Control of Energy Storage Devices Under Uncertainty Using Nonlinear Feedback Systems

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***Abstract*—We propose here a nonlinear control scheme for energy storage devices that is designed to operate under uncertainty conditions, but does not require a statistical representation of future signals. We first use Pontryagin's minimum principle to develop an optimal control law. This control law is shown to be unstable, and therefore only converges to the optimal solution if future values of the load signal are known. We then show a modified control law that is sub-optimal but stable, and therefore can work efficiently without any information on future signals. This modified controller is shown to work well in several practical test cases.**

I. Introduction

Over the past few years storage technologies are slowly emerging as an essential component of modern power systems [1]–[5]. A well-known challenge is how to optimally control storage devices to maximize the efficiency or reliability of a system [6]. As an example, for a grid-connected storage device the objective is usually to minimize the total cost, the total fuel consumption, or the peak of the generated power, while operating the device within its limits [7]–[9]. The problem in this case is to decide how much energy should be stored, and when to store it. Such optimal control problems are generally hard to solve due to their high numeric complexity, since finding an optimal solution requires computation of the stored energy at each and every point in time. In addition, such optimization problems are usually not convex, either since the objective function is not convex, or since it is not defined over a convex set [10].

Many energy storage optimization problems include elements of uncertainty, such as varying load curves, varying energy production from renewable sources, or time-varying price signals. Such uncertainties are often handled by stochastic control methods, which estimate the optimal control rule based on limited data. For instance, one approach is to formulate the problem as a Markov decision process, and to solve it based on stochastic dynamic programming [11], [12]. Several stochastic control strategies are summarized in Table I.

One disadvantage of stochastic control methods is that they require a relatively accurate statistical model of the system, which is often very complex or unavailable. In addition, stochastic dynamic programming may suffer from the "curse of dimensionality" [24], meaning that the numeric complexity of the optimal control law increases exponentially with the number of state variables. These two limitations may be crucial in practical energy management systems.

In this light, we propose here a nonlinear control scheme for energy storage devices that is designed to operate under uncertainty conditions, but does not require a statistical representation of future signals. We first use Pontryagin's minimum principle to develop an optimal control law. This control law is shown to be unstable, and therefore only converges to the optimal solution if future values of the load signal are known. We then show a modified control law that is sub-optimal but stable, and therefore can work efficiently without any information on future signals. This modified controller is shown to work well in several practical test cases.

The rest of the paper is organized as follows. Section II presents the optimal solution using Pontryagin's minimum principle. The proposed modified sub-optimal control law is presented in Section III, followed by illustrative examples in Section IV. Concluding remarks and a brief discussion are given in Section V.

II. Optimal Feedback Law Based on Pontryagin's Minimum Principle

We denote $\xi'(x) := \mathrm{d}\xi/\mathrm{d}x$ and $\xi''(x) := \mathrm{d}^2\xi/\mathrm{d}x^2$. When referring to signals the time argument t is sometimes omitted.

Consider a simplified power system that includes a fueled primary source, a load, and a storage device. This system may be a small part of a power network, a hybrid electric vehicle, or any other system that includes a fueled source and a storage device. The challenge is to determine the optimal power flowing into the storage device at every point in time t, such that the total cost is minimized:

$$\begin{aligned} \text{minimize} \quad & F_{tot} = \int_0^T F(u(t) + P_L(t))\mathrm{d}t \\ \text{subject to} \quad & \frac{\mathrm{d}}{\mathrm{d}t}x = u(t),\ x(0) = E_0, \\ & 0 \le x(t) \le E_{\max}, \end{aligned} \tag{1}$$

where $u(t)$ is the power flowing into the storage device, $P_L(t)$ is the load power, $u(t) + P_L(t)$ is the generated power, $x(t)$ is the stored energy, E_{max} is the device capacity, E_0 is the energy stored at time zero, and $F(\cdot)$ is a cost function representing fuel consumption or any other cost. It is assumed

TABLE I
SUMMARY OF OPTIMAL ENERGY STORAGE CONTROL METHODS THAT USE STOCHASTIC STRATEGIES.

