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Closed-Loop Identification of Fractional-order Models using FOMCON Toolbox for MATLAB

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Motivation, Contribution, and Outline

- The problem of closed-loop system identification is of major interest in the field of industrial control. Fractional calculus offers additional modeling flexibility allowing for more precise description of dynamical systems.
- The particular contribution of this paper is as follows:
  - An identification method is provided for arbitrary SISO FO transfer function models, in which any parameter may either be known *a priori*, or be identified by means of model output error minimization;
  - The implementation of the algorithm in FOMCON toolbox for MATLAB is detailed;
  - Experimental results: The algorithm is verified on an exemplary fractional-order closed-loop control system, running on a real-time prototyping platform, thereby testing both the direct and indirect closed-loop identification methods;
- Conclusions and further research perspectives are outlined.

Fractional Calculus Tools used in this Work

In the following, a summary of the most FOC tools used in this work is provided:

- Grünwald-Letnikov definition of the fractional operator:

$$ {}_a\mathscr{D}_t^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{1} $$

  where $a = 0$, $t = kh$, k is the $\#$ of steps and h is step size.

- For real-time applications we consider Oustaloup's approximation method, which allows to obtain a band-limited approximation of a fractional-order differentiator or integrator in the form $s^\alpha \approx H(s)$, where $\alpha \in (-1, 1) \subset \mathbb{R}$.

A transfer function representation of a fractional model with a delay term can be written as

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \mathrm{e}^{-Ls}, \tag{2}$$

where if $\beta_0 = \alpha_0 = 0$, then the static gain of the system is given by $K = b_0/a_0$. The parallel form of the fractional-order PID (FOPID) controller is given by

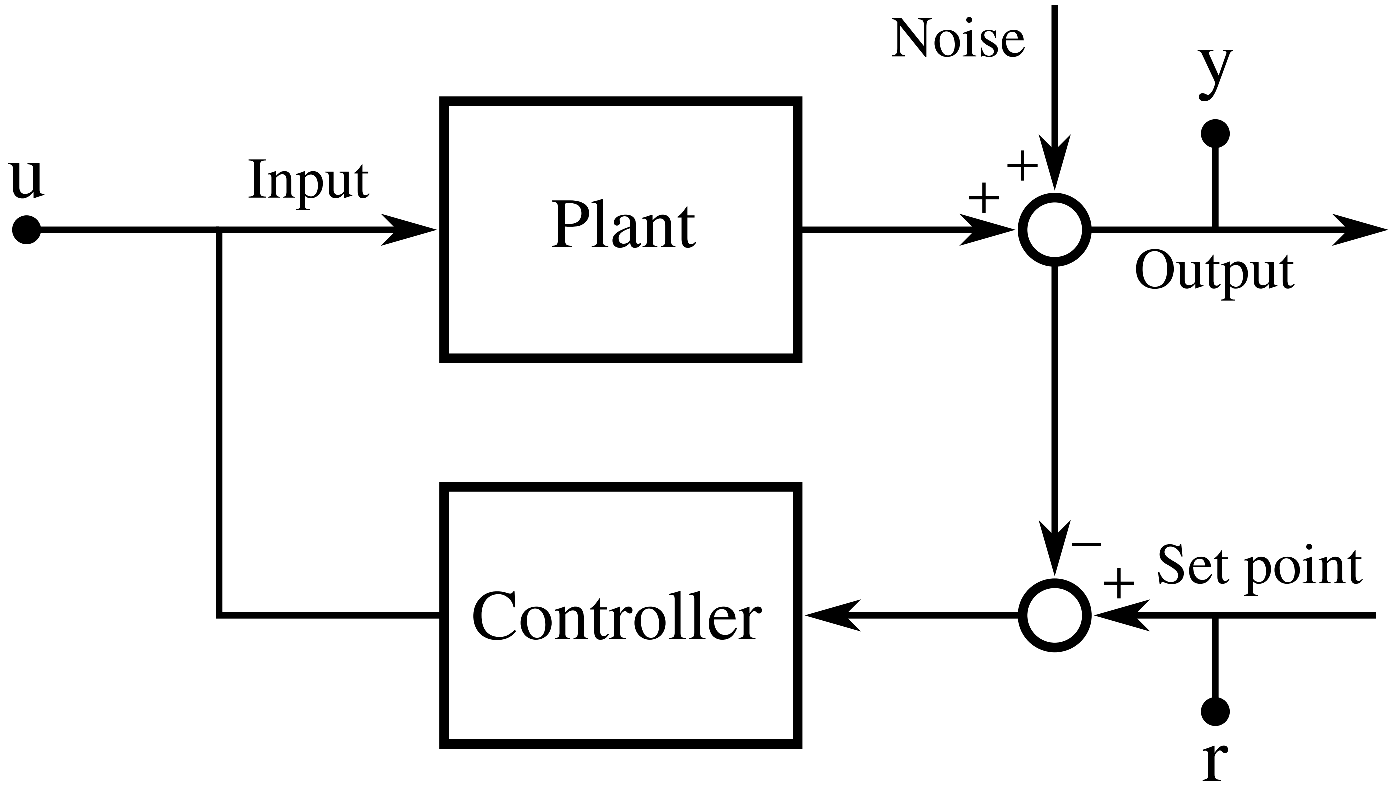
$$C(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu}. \tag{3}$$

