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TALLINN UNIVERSITY OF
TECHNOLOGY

Design of Retuning Fractional PID Controllers for a Closed-loop Magnetic Levitation Control System

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Motivation, contribution, and outline

- In this contribution, we propose an application of FOPID controller-based retuning method of an existing closed-loop control system to the Magnetic Levitation System (MLS) from INTECO. Particular points considered in the talk:
 - Overview of FOC tools used in the contribution;
 - Description and nonlinear model of the MLS;
 - Stability analysis of the FOPID control system, heuristic detection of stability regions, derivation of suboptimal FOPID controller settings;
 - Incorporation of fractional-order dynamics into an existing closed-loop PID-based control system;
 - Experimental results: Application of the method to a real-life system.
- Conclusions.



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The generalized operator

Fractional calculus is a generalization of integration and differentiation to non-integer order operator ${}_a\mathcal{D}_t^\alpha$, where a and t denote the limits of the operation and α denotes the fractional order such that

$${}_a\mathcal{D}_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0, \\ \int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0, \end{cases} \quad (1)$$

where generally it is assumed that $\alpha \in \mathbb{R}$, but it may also be a complex number.



Fractional-order transfer functions

A transfer function representation of a fractional dynamical model may be given by

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_0 s^{\alpha_0}}, \quad (2)$$

where usually $\beta_0 = \alpha_0 = 0$. The system in (2) has a commensurate order γ , such that $\lambda = s^\gamma$, if it can be represented as:

$$H(\lambda) = \frac{\sum_{k=0}^m b_k \lambda^k}{\sum_{k=0}^n a_k \lambda^k}, \quad (3)$$

where n is called the pseudo-order of the system.



Theorem 1. (*Matignon's stability theorem*) *The fractional transfer function $G(s) = Z(s)/P(s)$ is stable if and only if the following condition is satisfied in σ -plane:*

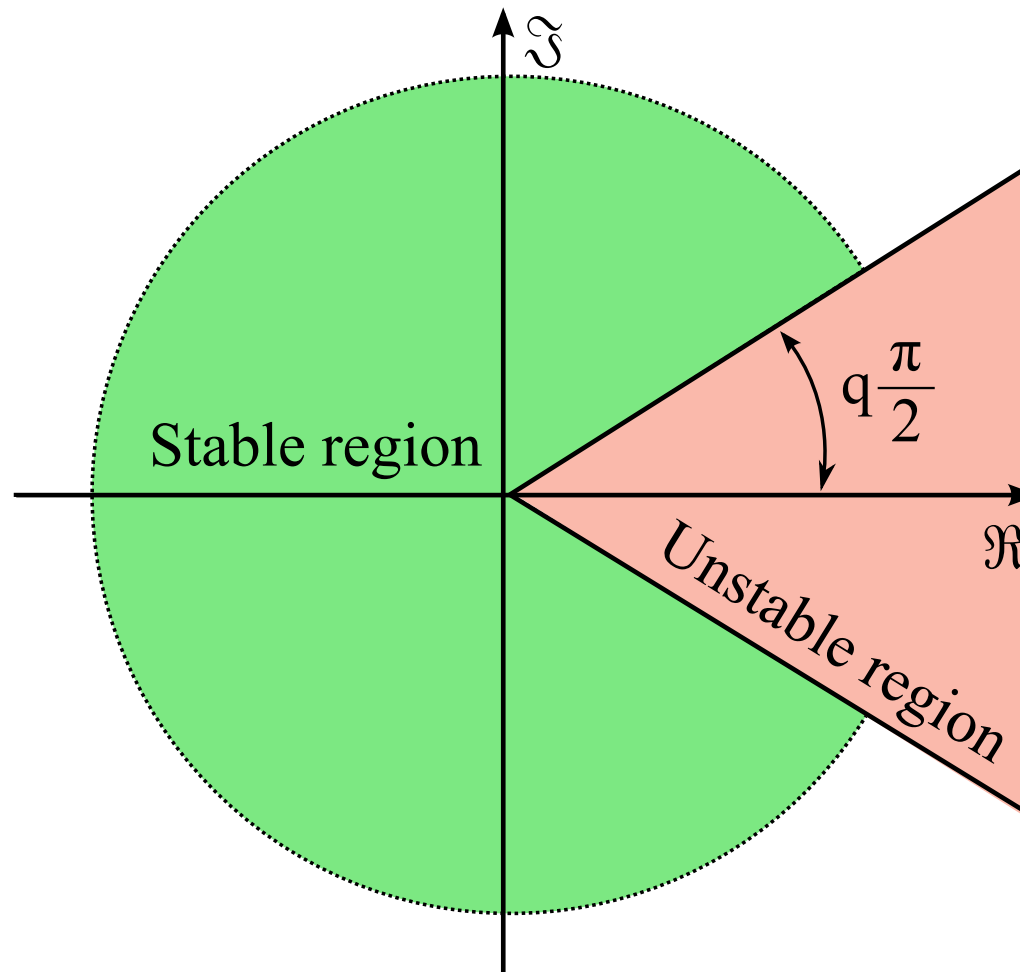
$$|\arg(\sigma)| > q\frac{\pi}{2}, \forall \sigma \in \mathbb{C}, P(\sigma) = 0, \quad (4)$$

