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FOPID Controller Tuning for Fractional FOPDT Plants subject to Design Specifications in the Frequency Domain

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Motivation, contribution, and outline

- PID controllers are widely used for industrial process control due to their relative simplicity and applicability to a wide range of industrial control problems. Fractional-order PID controllers offer more tuning flexibility;
- The main contributions of the presented paper:
  - First, a set of rules for selecting the orders of the integrator and differentiator of the FOPID controller is proposed;
  - A system of three nonlinear equations in three unknowns is constructed. The solution of this system grants the gains of the FOPID controller. Newton's method in multiple dimensions is adopted to the particular problem and a corresponding algorithm is outlined.
  - The algorithm is verified on an embedded device and in a hardware-in-the-loop control experiment;
- Finally, conclusions and further research perspectives are given.

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Basics of fractional calculus

Fractional calculus is a generalization of integration and differentiation to non-integer order operator ${}_a\mathscr{D}_t^\alpha$, where a and t are the limits of the operation and $\alpha \in \mathbb{R}$ is the fractional order

$$ {}_a\mathscr{D}_t^\alpha = \begin{cases} \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha} & \Re(\alpha) < 0. \end{cases} \tag{1} $$

The following relation holds for noninteger exponentiation of the imaginary unit j and is frequently encountered in fractional calculus. We shall make extensive use of it throughout this talk.

$$ j^\alpha = \cos\left(\frac{\alpha\pi}{2}\right) + j\sin\left(\frac{\alpha\pi}{2}\right) \tag{2} $$

Process models

Consider the following generalizations of conventional process models used in industrial control design.

(FO)FOPDT	$G(s) = \frac{K}{1+Ts}\mathrm{e}^{-Ls}$	$G(s) = \frac{K}{1+Ts^{\alpha}}\mathrm{e}^{-Ls}$
(FO)IPDT	$G(s) = \frac{K}{s}\mathrm{e}^{-Ls}$	$G(s) = \frac{K}{s^{\alpha}}\mathrm{e}^{-Ls}$
(FO)FOIPDT	$G(s) = \frac{K}{s(1+Ts)}\mathrm{e}^{-Ls}$	$G(s) = \frac{K}{s(1+Ts^{\alpha})}\mathrm{e}^{-Ls}$

Fractional-order controllers

The fractional PI$^\lambda$D$^\mu$ controller, where λ and μ denote the orders of the integral and differential components, respectively, is given by

$$C(s) = K_p + K_i s^{-\lambda} + K_d \cdot s^{\mu}. \tag{3}$$

The transfer function, corresponding to the fractional lead-lag compensator of order α, has the following form:

$$C_L(s) = K \left(\frac{1 + bs}{1 + as} \right)^{\alpha}. \tag{4}$$

When $\alpha > 0$ we have the fractional zero and pole frequencies $\omega_z = 1/b$, $\omega_h = 1/a$ and the transfer function in (4) corresponds to a fractional lead compensator. For $\alpha < 0$, a fractional lag compensator is obtained.

The FFOPDT process model

Recall, that the FO-FOPDT model is given by the following transfer function

$$G(s) = \frac{K\mathrm{e}^{-Ls}}{Ts^{\alpha} + 1}, \tag{5}$$

where it is assumed that $K > 0$, $T > 0$, $L > 0$ and $\alpha \in (0, 2]$. We suppose that all of the parameters of this plant are known *a priori*. They may be obtained, for instance, by employing an identification procedure of a real life process. We begin the analysis by deriving the equations to obtain the magnitude and phase angle of $G(j\omega)$. This is done by replacing $s = j\omega$ in (5), employing (2), and isolating the real and complex parts of the resulting expression as $z = a + jb$. Then, the magnitude A and phase angle φ are simply computed as $A = \sqrt{a^2 + b^2}, \varphi = \tan^{-1}(b/a)$.