In this work we assume, that the closed-loop system is represented by a typical negative unity feedback of the form

$$W(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}, \tag{4}$$

where $C(s)$ is the FOPID controller, and $G(s)$ is the plant under control.

Closed-loop Control System Structure



Closed-loop Identification: The Indirect Approach

- The controller is assumed to be known. The identification data set is given by

$$Z_N^i = \{r(1), y(1), \ldots, r(N), y(N)\}, \tag{5}$$

  where $r(k)$ and $y(k)$ denote the reference signal (set point) and plant output, respectively, and $k = 1, 2, \ldots, N$.

- In this work, we investigate the problem of identification of a closed-loop model in (4), where the parameters of $C(s)$ are known. The model of the plant $G(s)$ can be easily reconstructed, once the parameters of the closed-loop system are obtained.

Closed-loop Identification: The Direct Approach

- The feedback is ignored and open-loop identification is employed. The experimental data set used for identification is given by
$$Z_N^d = \{u(1), y(1), \ldots, u(N), y(N)\}, \qquad (6)$$
where $u(k)$ and $y(k)$ denote the plant input and output signal samples. The structure of the model to be identified is explicitly given by (2).

If we assume, that the controller under investigation is linear, time-invariant and noise free, then the two methods discussed above are equivalent. However, in industrial practice, such a controller is rarely realizable due to, e.g., actuator saturation and measurement noise. Therefore, the selection of the identification approach depends on the availability of necessary measurements.

Parametrization and Identification of the Fractional-order Model

The identification procedure for FO models is carried out by means of an optimization algorithm with the task of determining the set of parameters

$$\theta = \{p_1, p_2, \ldots, q_1, q_2, \ldots\} \tag{7}$$

of the model in (2), where p_i denote the zero and/or pole polynomial coefficients or the delay parameter L, and q_j denote the fractional powers of s. The algorithm uses the least-squares approach, such that the error norm $\left\|e\left(k\right)\right\|_2^2$ is minimized. We consider the output error method, therefore

$$e\left(k\right) = y\left(k\right) - \tilde{y}\left(k\right), \tag{8}$$

where $y(k)$ denotes the experimentally collected system output samples in (5) or (6), and $\tilde{y}(k)$ denotes the simulated system time-domain response to excitation signal samples in either $r(k)$ or $u(k)$. The response of the closed-loop system is computed numerically using a fractional-order differential equation solver based on (1).

```
[idpms, G] = pfid(fid, expr, pms, mpms, op)
```

Input arguments:

- fid—fractional-order identification data set;
- $expr$—a symbolic expression with the FOTF model structure;
- pms—model parameter information structure, the symbolic parameter names must correspond exactly to the ones in $expr$;
- $mpms$—a structure with additional model parameters, the numerical values of which are substituted into $expr$;
- op—additional optimization options.