where $\sigma := s^q$. When $\sigma = 0$ is a single root of $P(s)$, the system cannot be stable. For $q = 1$, this is the classical theorem of pole location in the complex plane: no pole is in the closed right plane of the first Riemann sheet.

Algorithm summary: Find the commensurate order q of $P(s)$, find a_1, a_2, \dots, a_n in (3) and solve for σ the equation $\sum_{k=0}^n a_k \sigma^k = 0$. If all obtained roots satisfy the condition (4), the system is stable.



Stability regions



Fractional-order Control: $PI^\lambda D^\mu$ controller

The parallel form of the $PI^\lambda D^\mu$ controller is given by

$$C(s) = K_p + K_i s^{-\lambda} + K_d s^\mu. \quad (5)$$

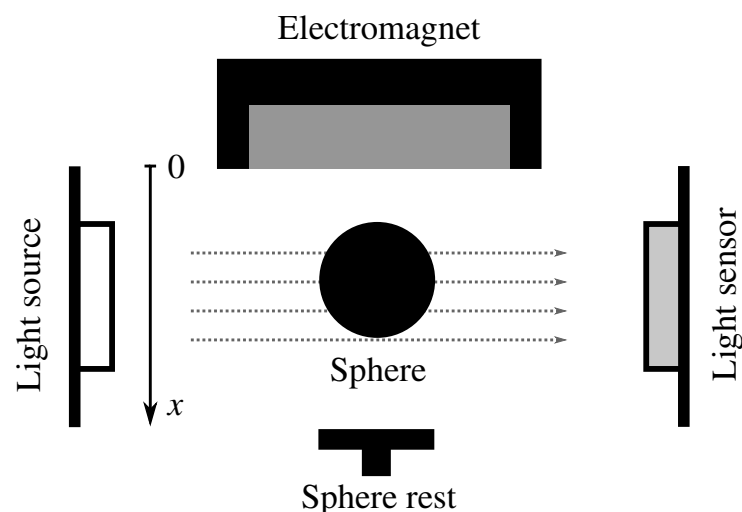
In this work, we consider the negative unity feedback closed loop system of the form

$$W(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}, \quad (6)$$

where $C(s)$ is the $PI^\lambda D^\mu$ controller, and $G(s)$ is the plant under control.



Model of the MLS



We use the following model of the MLS:

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{c(x_1)}{m} \frac{x_3^2}{x_1^2} + g, \\ \dot{x}_3 &= \frac{f_{ip2}}{f_{ip1}} \frac{i(u) - x_3}{e^{-x_1/f_{ip2}}},\end{aligned}\tag{7}$$

where x_1 is the position of the sphere, x_2 is the velocity of the sphere, and x_3 is the coil current, f_{ip1} and f_{ip2} are constants, $c(x_1)$ is a 4th order polynomial and $i(u)$ is a 2nd order polynomial.



Model Linearization and Stability Analysis

We will analyze the stability of linear approximation around a working point (u_0, x_{10}) . We linearize the model in (7) and obtain the following transfer function of the MLS:

$$G_M(s) = \frac{b_3 a_{23}}{s^3 - a_{33}s^2 - a_{21}s + a_{21}a_{33}}, \quad (8)$$

where

$$\begin{aligned} a_{21} &= \frac{(-2c_4 x_{10}^4 - c_3 x_{10}^3 + c_1 x_{10} + 2c_0)x_{30}^2}{m x_{10}^3}, \\ a_{23} &= -\frac{2c(x_{10})x_{30}}{m x_{10}^2}, \quad a_{33} = \frac{i(u_0) - x_{30}}{f_{ip1}} e^{x_{10}/f_{ip2}}, \\ b_3 &= \frac{f_{ip2}}{f_{ip1}} (k_1 + 2k_2 u_0) e^{x_{10}/f_{ip2}}. \end{aligned}$$



