Using DQ0 Signals based on the Central Angle Reference Frame to Model the Dynamics of Large-scale Power Systems

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Abstract—With increasing penetration of distributed and renewable sources into power grids, the dynamic behavior of large-scale power systems is becoming increasingly complex. These recent developments have led to several models attempting to simplify the analysis of dynamic phenomena, among them are models based on the dq0 transformation. A question that often arises when modeling interconnected systems based on dq0 quantities is how to choose the reference frame. One approach is to model the network and its components using a dq0 transformation that is based on a unified reference frame. However, when no generator is large enough to be considered an infinite bus, the unified reference frame may lead to unstable models, since the real frequency may deviate from the nominal frequency used as a reference. In this paper we propose to solve this problem by using the central angle as a basis for the dq0 transformation. Such an approach leads to models which are valid at high frequencies, and can also be used with systems with varying frequencies and no infinite bus. The proposed approach is demonstrated using networks with 9 and 57 buses.

Index Terms—power systems, dq0 transformation, stability, dynamics, time-varying phasors

I. INTRODUCTION

For many years the dynamics of power systems were dictated by large synchronous machines with high inertia and slow dynamic responses. Dynamic processes in such systems can be analyzed based on time-varying phasor models, in which a key assumption is that phasors are changing slowly in comparison to the system frequency, and the system is quasi-static [1]–[5]. However, in recent years, the increasing penetration of small distributed generators and power electronics based devices creates new challenges, one of them is that the system may no longer be quasi-static. A natural extension in this case is to use more general dq0-based models [6]–[12]. Such models combine two properties of interest: similarly to time-varying phasor models, dq0-based models map AC signals to quasi-constant signals, so the resulting model is often time-invariant, with a well-defined equilibrium point. In addition, dq0 models are inherently transient models, so they do not rely on the assumption of a quasi-static system, and remain accurate at high frequencies [1].

A question that often arises when modeling interconnected systems based on dq0 quantities is how to choose the reference frame, or in other words, how to link different machines that rotate at different frequencies. One approach is to model the network and its components using a dq0 transformation that is based on a unified (global) reference frame, rotating with a fixed frequency $\omega_s$, where this frequency is the frequency of a large machine or the frequency of another large network acting as an infinite bus [9], [13]. However, when no generator is large enough to be considered an infinite bus, the unified reference frame may lead to unstable models, since the real frequency may deviate from the nominal frequency used as a reference, and as a result the generator power angles may grow to infinity.

In classic time-varying phasor models this problem is solved by considering a reference frame that rotates with the system central angle, which is defined as the weighted average of the generator rotor angles [14]. The main idea motivating such a reference frame is that in steady state it rotates at the average system frequency. As a result, angles that are defined with respect to the central angle reference frame are constant at steady state, leading to a stable model. Another advantage of the central angle is that it enables a simple description of global system properties, similar to the way a central mass reference frame describes the motion of complex bodies in space. Specifically, in multi-machine systems operating under certain conditions, while the dynamics of each generator may be quite complex, the dynamics of the central frequency are simple, and may be predicted using a first-order differential equation.

In this paper we extend the idea of using the central angle reference frame, and propose to use it not with time-varying phasors, but as a basis for the dq0 transformation, where the objective is to model the fast dynamics of low inertia systems with high penetration level of distributed sources. Such an approach potentially combines several advantages, since the resulting models are valid at high frequencies, and have a well-
defined equilibrium point (due to the use of dq0 quantities), and are also applicable for systems with varying frequencies and no infinite bus (due to the use of the central angle). The proposed approach is demonstrated using several test-cases based on standard networks with 9 and 57 buses.

