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Lecture 4: Control of Energy Storage Devices

This lecture focuses on management and control of energy storage devices. We will consider several examples in which these devices are used for energy balancing, load leveling, peak shaving, and energy trading.

Basic parameters of storage devices

Two key parameters of energy storage devices are *energy density*, which is the capacity per unit mass or volume, and *power density*, which is the maximum output power per unit mass or volume. Common energy storage technologies include:

- ✓ **Mechanical**: hydroelectric energy storage (pumped storage)[1], flywheels, compressed air, thermal storage;
- ✓ **Electrochemical**: rechargeable batteries, flow batteries, fuel cells;
- ✓ **Electrical**: capacitors, inductors, superconducting magnetic coils.

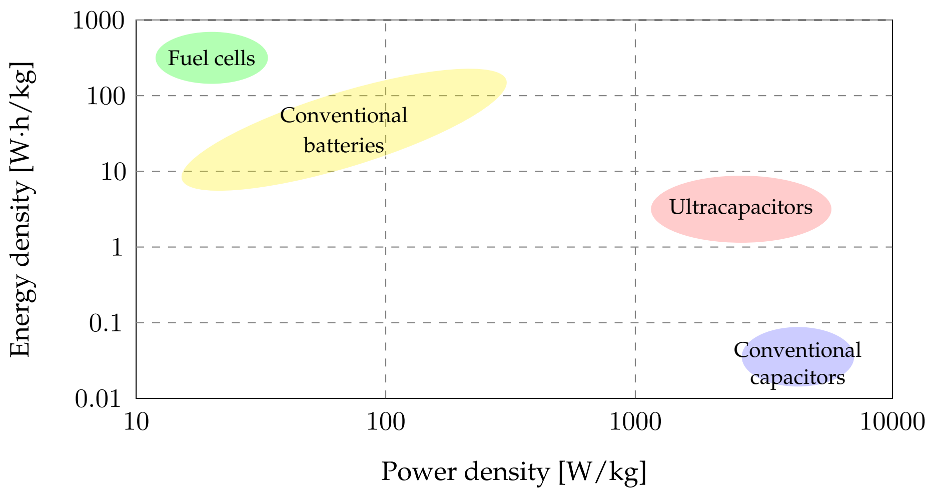


Figure 1: Energy density vs. power density.

Storage devices with high power density are crucial for stability of electric power systems. A classic example is the kinetic energy stored in the rotors of synchronous generators. As explained in previous lectures, this kinetic energy supports instantaneous changes in the load, and is essential for regulating the grid frequency. Today, one possible challenge associated with renewable energy integration is the low *rotational inertia* of renewable energy sources, which in several cases may lead to stability problems [1, 2]. To address this challenge the total system inertia may be increased by means of additional storage devices.

[1]Today the largest form of energy storage used in power grids is pumped storage in hydroelectric dams.

Storage devices with high capacity are mostly used for energy shifting and energy balancing. The main idea is to store surplus energy at times when the power demand is low, and then to use it when the main source cannot supply the energy needed, or when generation is difficult or expensive. Typical applications in power systems include:

- ✓ *Energy balancing*, *Load leveling*, or *Peak shaving*. In electric power systems the load is constantly varying. Storage devices may be used to shift generation from times of peak load to off-peak hours. This lowers the peak of the generated power, and improves the overall system efficiency.
- ✓ *Renewable energy integration*. Renewable sources provide variable power that is not always matched to the load. If the energy generated by renewable sources cannot be consumed immediately it can be stored and used later. Energy storage technologies seem to be essential for large-scale integration of renewable sources [3].
- ✓ *Energy trading*. Storage devices enable to buy energy at a low price, and then to sell this energy at a higher price. In addition to generating profit this also helps to match the power supply to the power demand, and stabilizes the energy cost.
- ✓ *Emergency preparedness*. Storage devices may supply energy in case of a malfunction in the generation or transmission systems. This function is vital for sensitive facilities such as hospitals, military bases, etc.

Basic dynamic model of a storage device

Consider a grid-connected storage device, as shown in Fig. 2.