Characteristic Polynomial of the Closed-loop Control System

To analyze the stability of the closed-loop fractional-order control system in (6) we shall use Matignon's theorem. The characteristic polynomial is given by

$$\begin{aligned} Q(s) = & s^{3+\lambda} - a_{33}s^{2+\lambda} - a_{21}s \\ & + (b_3a_{23}K_p + a_{21}a_{33})s^\lambda \\ & + b_3a_{23}K_d s^{\lambda+\mu} + b_3a_{23}K_i. \end{aligned} \quad (9)$$

Thus, a point of the form $(K_p, K_i, K_d, \lambda, \mu)$ in the $PI^\lambda D^\mu$ parameter space can be selected and the stability of the closed-loop control system can be verified.

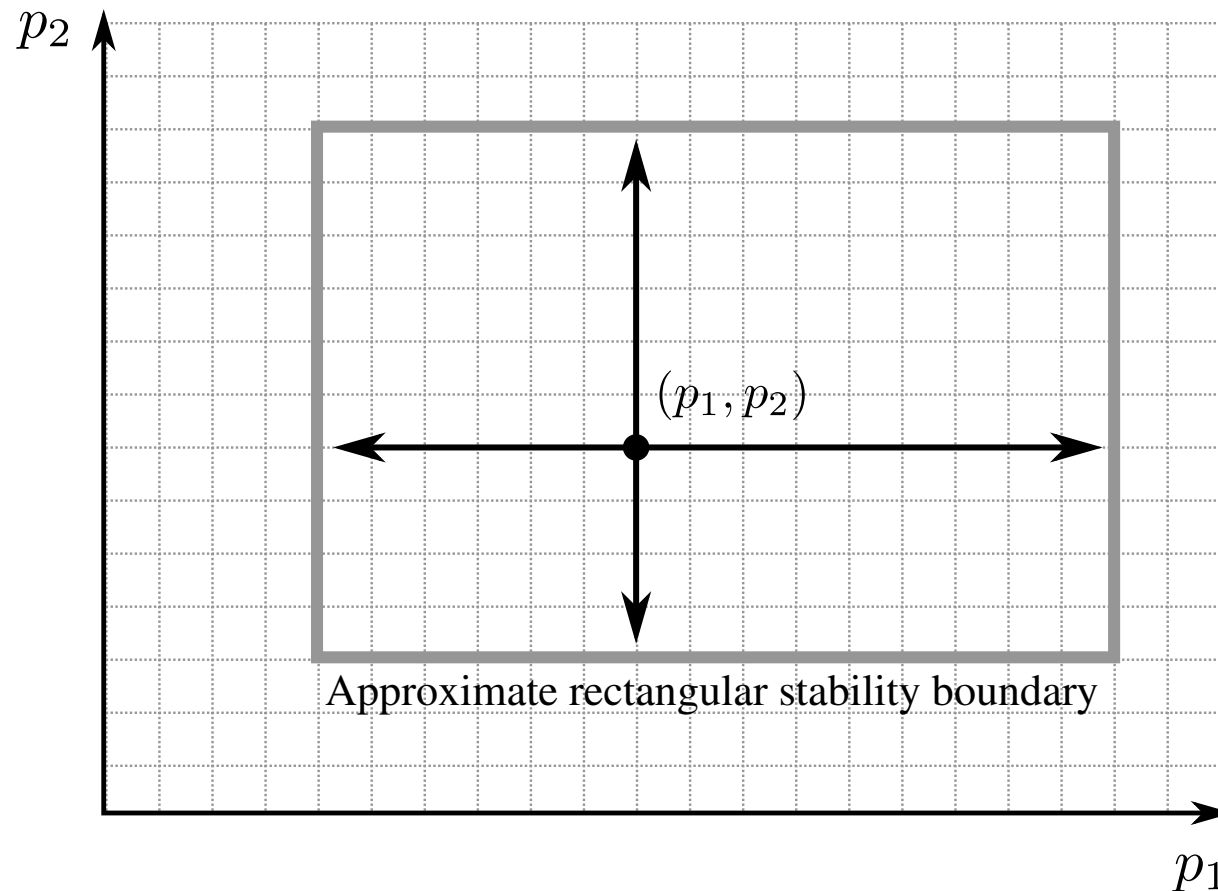


Determination and Optimization of Stabilizing FOPID Controllers

- To determine stabilizing controllers a randomized method may be used, where FOPID controller parameters are randomly selected from $K_p \in [K_p^l, K_p^u]$, $K_i \in [K_i^l, K_i^u]$, $K_d \in [K_d^l, K_d^u]$, $\lambda \in [\lambda^l, \lambda^u]$, $\mu \in [\mu^l, \mu^u]$.
- The choice of λ and μ must lead to a commensurate-order system, since only then the results of the stability test are reliable, otherwise they are only approximate.
- For example, one can choose a minimum commensurate order $q = 0.01$.
- The stability region is then heuristically determined by locating approximate rectangular boundaries and doing a sweep within the resulting grid.