II. THE CENTRAL ANGLE REFERENCE FRAME: REVIEW OF DEFINITION AND MAIN RESULTS

In this section we review the definition and key results of the central angle reference frame when applied to classic time-varying phasor models (quasi-static models). The central angle reference frame is useful when the system does not contain an infinite bus that defines the system frequency. In this case the steady-state frequency is slightly different than the nominal frequency \( \omega_n \), and the generator power angles \( \delta_i \) diverge over time. In other words, when a system with no infinite bus is described based on a reference frame rotating with \( \omega_n t \), the resulting model is typically unstable. The central angle reference frame solves this problem, since it rotates at a frequency equal to the average system frequency, such that the power angles \( \delta_i \) are bounded, and the resulting model is stable. The central angle is defined as:

\[
\theta_c(t) = \frac{1}{J_{\text{tot}}} \sum_{i=1}^{n} J_i \theta_i(t),
\]

(1)

where \( \theta_i \) is the angle of the \( i \)th generator, \( J_i \) is the moment of inertia, or virtual inertia of the generator, and \( J_{\text{tot}} = \sum_{i=1}^{n} J_i \).

Consider now a reference frame that rotates with the central angle \( \theta_c(t) \). The generator power angles are redefined with respect to this reference frame as:

\[
\delta_i(t) = \theta_i(t) - \theta_c(t) + \frac{\pi}{2},
\]

(2)

in which the constant \( \pi/2 \) is included to be consistent with the definition presented in Section III. We also define the following frequencies:

- rotor frequency: \( \omega_i(t) = d\theta_i/dt \);
- central frequency:

\[
\omega_c(t) = \frac{d}{dt} \theta_c = \frac{1}{J_{\text{tot}}} \sum_{i=1}^{n} J_i \omega_i(t).
\]

(3)

At steady-state all generators rotate at the same frequency, and therefore \( \omega_1 = \omega_2 = \cdots = \omega_n = \omega_c \), meaning that the generators rotate at a frequency that is equal to the central frequency. In addition,

\[
\frac{d}{dt} \delta_i = \frac{d}{dt} \theta_i - \frac{d}{dt} \theta_c = \omega_i - \omega_c = 0,
\]

(4)

meaning that at steady-state the power angles \( \delta_i \) are constant.

A well-known result that is closely related to the central angle reference frame is the aggregated swing equation [15], which under certain assumptions describes the joint dynamics of complex systems using a first-order differential equation [14]. Consider the power system represented in Fig. 1. It is assumed that the network, generators, and loads are modeled based on time-varying phasors. Furthermore, the transmission network is balanced three-phase, lossless, and can deliver unlimited power. The generators are modeled as simplified synchronous machines with 2 poles, based on the swing equation, including a standard droop control mechanism. Under these assumptions, the generators are modeled by:

\[
\frac{d}{dt} \omega_i = \frac{1}{J_i \omega_s} \left(-3P_i + 3P_{\text{ref},i} - \frac{1}{D_i} (\omega_i - \omega_s)\right)
\]

(5)

and

\[
\frac{d}{dt} \delta_i = \omega_i - \omega_c = \omega_i - \frac{1}{J_{\text{tot}}} \sum_{i=1}^{n} J_i \omega_i,
\]

(6)

where \( \omega_i \) is the electric frequency of the rotor, \( P_i \) is the generator single-phase active power output, \( P_{\text{ref},i} \) is the reference power for droop control mechanism, and \( D_i \) is the slope of droop control mechanism. In addition, define \( P_{\text{ref},\text{tot}} = \sum_i P_{\text{ref},i} \) to be the total reference power, \( P_{L,\text{tot}} = \sum_n P_{L,n} \) to be the total load power, \( \Delta \omega = \omega_c - \omega_s \) to be the deviation of central frequency from nominal frequency, and \( \Delta P_L = P_{L,\text{tot}} - P_{\text{ref},\text{tot}} \) to be the deviation of total load from the reference power.

In order to derive a first-order differential equation describing the central frequency, assume there is an inverse proportion\(^1\) between the moment of inertia \( J_i \) and the droop control constants \( D_i \), such that \( J_i D_1 = J_2 D_2 = \cdots = J_n D_n = \text{const} \). Following this assumption and some basic algebraic manipulations yields

\[
\frac{d}{dt} \Delta \omega = -\frac{1}{\omega_c J_1 D_1} \Delta \omega - \frac{3}{\omega_s J_{\text{tot}}} \Delta P_L.
\]

(7)

This equation (which may appear in several different forms) is known as the aggregated swing equation. Following a transient, the dynamic behavior of each specific generator is related to all other generators, and may be quite complex. However, the dynamic behavior of the central frequency, which describes the average frequency of all generators, is described by the first-order differential equation (7).