Ref.	Application	Method details	Objective
[13]	Households	Markov decision process	Minimize the total cost by charging storage from purchased power and discharging it for personal consumption
[14]	Residential	Stochastic dynamic program	Minimize the average cost of conventional generation, and the investment cost in storage
[15]	Households	Model predictive control	Maximize battery lifetime, including forecasts of generation and demand
[16]	Smart home	Stochastic dynamic programming	Minimize a consumer's energy charges under a time-of-use tariff, while satisfying home power demand and plug-in electric vehicle charging requirements, and accommodating the variability of solar power
[17]	Grid-connected storage device	Online energy procurement algorithm based on Lyapunov optimization technique	Mitigate mismatches between the forecasted and actual renewable energy generation
[18]	Grid-connected wind farm	Adaptive optimal policy with approximated objective function	Maximize expected daily profit by time-shifting wind energy, considering uncertainties in generation and prices
[19]	Generic grid-connected storage device	Stochastic dynamic programming	Co-optimize the use of a storage device that is put to multiple uses
[20]	Grid-connected storage device	Stochastic dynamic programming	Maximize value of energy storage system under uncertain energy prices
[21]	Grid-connected storage device	Stochastic dynamic programming	Optimal bidding mechanism for storage devices selling both energy and reserve
[22]	MicroGrid	Stochastic dual dynamic programming	Minimize the expected total energy costs subject to storage capacity, line capacity, and other physical constraints
[23]	Energy management system for a telecommunication operator	Markov decision process with dynamic programming	Minimize the energy bill while serving customers requests

that $F(\cdot)$ is twice continuously differentiable and strictly convex. Note that in (1) the signal $u(t)$ is the decision variable, and it is assumed that the function $P_L(t)$ is given. These definitions and others are summarized in the Appendix.

We will now show that this optimization problem may be efficiently solved using Pontryagin's minimum principle, and that the solution may be expressed as a nonlinear feedback system. To this end, it is convenient to define a modified running cost function that incorporates the constraint on $x(t)$:

$$J = \int_0^T \left[F(u(t) + P_L(t)) + \phi(x(t))\right] \mathrm{d}t, \tag{2}$$

where

$$\phi(x) = \begin{cases} \frac{\alpha}{2E_{\max}}(x - E_{\max})^2, & x > E_{\max}, \\ 0, & 0 \le x \le E_{\max}, \\ \frac{\alpha}{2E_{\max}} x^2, & x < 0, \end{cases} \tag{3}$$

with $\alpha > 0$. If the constant α is very large then the stored energy $x(t)$ is practically limited to the range $0 \le x \le E_{\max}$, so the bounds on $x(t)$ in the original problem may be ignored. Using this running cost, (1) can be rewritten as

$$\begin{aligned} &\text{minimize} \quad J = \int_0^T \left[F(u(t) + P_L(t)) + \phi(x(t))\right] \mathrm{d}t \\ &\text{subject to} \quad \frac{\mathrm{d}}{\mathrm{d}t} x = u(t),\ x(0) = E_0. \end{aligned} \tag{4}$$

Based on the minimum principle, an optimal solution must obey the following conditions:

$(a1)$ $\frac{\mathrm{d}}{\mathrm{d}t} x^* = u^*$, $x^*(0) = E_0$;

$(b1)$ $\frac{\mathrm{d}}{\mathrm{d}t}\lambda^* = -\phi'(x^*)$, $\lambda^*(T) = 0$;

$(c1)$ $\frac{\partial}{\partial u}\left\{F(u + P_L) + \lambda^* u\right\}\big|_{u=u^*} = 0$.

Replacing λ^* with $-\lambda^*$, and simplifying $(c1)$ leads to

$(a2)$ $\frac{\mathrm{d}}{\mathrm{d}t} x^* = u^*$, $x^*(0) = E_0$;

$(b2)$ $\frac{\mathrm{d}}{\mathrm{d}t}\lambda^* = \phi'(x^*)$, $\lambda^*(T) = 0$;

$(c2)$ $u^* = (F')^{-1}(\lambda^*) - P_L$.

Note that in $(c2)$ the inverse function $(F')^{-1}(\lambda^*)$ is well-defined since we assumed $F''(\lambda^*) > 0$, and therefore $F'(\lambda^*)$ must be monotonically increasing. Substitution of $(c2)$ in $(a2)$ yields the equivalent conditions:

$(a3)$ $\frac{\mathrm{d}}{\mathrm{d}t} x^* = u^* = (F')^{-1}(\lambda^*) - P_L$, $x^*(0) = E_0$;

$(b3)$ $\frac{\mathrm{d}}{\mathrm{d}t}\lambda^* = \phi'(x^*) = \frac{\alpha}{E_{\max}}\zeta(x^*)$, $\lambda^*(T) = 0$,

where $\zeta(x^*)$ is the *dead-zone* function defined as

$$\zeta(x^*) = \begin{cases} x^* - E_{\max}, & x^* > E_{\max}, \\ 0, & 0 \le x^* \le E_{\max}, \\ x^*, & x^* < 0. \end{cases} \tag{5}$$

This solution may be expressed as a *nonlinear feedback system*, as shown in Fig. 1. In this system the initial condition of $x^*(t)$ is E_0, and the initial condition of $\lambda^*(t)$ is unknown. Therefore, a low-complexity method to compute the optimal solution is to scan possible values of $\lambda^*(0)$ until the final condition $\lambda^*(T) = 0$ holds. This may be done efficiently using a simple line-search algorithm.