Determination and Optimization of Stabilizing FOPID Controllers: Illustration



Determination and Optimization of Stabilizing FOPID Controllers: Design

- Once the procedure is complete, stable parameter ranges are obtained for all controller parameter pairs and may be used in FOPID controller optimization as lower and upper bounds for corresponding controller parameters.
- Optimizing only two parameters at a time can be beneficial from the perspective of conditioning the problem.
- It is difficult to impose feasible robustness specifications for the MLS in the frequency domain.
- The performance of the system will be evaluated experimentally, settling time τ_s , percent overshoot θ , and percent maximum deviation from reference due to disturbance θ_d are used as performance measures. We consider time-domain simulations of the nonlinear model in (7) and minimize a cost defined by

$$ISE = \int_0^t |e(\tau)| d\tau. \quad (10)$$



The FOPID Controller Retuning Method

Consider the original integer-order PID controller of the form

$$C_{PID}(s) = K_P + K_I s^{-1} + K_D s. \quad (11)$$

Let $C_R(s)$ be a controller of the form

$$C_R(s) = \frac{K_2 s^\beta + K_1 s^\alpha - K_D s^2 + (K_0 - K_P)s - K_I}{K_D s^2 + K_P s + K_I}, \quad (12)$$

where the orders α and β are such, that $-1 < \alpha < 1$ and $1 < \beta < 2$. The $PI^\lambda D^\mu$ controller resulting from a classical PID controller will have the following coefficients

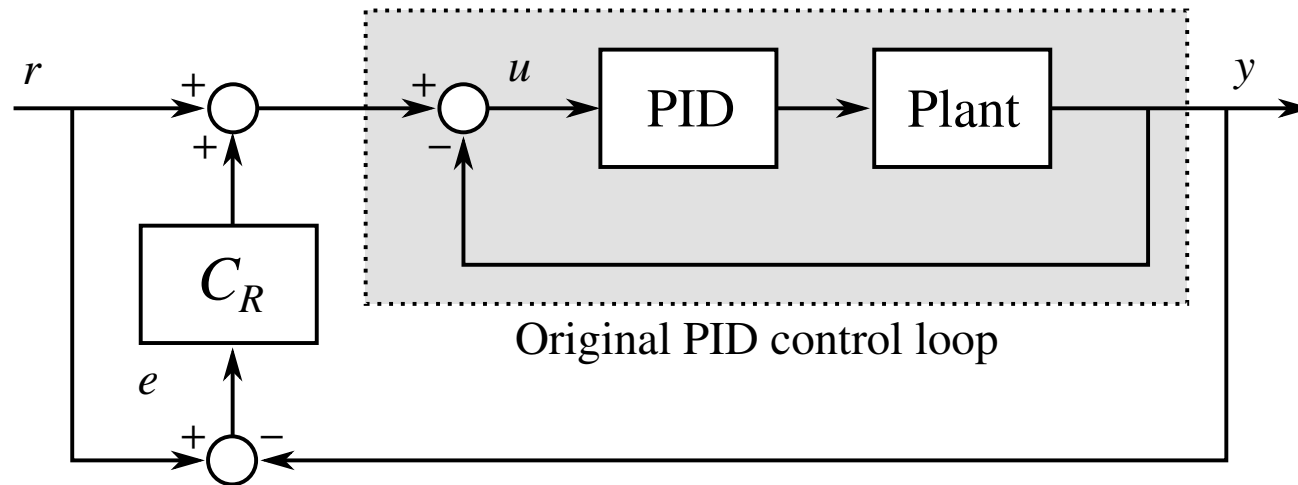
$$K_P^* = K_0, \quad K_I^* = K_1, \quad K_D^* = K_2, \quad (13)$$