Output arguments:

- $idpms$—a structure that contains the identified parameters;
- G—the identified fractional-order transfer function, s.t. $idpms \to expr$.

Experimental results: Model and Controller

We consider the following fractional-order transfer function model

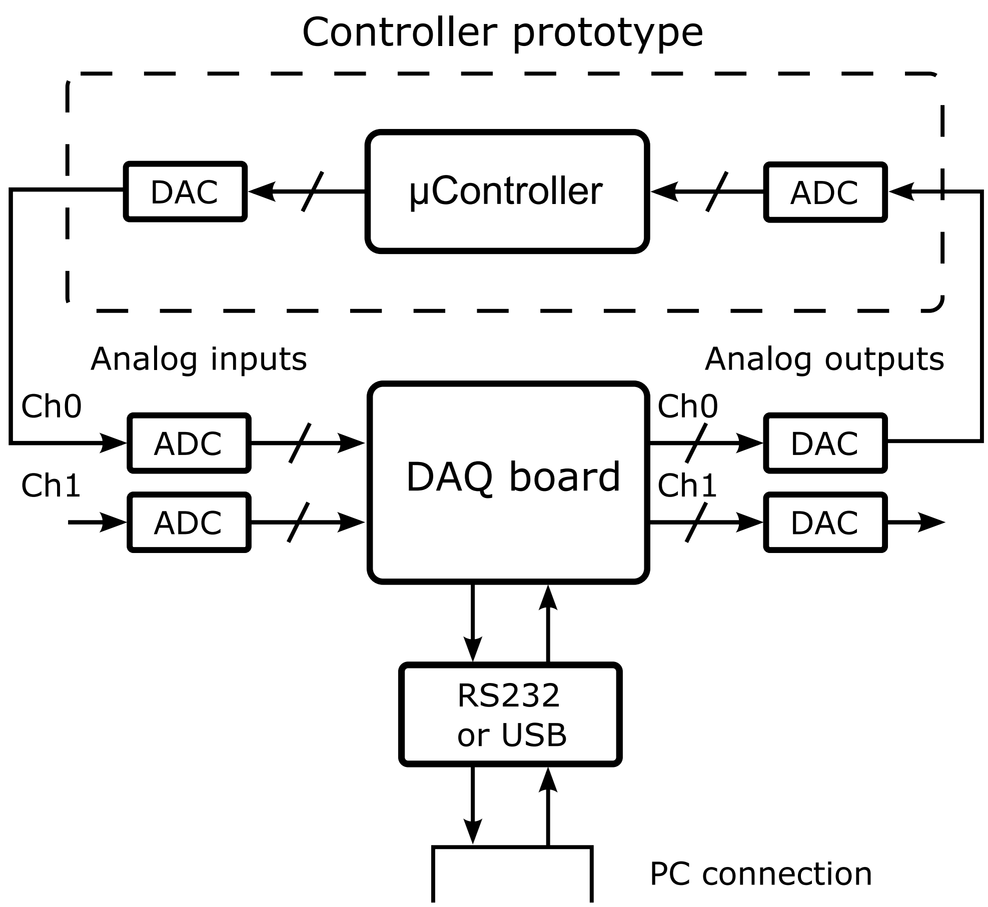
$$G(s) = \frac{1}{0.8s^{2.2} + 0.5s^{0.9} + 1}, \tag{9}$$

which is controlled by means of a fractional-order PID controller, implemented on a hardware prototype, having the form

$$C(s) = 0.01 + \frac{0.53795s^{0.1}}{s} + 0.84749s^{0.75}. \tag{10}$$

A constant reference signal $r(k) = 0.5$ is chosen, and experimental data forming the sets in (5) and (6) is collected. In the following, we supply this data to the identification algorithm with the aim of reconstructing the nominal transfer function in (9) thereby verifying the identification algorithm.