III. A STABLE AND SUB-OPTIMAL FEEDBACK SYSTEM

The nonlinear feedback system from the previous section converges to the optimal solution. Unfortunately, this system

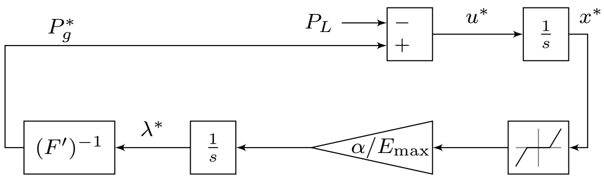

Fig. 1. The optimal energy management strategy may be obtained using a nonlinear feedback system.

may be unstable, and will converge to the optimal solution only from a specific initial condition $\lambda^*(0)$. To find this initial condition complete knowledge of $P_L(t)$ is required, and therefore this feedback system is not practical in most applications. A natural question is whether there exists a modified feedback system which is stable, and therefore converges to a sub-optimal solution from any initial point, and without needing any statistical information on $P_L(t)$. Obviously, such a controller may be very useful in practical energy management problems. In this section, we show an example for such a modified (and sub-optimal) feedback system.

Intuitively, the system in Fig. 1 is unstable due to positive feedback: increasing values of $x^*(t)$ lead to increasing values of $u^*(t)$ and *vice-versa*. Therefore, a possible method to stabilize this system is to include a negative feedback mechanism between $x^*(t)$ and $u^*(t)$. Such negative feedback is represented in Fig. 2, and is described as follows:

$(a4)$ $\frac{\mathrm{d}}{\mathrm{d}t}\tilde{x}^* = \tilde{u}^*$, $\tilde{x}^*(0) = E_0$;

$(b4)$ $\tilde{u}^* = (F')^{-1}(\lambda^*) - \beta r(\tilde{x}^*) - P_L$;

$(c4)$ $\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\lambda}^* = \frac{\alpha}{E_{\max}}\zeta(\tilde{x}^*)$,

where

$$r(\tilde{x}^*) = \left(\frac{\tilde{x}^*}{E_{max}} - \frac{1}{2}\right) / \left(\frac{1}{2} - \left|\frac{\tilde{x}^*}{E_{max}} - \frac{1}{2}\right|\right). \quad (6)$$

This function is plotted in Fig. 3.

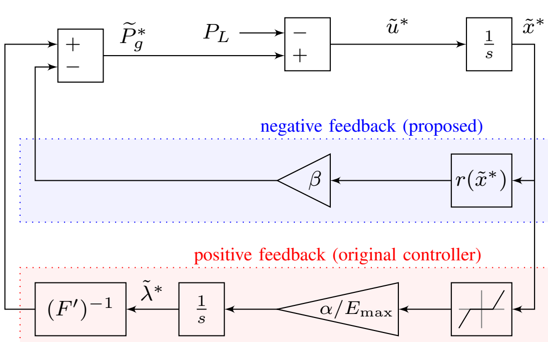

Fig. 2. Modified sub-optimal system with positive and negative feedback mechanisms.

In this modified system the negative feedback mechanism is governed by the function $r(\tilde{x}^*)$. When the stored energy $\tilde{x}^*$ is close to the middle of the range $[0, E_{max}]$ the function $r(\tilde{x}^*)$ is relatively low. In this case the negative feedback branch has little effect, so the trajectory is dictated by the positive feedback and is similar to the optimal one. However, when

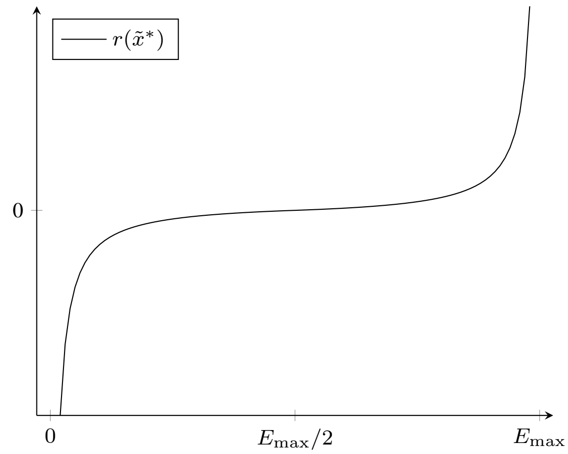

Fig. 3. The function $r(\tilde{x}^*)$.

the stored energy $\tilde{x}^*$ is close to either 0 or E_{max} the value of $r(\tilde{x}^*)$ is high. In this case the negative feedback branch is the dominant one, and the solution tends to stabilize. The controller can be tuned using the parameters α and β.