and the orders will be

$$\lambda = 1 - \alpha, \quad \mu = \beta - 1. \quad (14)$$



The FOPID Controller Retuning Method: Illustration

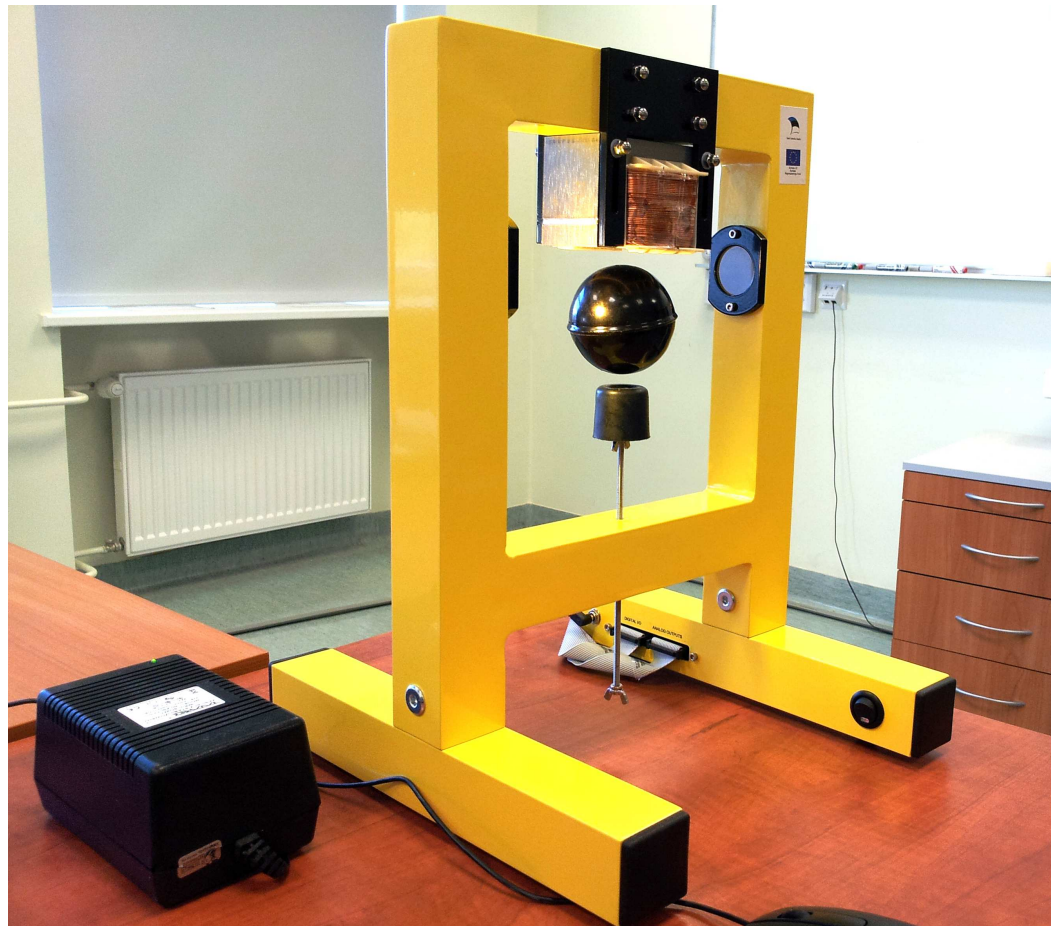


It can be shown, that this structure may be replaced by a feedback of the form (6), where

$$C(s) = (C_R(s) + 1) \cdot C_{PID}(s). \quad (15)$$



Experimental Results: Real-life MLS Model



Experimental Results: Nonlinear Model Identification: Original Control Loop

- Because MLS is open-loop unstable, only closed-loop identification is applicable. Our approach is to use the existing PID control loop with

$$K_P = -39, \quad K_I = -10, \quad K_D = -2.05 \quad (16)$$

provided by INTECO. It should be noted, that a constant input $u_c = 0.38$ is added to the control law $u_{PID}(t)$ in (16), that is the full control law $u(t)$ is such that $u(t) = u_{PID}(t) + u_c$.