Experimental Real-time Prototyping Platform Structure



Experimental Results: The Indirect Approach

Suppose, that the structure of a fractional-order model to be identified is known and may be parametrized as

$$G_p(s) = \frac{p_1}{p_2 s^{q_1} + p_3 s^{q_2} + p_4}. \tag{11}$$

The closed-loop transfer function in (4) used in the identification procedure is then given by

$$G_{cl}(s) = \frac{p_1 G_{pid}(s)}{s\left(p_2 s^{q_1} + p_3 s^{q_2} + p_4\right) + p_1 G_{pid}(s)},$$

where $G_{pid}(s) = \left(K_p s + K_i s^{1-\lambda} + K_d s^{1+\mu}\right)$ and the parameters of the FOPID controller are assumed to be known and correspond to those in (10).

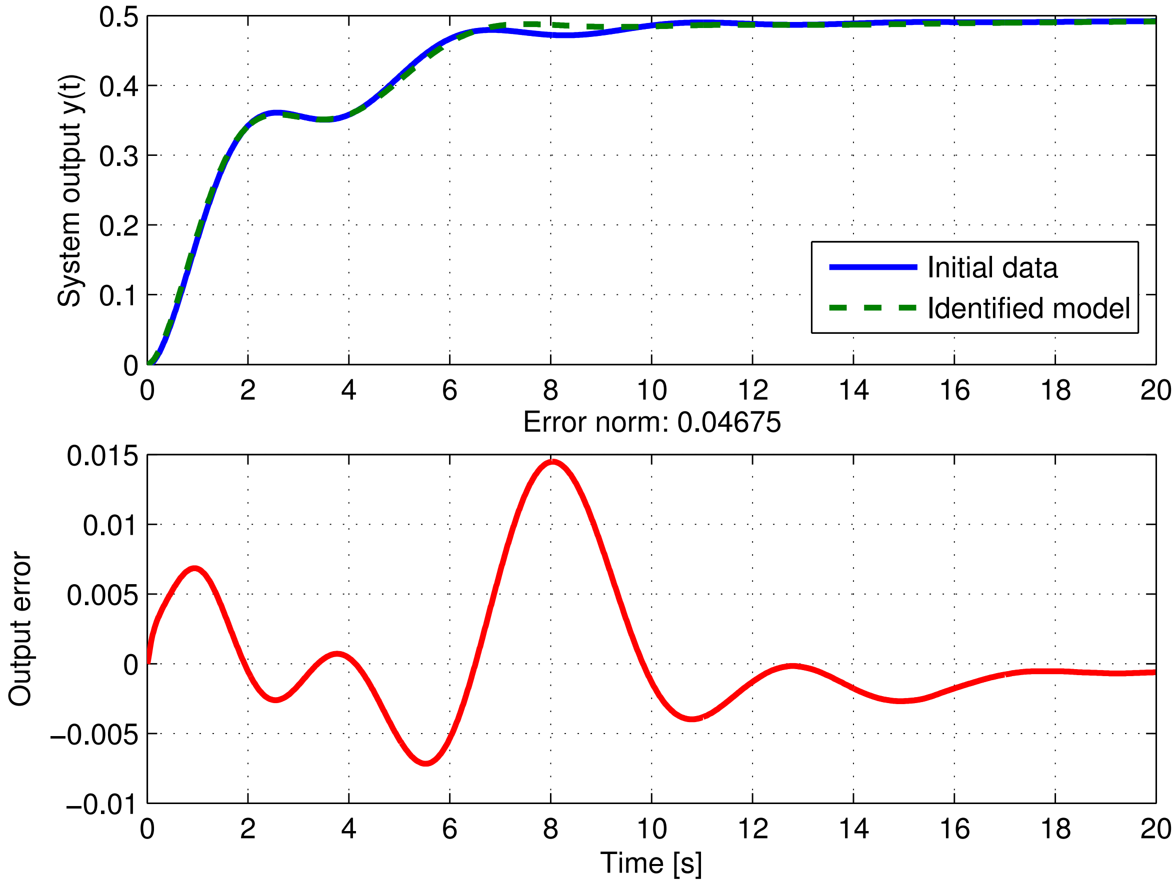
Experimental Results: The Indirect Approach: MATLAB Code and Result

```
% Linear controller parameters
m.Kp = 0.01; m.Ki = 0.53795; m.Kd = 0.84749;
m.Lambda = 0.9; m.Mu = 0.75;

% Closed-loop system
g_cl=['(p1*(Kp*s+Ki*s^(1-Lambda)+Kd*s^(1+Mu)))/'...
      '(s*(p2*s^q1+p3*s^q2+p4)+' ...
      '(p1*(Kp*s+Ki*s^(1-Lambda)+Kd*s^(1+Mu))))'];

% Do the identification
[prms, G] = pfid(fid_ind, g_cl, [], m);
```