The FFOPDT process model: Magnitude and phase response

Magnitude:

$$\left|G(j\omega)\right| = \frac{|K|}{\sqrt{1 + T^2\omega^{2\alpha} + 2T\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}} \tag{6}$$

Phase angle:

$$\arg\left(G(j\omega)\right) = -L\omega - \tan^{-1}\left(\frac{T\sin\left(\frac{\alpha\pi}{2}\right)}{\omega^{-\alpha} + T\cos\left(\frac{\alpha\pi}{2}\right)}\right). \tag{7}$$

The obtained relations will be used in the following calculations.

FOPID controller: Magnitude response

We derive the expression for the magnitude response as

$$\left|C(j\omega)\right| = \sqrt{C_R^2(\omega) + C_I^2(\omega)}, \tag{8}$$

where

$$C_R(\omega) = K_p + \omega^{-\lambda} K_i \cos\left(\frac{\lambda\pi}{2}\right) + \omega^{\mu} K_d \cos\left(\frac{\mu\pi}{2}\right) \tag{9}$$

and

$$C_I(\omega) = -\omega^{-\lambda} K_i \sin\left(\frac{\lambda\pi}{2}\right) + \omega^{\mu} K_d \sin\left(\frac{\mu\pi}{2}\right). \tag{10}$$

FOPID controller: Phase angle

The phase angle of the FOPID controller may be computed using

$$\arg\left(C(j\omega)\right) = \tan^{-1}\left(\frac{C_N(\omega)}{C_D(\omega)}\right), \tag{11}$$

where

$$C_N(\omega) = \omega^{\lambda+\mu} K_d \sin\left(\frac{\mu\pi}{2}\right) - K_i \sin\left(\frac{\lambda\pi}{2}\right) \tag{12}$$

and

$$C_D(\omega) = K_i \cos\left(\frac{\lambda\pi}{2}\right) + \omega^{\lambda}\left(\omega^{\mu} K_d \cos\left(\frac{\mu\pi}{2}\right) + K_p\right). \tag{13}$$

Tuning FOPID controllers for the FFOPDT model

We have five parameters to tune, out of which two are selected based on some concrete rule. The other three parameters must be optimized to satisfy three design specifications. Current implementation is as follows:

- Choose λ—the order of the fractional integrator—according to the F-MIGO rule;
- Choose μ—the order of the fractional differentiator—according to control system output signal measurement SNR (no concrete relation yet);
- Select three design specifications, form three equations for K_p, K_i and K_d—the FOPID controller gains—and use the Newton method to solve the system of these equations.

FOPID control for FFOPDT: F-MIGO

The F-MIGO method has been developed based on observations using a test batch of regular FOPDT models used for FOPI controller design. Based on the relative dead time parameter

$$\tau = \frac{L}{L+T} \tag{14}$$

the following rule was established for selecting the order of the fractional integrator

$$\lambda = \begin{cases} 1.1, & \tau \geqslant 0.6, \\ 1.0, & 0.4 \leqslant \tau < 0.6, \\ 0.9, & 0.1 \leqslant \tau < 0.4, \\ 0.7, & \tau < 0.1. \end{cases} \tag{15}$$

FOPID control for FFOPDT: The Newton method

We have $x = \begin{bmatrix} K_p & K_i & K_d \end{bmatrix}^\top$ and must solve for Δx the matrix equation

$$J\Delta x = -F. \tag{16}$$

The next iterate is computed as $x^+ = x + \Delta x$, where

$$J = \begin{bmatrix} \frac{\partial \kappa_1(\cdot)}{\partial K_p} & \frac{\partial \kappa_1(\cdot)}{\partial K_i} & \frac{\partial \kappa_1(\cdot)}{\partial K_d} \\ \frac{\partial \kappa_2(\cdot)}{\partial K_p} & \frac{\partial \kappa_2(\cdot)}{\partial K_i} & \frac{\partial \kappa_2(\cdot)}{\partial K_d} \\ \frac{\partial \kappa_3(\cdot)}{\partial K_p} & \frac{\partial \kappa_3(\cdot)}{\partial K_i} & \frac{\partial \kappa_3(\cdot)}{\partial K_d} \end{bmatrix} \tag{17}$$

is the Jacobian matrix and

$$F = \begin{bmatrix} \kappa_1(\cdot) & \kappa_2(\cdot) & \kappa_3(\cdot) \end{bmatrix}^\top \tag{18}$$

is the specifications vector. Both J and F are evaluated at the current value of x. Then $x \leftarrow x^+$ and the process continues until either the design goal is satisfied, or divergence or excess of allowed number of iterations is detected.