III. MODELING SYSTEMS USING A DQ0 TRANSFORMATION BASED ON THE CENTRAL ANGLE

The results in the previous section are obtained by assuming that the system is quasi-static. In this section we extend these results, and present models that use the central angle reference frame as a basis for a general dq0 transformation. Consider

\(^1\)This assumption is typically reasonable for large synchronous generators, but may not hold for renewable and distributed power sources.
To illustrate the above definitions, we now develop a model of a simple synchronous machine using a \(dq0\) transformation based on the central angle. Assume a simplified synchronous machine represented as an ideal voltage source behind a synchronous inductance. The synchronous machine voltage is \(\tilde{v}_d = 0, \tilde{v}_q = V_c, v_0 = 0\), with a reference angle of \(\theta\). In this example \(\theta\) is the electric angle of the rotor with respect to the stator. Now to represent the synchronous machine in the central angle reference frame, we convert the voltage from one reference frame to another using (11), which yields

\[
\begin{bmatrix}
\tilde{v}_d \\
\tilde{v}_q \\
v_0
\end{bmatrix} = \begin{bmatrix}
V_c \cos (\delta) \\
V_c \sin (\delta) \\
0
\end{bmatrix}, \quad \delta = \theta - \theta_c + \pi/2.
\]

In addition, the dynamic behavior of the angle \(\theta\) is

\[
\frac{d^2}{dt^2} \theta = \frac{d}{dt} \omega = \frac{1}{\omega_s} \left(3P_{ref} - P_{dq0} - \frac{1}{D} (\omega - \omega_s)\right),
\]

which is the swing equation combined with the droop control mechanism. The combination of (13) and (14) results in a state-space model given as

\[
\begin{align}
\frac{d}{dt} \delta &= \omega - \omega_c, \\
\frac{d}{dt} \omega &= \frac{1}{\omega_s} \left( -\frac{3}{2} V_c (\cos(\delta)i_d + \sin(\delta)i_q) \\
&\quad + 3P_{ref} - \frac{1}{D} (\omega - \omega_s)\right),
\end{align}
\]

where the state variables are \(\delta, \omega\), the inputs are \(i_d, i_q\), \(\omega_c\), and the outputs are \(v_d, v_q, v_0, \omega\). The model represents only the machine’s voltage source, and does not include the synchronous inductance, which is modeled separately in the next section. More complex models of synchronous machines may be obtained following a similar procedure, using (11), its inverse and Fig. 2.

A. Modeling Passive Components and Transmission Networks based on the Central Angle

In this section we develop models of single passive components based on the \(dq0\) transformation, and using the central angle reference frame.

Consider a balanced three-phase inductor, which is modeled in the abc reference frame as

\[
L \frac{d}{dt} I_{abc} = V_{abc,1} - V_{abc,2}.
\]

Observe that the differentiation of (8) results in

\[
\frac{d}{dt} I_{dq0} = \frac{d}{dt} \tilde{\theta}_c \tilde{I}_{dq0} + \tilde{\theta}_c \frac{dI_{abc}}{dt},
\]
and the derivative of $T_{θc}$, using (3), can be expressed as
\[
\frac{dT_{θc}}{dt} = -\frac{2ωc}{3} \begin{bmatrix}
\sin(θc) & \sin(θc + \frac{2π}{3}) & \sin(θc + \frac{4π}{3}) \\
\cos(θc) & \cos(θc - \frac{2π}{3}) & \cos(θc - \frac{4π}{3}) \\
0 & 0 & 0
\end{bmatrix},
\]
\[
= \begin{bmatrix}
0 & ωc & 0 \\
-ωc & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} T_{θc} = W_c T_{θc}.
\] (18)

Substitute (16) and (18) into (17), and use (8) to get
\[
\frac{d}{dt}I_{dq0} = \frac{1}{L} (V_{dq0,1} - V_{dq0,2}) + W_c I_{dq0}.
\] (19)