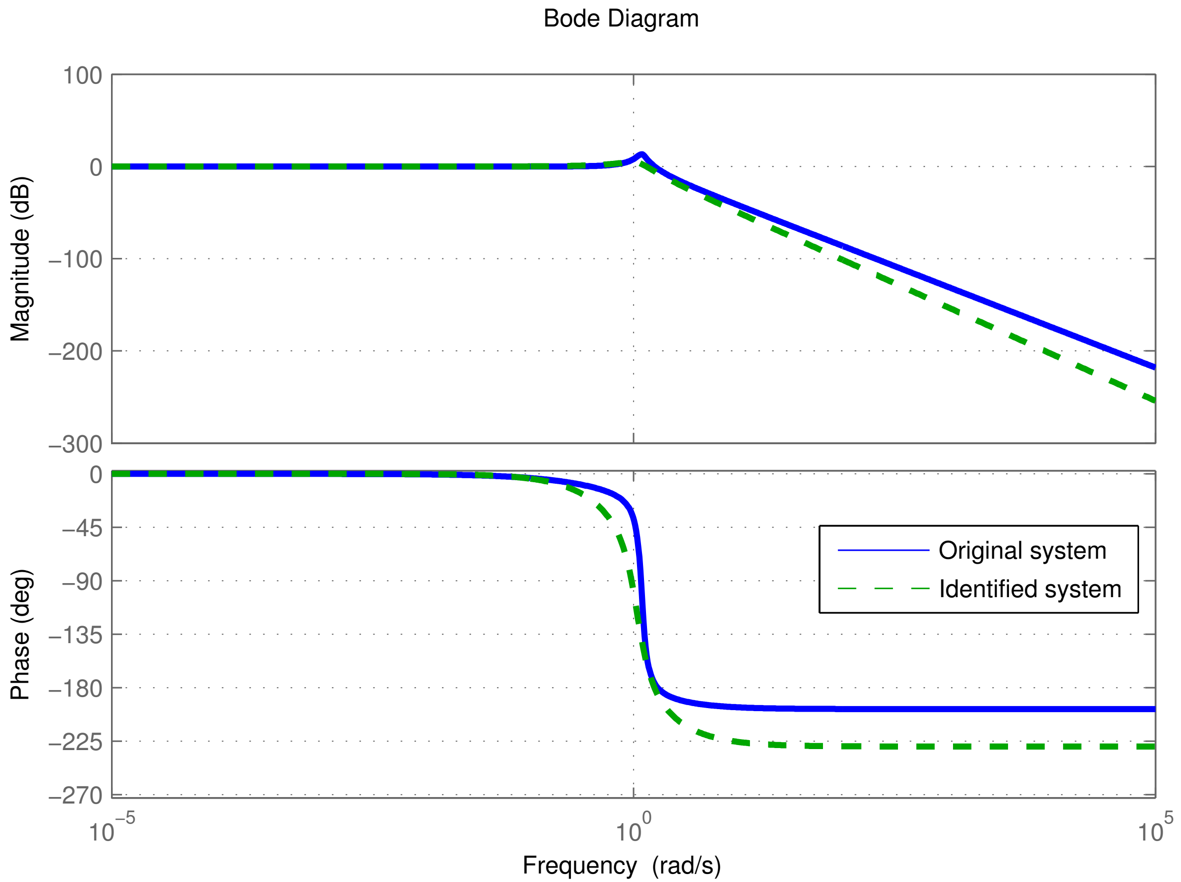
The following model is obtained:

$$G_1(s) = \frac{0.980}{0.886s^{2.550} + 1.328s^{1.254} + 1.000}.$$

Experimental Results: The Indirect Approach: Results (Time Domain)