FOPID control for FO-FOPDT: Chosen specifications

The following specifications are considered:

- Gain crossover frequency ω_c;
- Phase margin φ_m (in radians) which is computed using knowledge of ω_c;
- Robustness to gain variations $\psi'_{gm}(\omega_c) = 0$.

The following functions are thus constructed:

$$
\begin{aligned}
\kappa_1(\cdot) &= |C(j\omega_c)| \cdot |G(j\omega_c| - 1, & (19)\\
\kappa_2(\cdot) &= \arg(C(j\omega_c)) + \arg(G(j\omega_c)) + \pi - \varphi_m, & (20)\\
\kappa_3(\cdot) &= \psi'_{gm}(\omega_c). & (21)
\end{aligned}
$$

Algorithm: Determination of FOPID Controller Gains

procedure $\text{FOPIDDESIGN}(g_0, \omega_c, \varphi_m, G_m)$
    $\epsilon \leftarrow$ Tolerance, $\epsilon_m \leftarrow$ MachineTolerance
    $g \leftarrow g_0, k \leftarrow 0, \nu \leftarrow$ MaxIterations
    while $k < \nu$ **do**
        if $\det J < \epsilon_m$ **then return** $\{-1, g\}$
        end if
        if $G_m^* < G_m$ **then return** $\{-2, g\}$
        end if
        if $\|F_s\|_2 < \epsilon$ **then return** $\{1, g\}$
        end if
        $g \leftarrow g - J^{-1}F_s$
        $k \leftarrow k + 1$
    end while
    return $\{0, g\}$
end procedure

Algorithm: Determination of FOPID Controller Gains: Return Codes

Table 1: Meaning of optimization procedure return codes

Code	Description
-2	Additional condition not satisfied—the gain margin G_m^* computed for the control system is less than the value given in G_m.
-1	Singular Jacobian matrix—local minimum possible.
0	Maximum number of algorithm iterations reached.
1	All conditions satisfied, successful termination.

FOPID control for FFOPDT: Example

Consider a plant

$$G(s) = \frac{66.16\mathrm{e}^{-1.93s}}{12.72s^{0.5} + 1} \tag{22}$$

which represents a model of a heating process. In the following, we illustrate the controller design procedure. Specifications are $\omega_c = 0.1$, $\varphi_m = 60^\circ$, and $\psi'_{gm}(\omega_c) = 0$.