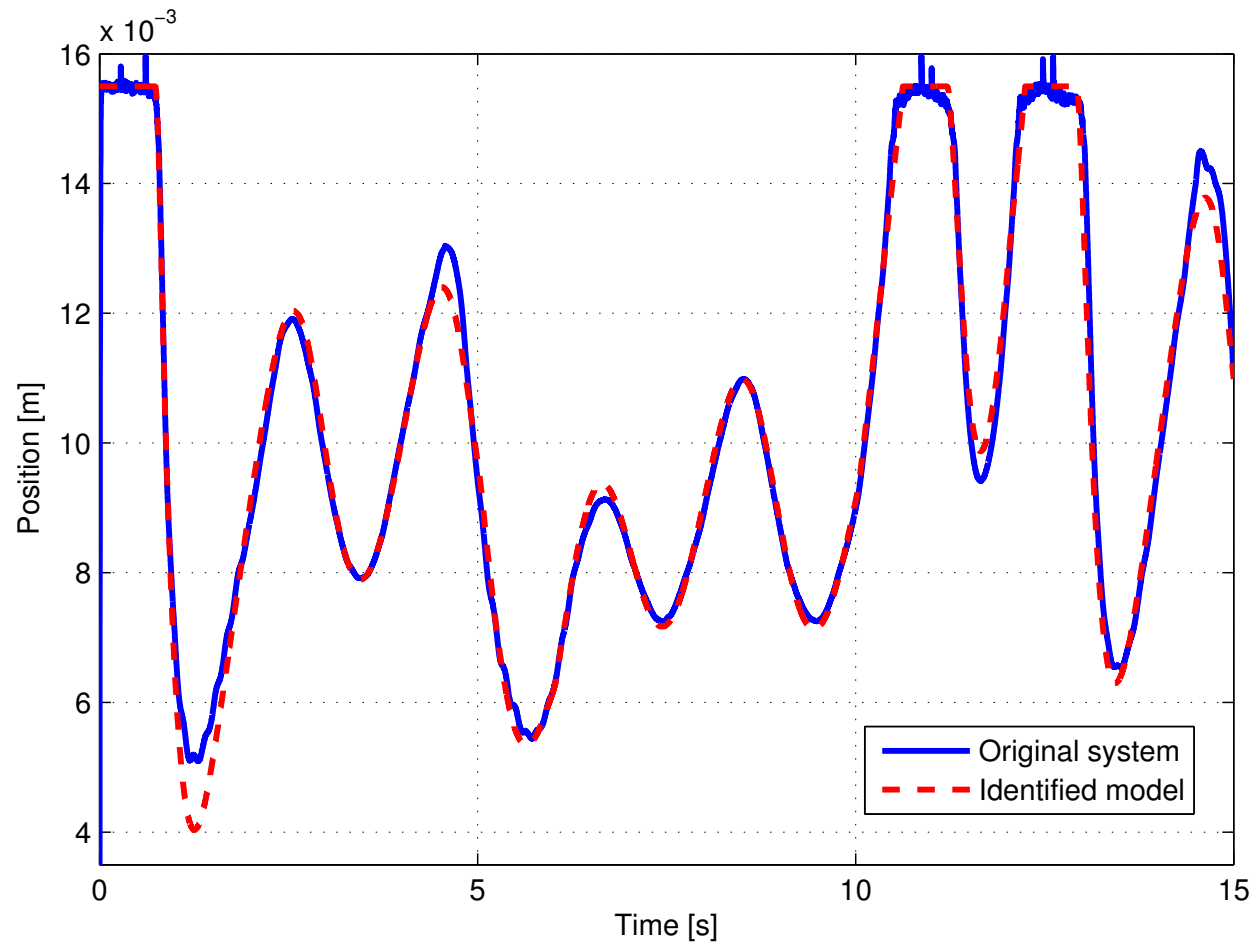
- With the conventional PID controller the following results are achieved:

$$\tau_s = 3.34 \text{ s}, \quad \theta = 66.0\%, \quad \theta_d = 60.6\%.$$

- In order to determine the values of the parameters, we employ time-domain simulations and minimize the model output error by means of the least-squares method.



Experimental Results: Nonlinear Model Identification Accuracy



Experimental Results: Design of FOPID Controllers

- We first obtain a linear approximation. We choose a working point $u_0 = 0.3726, x_{10} = 9.84 \cdot 10^{-3}$ and obtain

$$G_M(s) = -\frac{1788}{s^3 + 34.69s^2 - 1737s - 60240}. \quad (17)$$

- Next, we apply the random controller generation method. First, we randomly generate FOPID controllers using the ranges $K_p \in [-100, 0]$, $K_i \in [-50, 0]$, $K_d \in [-25, 0]$, $\lambda \in [0.8, 1.2]$, $\mu \in [0.5, 1.0]$. On the average, about 20 out of 100 tested controllers are found to produce a stable closed-loop system.
- After inspection, three of them are selected for the optimization phase. For each controller in this set, we find stability boundaries in different parameter planes, that is in (K_p, K_i) , (K_p, K_d) , and (K_i, K_d) , so that we can obtain a wider set of results.