Experimental Results: The Indirect Approach: Results (Frequency Domain)



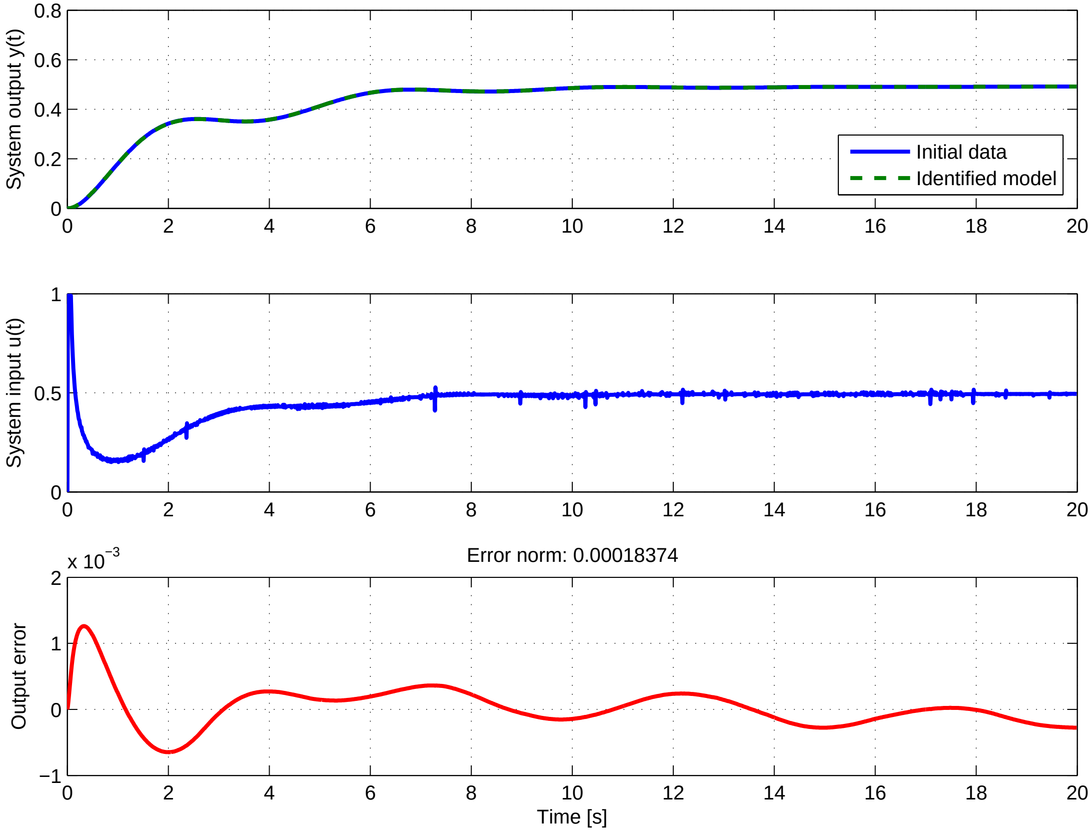
Experimental Results: The Direct Approach

We proceed along the lines of the previous subsection. The model strtucture is once again assumed to be known and of the form (11). Running the same algorithm with this structure yields the model