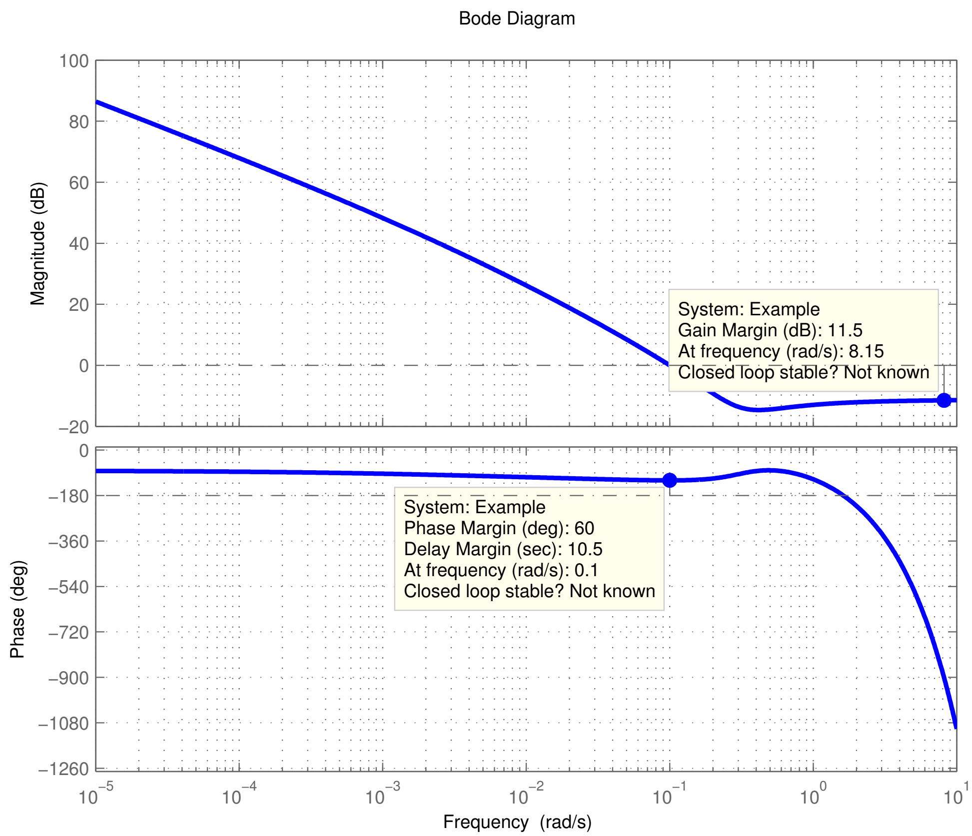
In this case, F-MIGO yields $\lambda = 0.9$ and we choose $\mu = 0.5$. For this problem we take the initial solution as

$$K_p = K_i = K_d = 1/K = 1/66.16 = 0.0151 \tag{23}$$

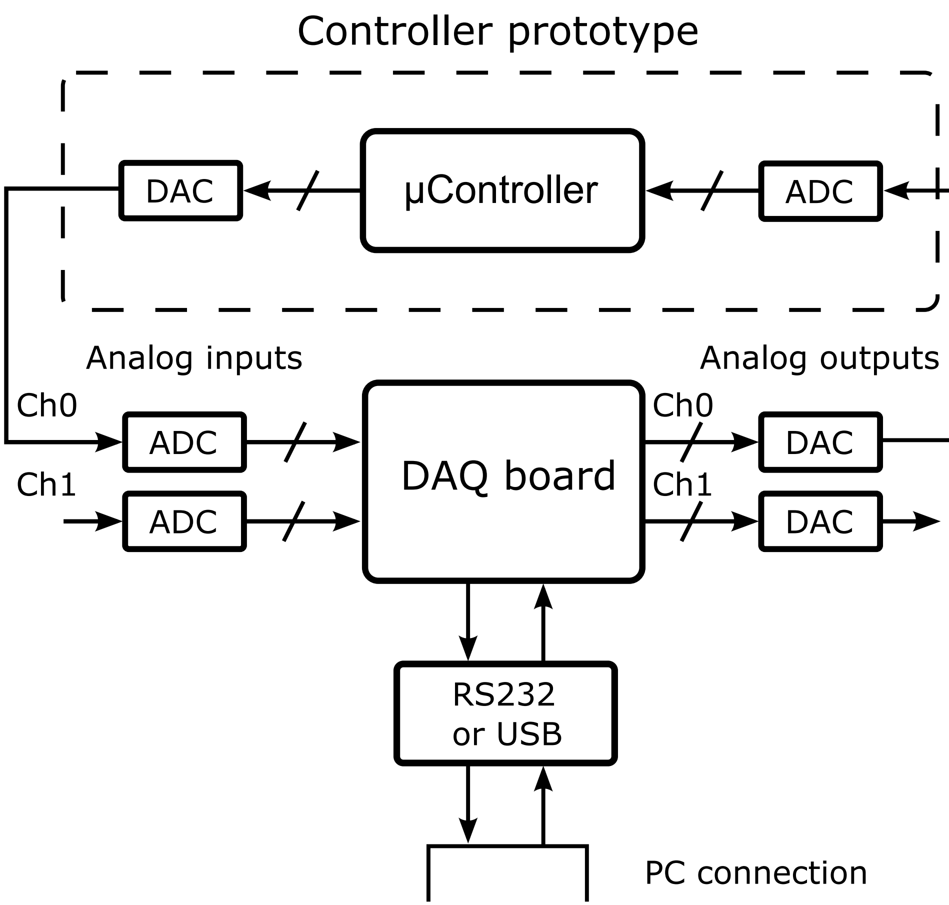
and apply Newton's method. The following gains are obtained:

$$K_p = -0.002934,\ K_i = 0.01030,\ K_d = 0.05335. \tag{24}$$

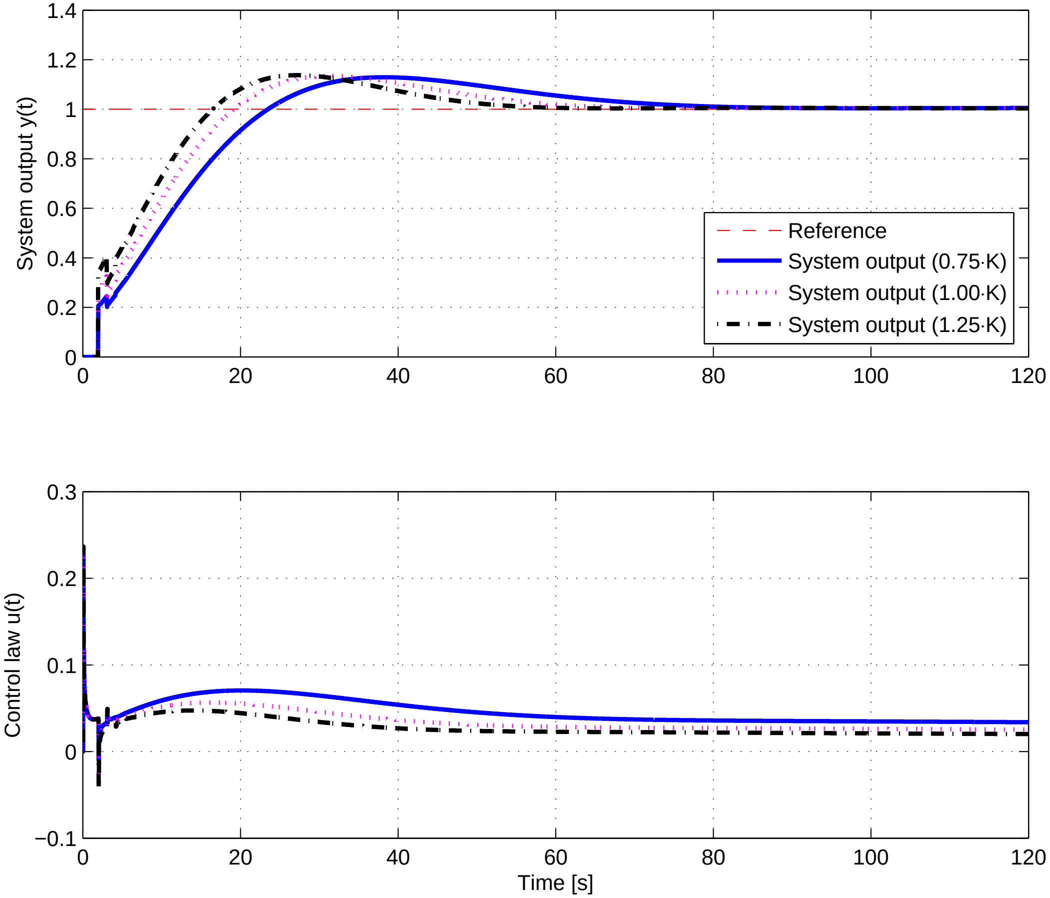
FOPID Control for FFOPDT: Frequency Response