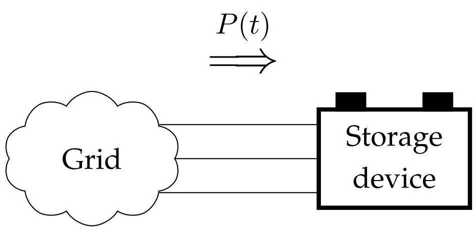


Figure 2: A grid-connected storage device.

A basic dynamic model of the device is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \begin{cases} \eta_c(E,P)P, & P \geq 0, \\ \eta_d^{-1}(E,P)P, & P < 0, \end{cases} \tag{1}$$

where

- ✓ $E(t)$ is the *stored energy*;
- ✓ $P(t)$ is the total power flowing into the device;
- ✓ $\eta_c(E,P)$ is the *charging efficiency*;
- ✓ $\eta_d(E,P)$ is the *discharging efficiency*.

Both $\eta_c(E,P)$ and $\eta_d(E,P)$ have values in the range $(0,1]$.
As an example assume that $P = -|P|$ and $\eta_d = 0.8$. In this case

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{1}{0.8}P = -1.25|P|. \tag{2}$$

The energy derivative is higher (in absolute value) than the power flowing into the grid.

In addition, the *round trip* efficiency (η_r) is the efficiency of a charge-discharge cycle. Consider a device which is charged from E_{low} to E_{high}, and then discharged from E_{high} to E_{low}. The round trip efficiency is the ratio between the energy extracted from the device to the energy stored in the device:

$$\eta_r = \frac{-\int_{P(t)<0} P(\tau)\mathrm{d}\tau}{\int_{P(t)\geq 0} P(\tau)\mathrm{d}\tau}. \tag{3}$$

Changing the integration variable based on (1) yields

$$\eta_r = \frac{\int_{E_{low}}^{E_{high}} \eta_d(E,P)\mathrm{d}E}{\int_{E_{low}}^{E_{high}} \eta_c^{-1}(E,P)\mathrm{d}E}, \tag{4}$$

and if η_c and η_d are constants then

$$\eta_r = \frac{\eta_d \int_{E_{low}}^{E_{high}} \mathrm{d}E}{\eta_c^{-1} \int_{E_{low}}^{E_{high}} \mathrm{d}E} = \frac{\eta_d \left(E_{high} - E_{low}\right)}{\eta_c^{-1} \left(E_{high} - E_{low}\right)} = \eta_c \eta_d. \tag{5}$$

In this case the round-trip efficiency is the multiplication of the charging and discharging efficiencies, as expected.

Managing storage devices connected to non-controllable power sources

Consider a power system with a generator and a load that are non-controllable. A classic example is a standalone system powered by a photovoltaic source, as found in many commercial applications (streetlights, bus stations ...). Another example may be a remote area in a power grid that includes mainly loads and renewable sources. In such systems the generated power is usually different from the momentary load, so a storage device must be used for energy balancing.

As an example consider the simple power system shown in Fig. 3. The system includes a photovoltaic array with a given output power $P_g(t)$, a load with a given power $P_L(t)$, and a storage device with power $P_s(t)$. The storage device is assumed to be lossless.

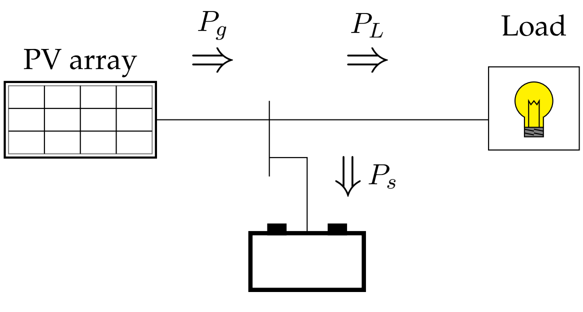


Figure 3: Solar cell connected to a storage device and load.