Experimental Results: Optimization of FOPID Controllers

The initial controllers are thus

$$\begin{aligned}C_1(s) &= -42.8642 - 18.5653s^{-1.06} - 3.0559s^{0.94}, \\C_2(s) &= -54.3649 - 47.6078s^{-0.82} - 6.5436s^{0.98}, \\C_3(s) &= -45.3118 - 4.24932s^{-0.86} - 3.51115s^{0.98}.\end{aligned}$$

We then proceed directly to the optimization procedure. Pairs of controller gains are tuned constrained by the stability region. The results are as follows:

$$\begin{aligned}C_1^*(s) &= -45.839 - 18.504s^{-1.06} - 3.0559s^{0.94}, \\C_2^*(s) &= -54.444 - 47.6078s^{-0.82} - 3.7773s^{0.98}, \\C_3^*(s) &= -45.3118 - 4.916s^{-0.86} - 2.9074s^{0.98}.\end{aligned}$$



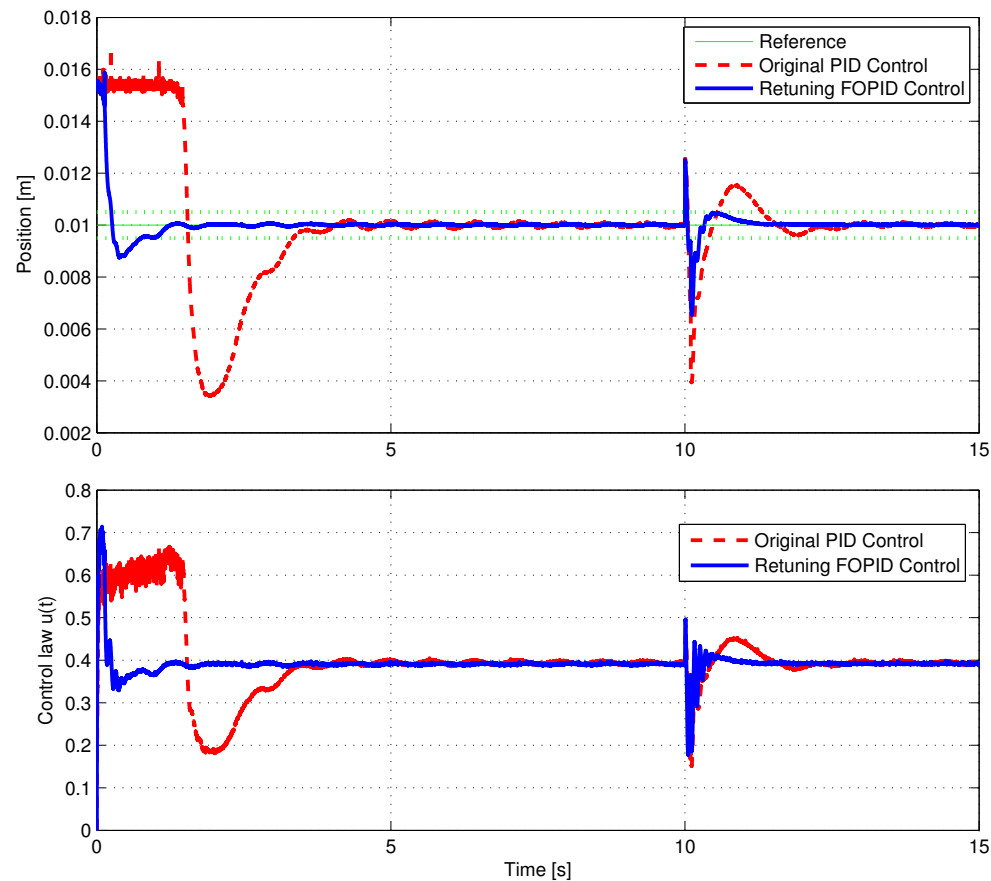
Experimental Results: Controller Performance Comparison

| FOPID | $\tau_s[s]$ | $\theta[\%]$ | $\theta_d[\%]$ | FOPID* | $\tau_s[s]$ | $\theta[\%]$ | $\theta_d[\%]$ |
|----------|-------------|--------------|----------------|------------|-------------|--------------|----------------|
| $C_1(s)$ | 1.85 | 24.0 | 60.3 | $C_1^*(s)$ | 1.68 | 14.8 | 56.4 |
| $C_2(s)$ | 1.39 | 19.4 | 37.5 | $C_2^*(s)$ | 0.86 | 11.6 | 34.6 |
| $C_3(s)$ | 4.68 | 14.6 | 55.7 | $C_3^*(s)$ | 3.84 | 15.0 | 58.3 |