Illustrative Example: Hardware-in-the-Loop Real-time Experiment



Illustrative Example: Pure Software Simulations of the FOPID Control System



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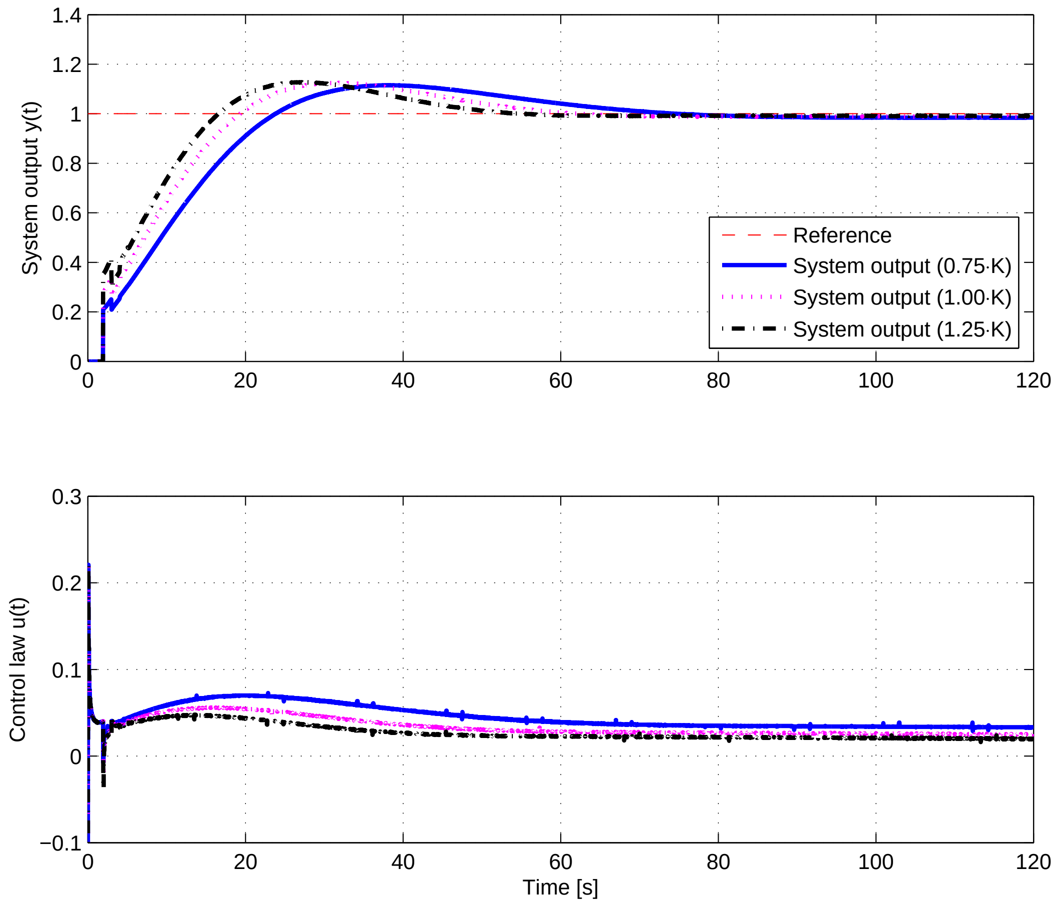
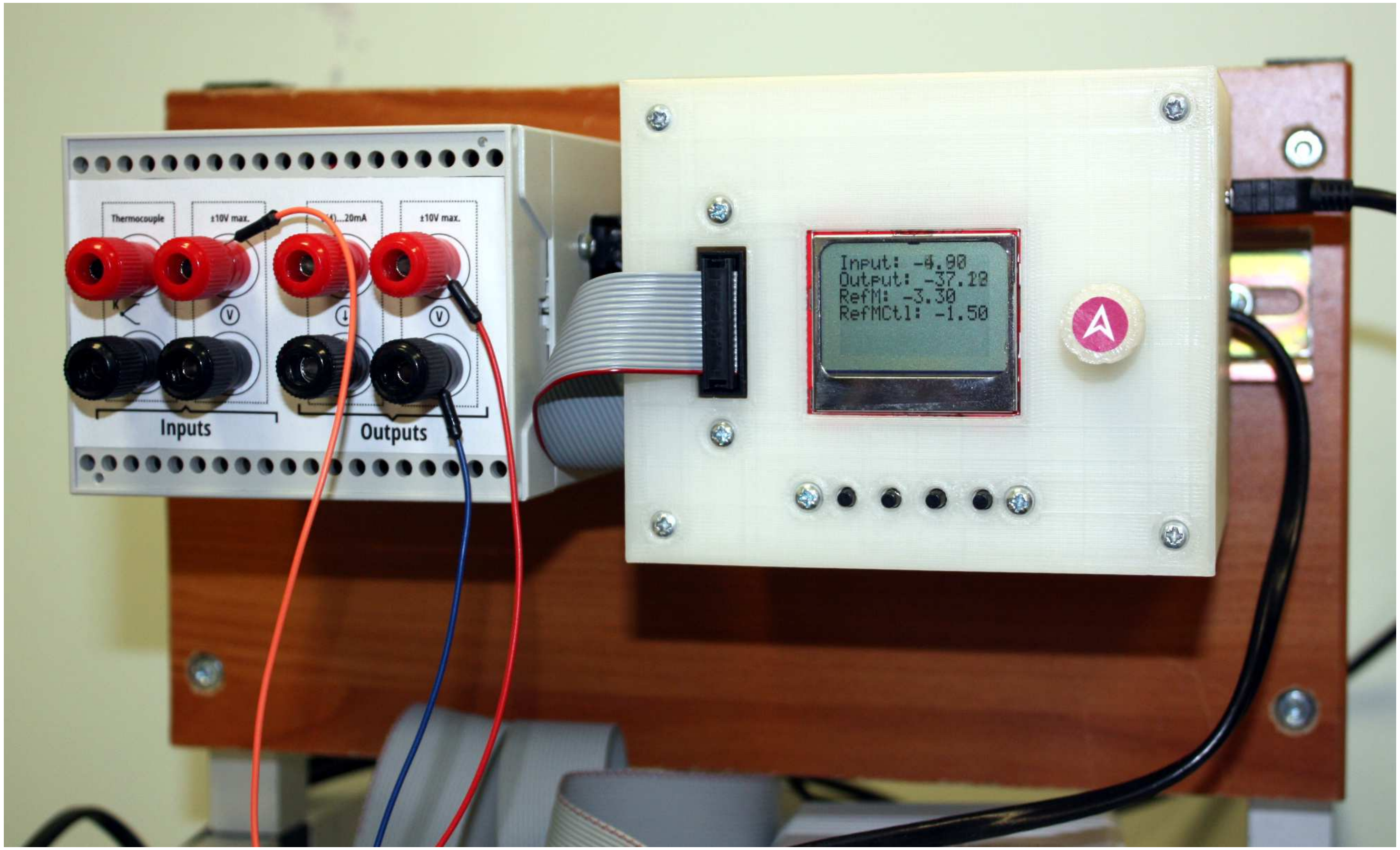


Conclusions and Further Perspectives

- In this paper, we have presented a method for designing FOPID controllers for FFOPDT plants based on available autotuning data.
- The tuning method was validated on a model of a heating process and successfully implemented on an embedded device.
- Experimental results involving real-time hardware-in-the-loop simulations confirm the validity of the proposed approach.
- A problem with conventional tuning approach was found, where the time constant of the system was not correctly identified. A more sophisticated tuning approach, which falls outside of the scope of this paper, may be used to tackle the issue.
- Research efforts should also be dedicated to developing a set of rules for selecting the orders of the FOPID controller integrator and differentiator components.

Future work: FOPID Controller Hardware Prototype



FOMCON project: Fractional-order Modeling and Control

FOMCON

- Official website: `http://fomcon.net/`
- Toolbox for MATLAB available;
- An interdisciplinary project supported by the Estonian Doctoral School in ICT and Estonian Science Foundation grant nr. 8738.

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`http://www.itakadeemia.ee/`

Thank you for listening!

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