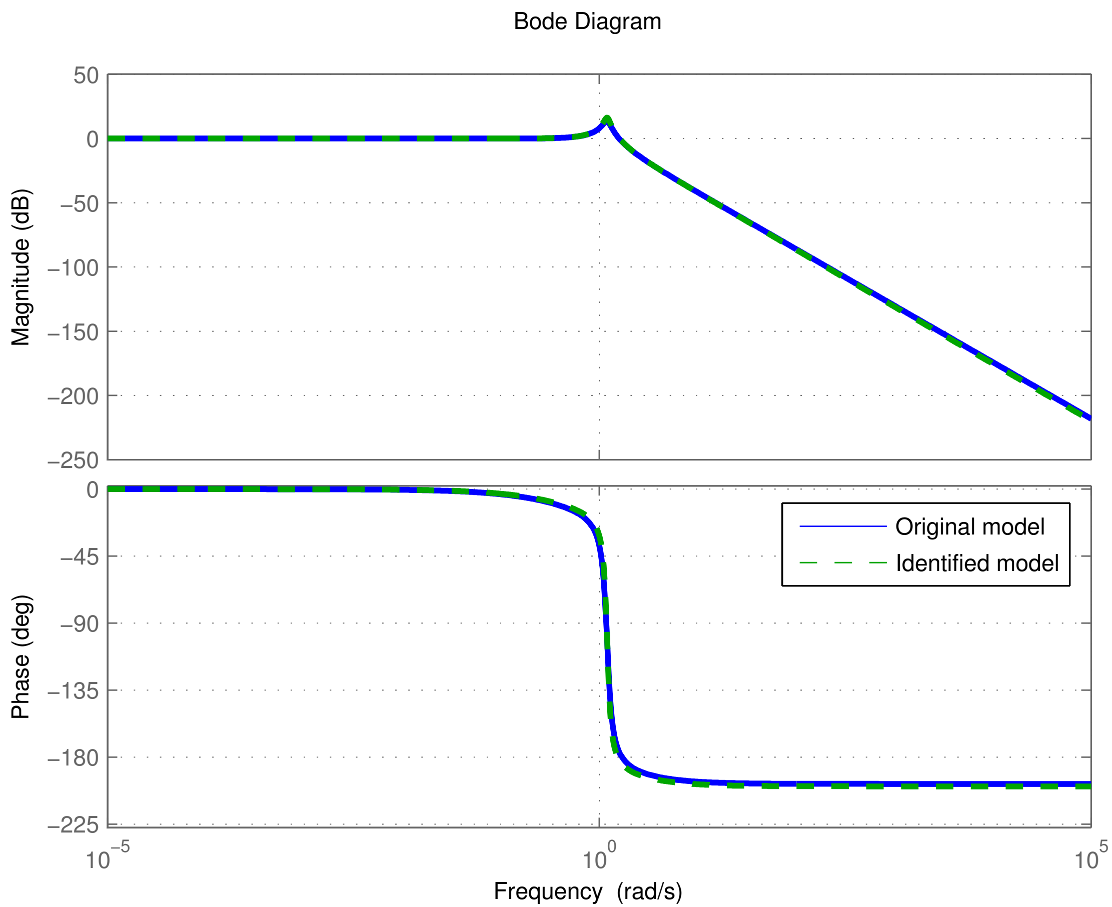
$$G_2(s) = \frac{0.999}{0.779s^{2.218} + 0.465s^{0.916} + 1.000}, \qquad (12)$$

which is very close to the original nominal model with the error norm $\xi = 1.8374 \cdot 10^{-4}$. The result has improved compared to the one in case of the indirect approach. Slight discrepancies in the model parameters can be observed, however, they are in no way essential in terms of capturing the dynamics of the system under study. The results are also illustrated in the following figures.

Experimental Results: The Direct Approach: Results (Time Domain)



Experimental Results: The Direct Approach: Results (Frequency Domain)



Conclusions and further perspectives

- In this paper, we have presented a method for closed-loop system model identification and succesfully validated the algorithm by considering an exemplary fractional control system and two identification approaches;
- The experiment was carried out on a prototyping platform with the controller running on a hardware prototype;
- The quality of identification was superior in case of the direct method;
- Future work in this direction should involve research of identification methods for nonlinear fractional-order models, as well as implementation of corresponding software applications.

FOMCON project: Fractional-order Modeling and Control

FOMCON

- Official website: http://www.fomcon.net/
- Toolbox for MATLAB available;
- An interdisciplinary project supported by the Estonian Doctoral School in ICT.

Thank you for listening!

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