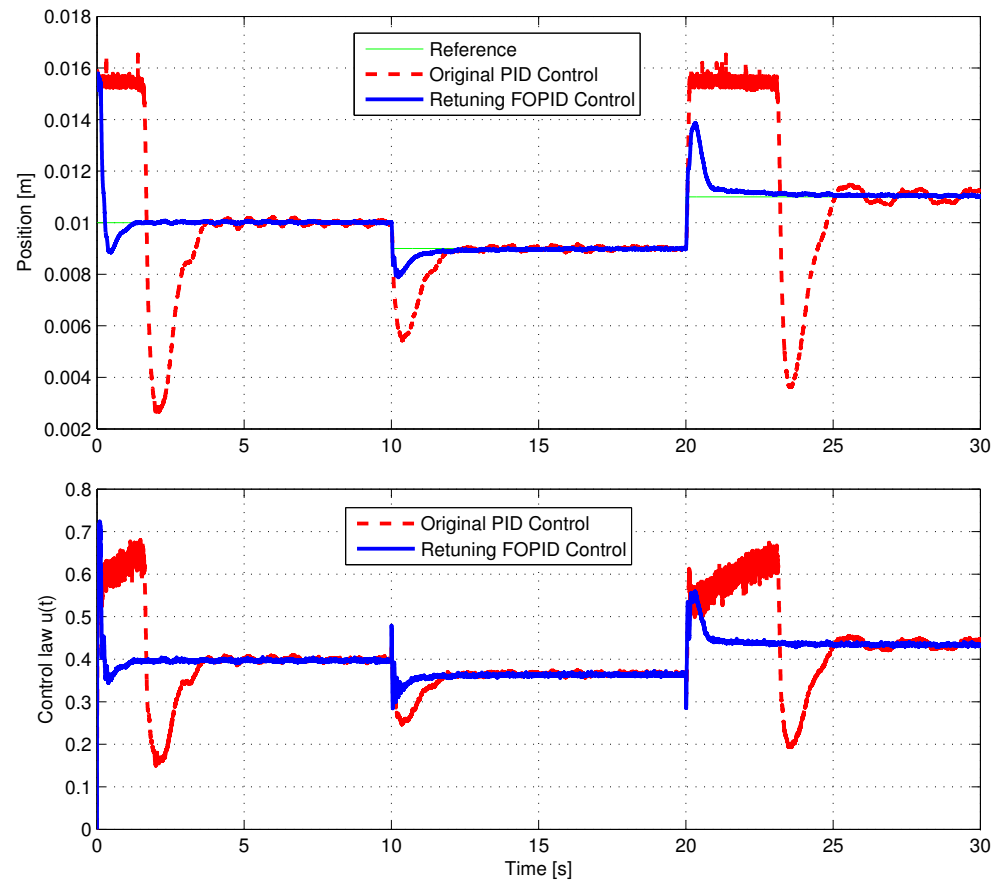
- It can be seen, that the best performance is achieved, when controller $C_2^*(s)$ is used.
- The controller $C_3(s)$ outperforms the original PID only in terms of overshoot, while $C_3^*(s)$ offers similar settling time with a much smaller overshoot.



Experimental Results: Controller Performance: Step experiment



Experimental Results: Controller Performance: Variable Set-point



Conclusions

- In this paper, we have presented a method for FOPID controller design that allows incorporating fractional-order dynamics into existing PID control loops.
- An unstable plant, namely the MLS system was considered and a nonlinear model of this plant was identified from a closed-loop experiment.
- Linear analysis methods were employed to determine stabilizing FOPID controllers and stability boundaries in two-dimensional parameter planes thereof.
- The controllers were then evaluated, and those with best performance were optimized.
- In all cases, the optimization procedure enhanced the performance of the control loop.
- Virtually all retuning controllers offer superior performance compared to the original control loop, thereby establishing the validity of the proposed approach.



FOMCON project: Fractional-order Modeling and Control



- Official website: <http://fomcon.net/>
- Toolbox for MATLAB available;
- An interdisciplinary project supported by the Estonian Doctoral School in ICT and Estonian Science Foundation grant nr. 8738.



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<http://www.itakadeemia.ee/>



Questions?

Thank you for listening!



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