This equation describes a state-space model of the three-phase inductor in the central angle reference frame. Note that the model is nonlinear, since $ωc$ in $W_c$, which is a sum of state variables, is being multiplied with the $dq0$ currents. Similarly, the model of a capacitor $C$ is
\[
\frac{d}{dt} (V_{dq0,1} - V_{dq0,2}) = \frac{1}{C} I_{dq0} + W_c (V_{dq0,1} - V_{dq0,2}),
\] (20)
and for a resistor $R$ the model is
\[
V_{dq0} = I_3 R I_{dq0},
\] (21)
with $I_3$ being the $3 \times 3$ identity matrix. This analysis can be extended to model large-scale networks as done in [7], where $ωs$ is replaced with $ωc$.

### IV. NUMERIC RESULTS

This section demonstrates application of the presented $dq0$-based models defined with respect to the central angle reference frame. Two standard networks with 9 and 57 buses are considered, for which parameters are taken from [17]. In the first example, the transient response of the 9-bus network is addressed. The second example is devoted to the small-signal stability analysis of the 57 bus network.

#### A. 9-bus Network: Transient Analysis

Consider the 9-bus network depicted in Fig. 3. All models are constructed based on $dq0$ quantities using the software package [18]. We compare the proposed central angle models with the models defined with respect to the unified reference frame (rotating with $ωs = 2π/50$ rad/s). The model of the network is constructed in state-space form using the equations presented in Section III-A. Loads are represented as simple $dq0$ state-space models based on balanced series $RL$ impedances located on buses 5, 7, and 9. Synchronous machines are connected to buses 1, 2, and 3. Observe that in case of the unified reference frame, bus 1 represents an infinite bus, modeled by voltage source with $v_d = 2.83 \times 10^4$ V and $v_q = v_0 = 0$. To make the simulations more realistic we use the physical models of synchronous machines based on [19]. Simple droop controller based on the frequency variation is used to adjust the input mechanical power.

Consider the case when the single phase reference powers $P_{ref,2}$ and $P_{ref,3}$ are stepped by additional 120 MW from the nominal values at time $t = 10$ s. Once the transient process is complete, both systems are linearized around an operating point and the eigenvalues with the largest real part are 0.5369 (unified) and −0.4005 (central). Observe that the $unified$ model predicts that the system is unstable, while the $central$ model predicts that it is stable. Simulation results are presented in Figs. 4 and 5, which show variations in active powers, frequencies, and angles of generators. Note that the frequency variable is plotted on a shorter interval to better illustrate the transient process.

![Fig. 3. Single-line diagram of the 9-bus system.](image)

![Fig. 4. Unified reference frame, 9-bus. Top plot corresponds to the active power, the middle plot shows frequency variations, and the bottom plot depicts evolution of the power angle.](image)

![Fig. 5. Central angle reference frame, 9-bus. Top plot corresponds to the active power, the middle plot shows frequency variations, and the bottom plot depicts evolution of the power angle.](image)
interconnected system, which is done in several steps. All the models are constructed based on \(dq0\) quantities. Once the transient process is complete, the obtained closed loop model is linearized around the system’s operating point, so the resulting model is linear, and the inputs and outputs are small-signals. An array of Bode plots representing the resulting model is linear, and the inputs and outputs are small-signals. In this light, the \(dq0\) transformation provides several important advantages that simplify the system dynamics, especially when modeling balanced or symmetrically configured power systems.

A question that often arises when modeling interconnected systems based on \(dq0\) quantities is how to choose the reference frame, or in other words, how to link different machines that rotate at different frequencies. One approach is to model the network and its components using a \(dq0\) transformation that is based on a unified reference frame, rotating with a fixed frequency \(\omega_s\). However, when no generator is large enough to be considered an infinite bus, the unified reference frame may lead to unstable models, since the real frequency may deviate from the nominal frequency used as a reference. In this paper we propose to solve this problem by using the central angle as a basis for the \(dq0\) transformation. Such an approach leads to models which are valid at high frequencies, have a well-defined equilibrium point, and can also be used with systems with varying frequencies and no infinite bus (due to the use of the central angle). The proposed approach is demonstrated using several test-cases based on standard networks.

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