \mathrm{tansig}\left(W_2 \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}} \right). \end{aligned}$$

Simulation results: quality of the control algorithm

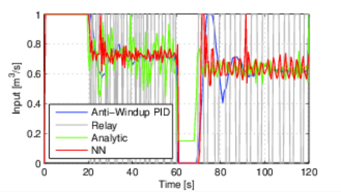


Comparison

Comparison results: outputs



Comparison results: control signals



Comparison: cont.

Statistical measure of performance in steady-state:

Method	AW PID	Relay	Analytic	NN
MSE	$3.15 \cdot 10^{-5}$	$2.47 \cdot 10^{-5}$	$\mathbf{2.6 \cdot 10^{-6}}$	$4.73 \cdot 10^{-6}$
SSE	0.2681	0.2103	**0.0221**	0.0402
$\sum \lvert v - y \rvert$	26.5788	34.2955	**10.3010**	11.0672

The evaluation of each method is summarized:

Criteria	AW PID	Relay	Analytic	NN
complexity	medium	low	high	medium
versatility	medium	high	low	high
robustness	high	medium	medium	high
model	–	–	required	–
quality of u	medium	low	medium	medium

Thank you very much for your attention!
Any questions?