IV. Numeric Results

Consider an example in which a load, an energy storage device, and a PV array are connected to the grid, as illustrated in Fig. 4.

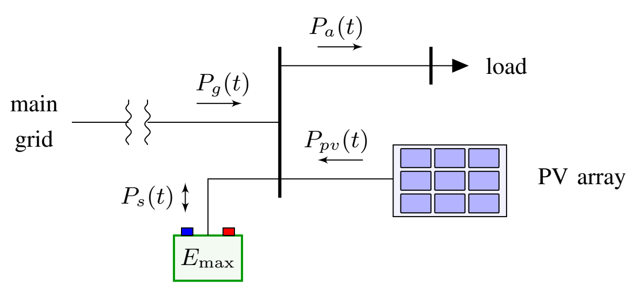

Fig. 4. A system with a load, a photovoltaic array, and a storage device.

The reference load power can be computed as $P_L(t) = P_a(t) - P_{pv}(t)$. The parameters in this example are $T = 48$, $E_{\max} = 3$, $\alpha = 200$, $\beta = 0.002$, and the initial conditions are selected as $x(0) = E_{\max}/2 = 1.5$, $\lambda(0) = P_{L,avg} = 0.306$. The cost function is selected to be $F := (\lambda^*)^2$. Simulations are performed in the Matlab/Simulink environment using the `ode45` solver with a maximum step size of 1×10^{-3} and a relative tolerance of 1×10^{-4}. All quantities are specified in per-unit (pu).

Figure 5 shows the resulting optimal and sub-optimal generated and stored energies. The respective power flows are shown in Fig. 6. Recall that the optimal quantities obtained using Pontryagin's minimum principle from Section II are denoted using *star* notation, and the respective sub-optimal solutions from Section III are denoted using *tilde* notation. In addition, recall that the optimal method assumes the full knowledge of the load profile, whereas the proposed sub-optimal solution utilizes only the current value of $P_L(t)$ at each time instance. It can be seen that both approaches respect the imposed constraints as defined in (3), and the sub-optimal solution closely follows the optimal path. Summary of several statistical measures is presented in Table II for different capacity levels of the energy storage device. The selected measures are further emphasized in Fig. 7. It is interesting to note that the accuracy does not decrease monotonically with the increase in capacity level, but has a local minimum around $E_{\max} = 3$. This observation triggers several questions about the accuracy of the sub-optimal solution, which may be the subject of future research.

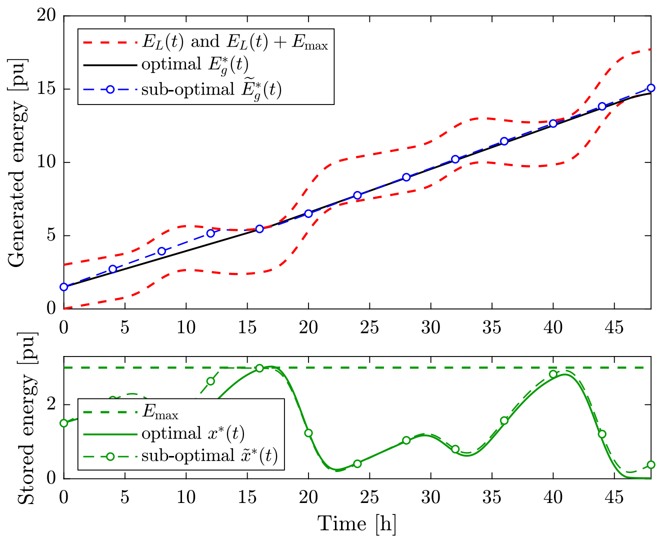

Fig. 5. The upper plot shows the generated optimal energy ('—') limited by its upper and lower bounds ('- -'), and the proposed sub-optimal solution ('- o -'). The bottom plot depicts the optimal stored energy ('—'), sub-optimal stored energy ('- o -), and storage capacity ('- -').

TABLE II
SUMMARY OF STATISTICAL MEASURES FOR DIFFERENT CAPACITY LEVELS OF THE ENERGY STORAGE DEVICE.

	Measures	Capacity ($E_{\max}$) 1	2	3	5	8
	$\max(P_g^*)$	0.7156	0.4577	0.2974	0.2570	0.2247
	$\max(\widetilde{P}_g)$	1.200	1.0798	0.3212	0.3064	0.3061
generated E_g^*, $\widetilde{E}_g$	RMSE	0.3276	0.4296	0.2647	1.2219	2.2327
	SSE	51.6314	88.7914	33.7063	718.1372	2397
	ME	-0.1372	-0.2379	-0.1654	-1.0610	-1.9330
stored E_s^*, $\widetilde{E}_s$	RMSE	0.3270	0.4221	0.2588	1.2213	2.2349
	SSE	51.4175	85.6814	32.2142	717.4911	2403
	ME	-0.1304	-0.2322	-0.1607	-1.0583	-1.9335

*RMSE–root mean squared error, SSE–sum of squared estimate of errors, ME–mean error

V. CONCLUSION

We propose a nonlinear control scheme for energy storage devices that is designed to operate under uncertainty conditions, but does not require statistical representation of future signals. The solution is based on Pontryagin's minimum principle, which describes the optimal energy management strategy as a trajectory of a nonlinear feedback system. However this optimal feedback system may be unstable, and so has little practical value in most applications. To stabilize this system we propose to add a negative feedback component, which drives the power flowing to the storage device in inverse proportion to the stored energy. This modified feedback system resembles the optimal one, and therefore produces sub-optimal solutions. This approach is tested in simulation, and is shown to work well in several practical test cases.

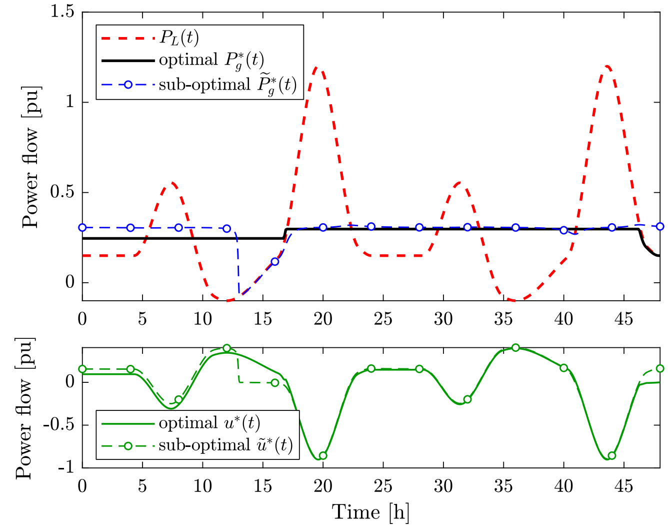

Fig. 6. Optimal power flow. The upper plot shows the load ('- -'), optimal ('—') and sub-optimal ('- o -') generated powers. The bottom plot depicts the optimal ('—') and sub-optimal ('- o -) powers flowing into the storage device.

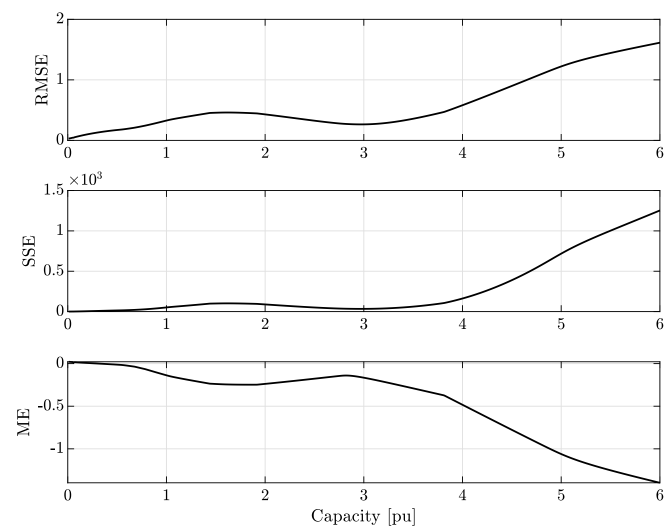

Fig. 7. Evolution of RMSE, SSE, and ME for the generated energy.

APPENDIX

Summary of the variables.

TABLE III
NOMENCLATURE.

Variable	Physical meaning
$P_s(t) = u(t)$	storage power
$P_g(t) = P_s(t) + P_L(t)$	generated power
$P_L(t)$	load power
$E_s(t) = \int_0^t P_s(\tau)\mathrm{d}\tau + E_0 = x(t)$	stored energy
$E_g(t) = \int_0^t P_g(\tau)\mathrm{d}\tau + E_0$	generated energy
$E_L(t) = \int_0^t P_L(\tau)\mathrm{d}\tau$	load energy

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