The generated power is given by

$$P_g(t) = \begin{cases} P_L(t), & \text{if } E \geq E_{\max} \text{ and } P_{pv}(t) \geq P_L(t), \\ P_{pv}(t), & \text{otherwise,} \end{cases} \tag{6}$$

where $P_{pv}(t)$ is the maximum power point of the photovoltaic array, and $E_{\max}$ is the capacity of the storage device. According to this equation if the storage device is full then the surplus power must be lost. As a result, the power flowing into the storage device is

$$P_s(t) = P_g(t) - P_L(t) = \begin{cases} 0, & \text{if } E \geq E_{\max} \text{ and } P_{pv}(t) \geq P_L(t), \\ P_{pv}(t) - P_L(t), & \text{otherwise,} \end{cases} \tag{7}$$

and the stored energy is

$$E(t) = \int_0^t P_s(\tau)\mathrm{d}\tau = \int_0^t \left(P_g(\tau) - P_L(\tau)\right)\mathrm{d}\tau. \tag{8}$$

Typical waveforms are illustrated in Fig. 4.

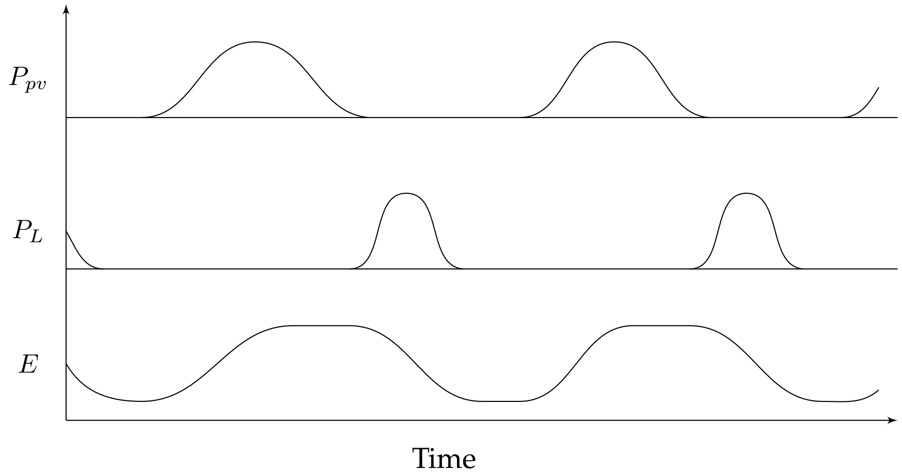


Figure 4: Typical waveforms for a system with a storage device and a non-controllable power source. If the storage device is full then the surplus power must be lost.

Managing storage devices connected to controllable power sources

Consider now a system consisting of generators, storage devices and loads, in which the generated power can be controlled. In such a system it is necessary to decide at every moment how much energy should be generated, and how much energy should be stored. The best mix is found by solving an optimization problem, in which the objective is to maximize the efficiency or to minimize the overall cost. Typical applications include:

✓ *Grid energy storage*. The storage device is connected to the grid, and operates alongside traditional generators. This enables to minimize the total cost, the total fuel consumption, or the peak of the generated power. The problem here is to decide how much energy to store, and when to store it.

✓ *Hybrid electric vehicles*. The storage device, often a battery, is connected to a fueled engine. When the mechanical load is low energy is stored, and when the mechanical load is high the stored energy is transferred to the load. This enables the main engine to operate with a lower peak power and at a higher efficiency. The challenge here is again to decide how much energy to store, and when to store it.

We will now develop the optimal energy management strategy in a system with a single controllable generator and a single storage device, as shown in Fig. 5.

To simplify the analysis it is assumed that the load profile can be estimated with reasonable accuracy, meaning that the power $P_L(t)$ consumed over the time interval $[0, T]$ is known. This assumption is justified for some types of loads, but may not apply for others.

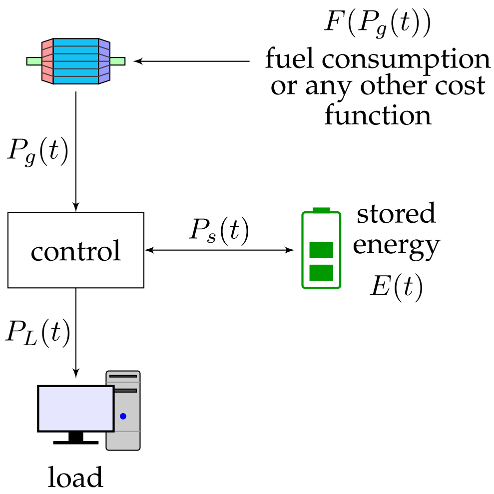


Figure 5: A general system consisting of a controllable generator (source), a load, and an energy storage device.

The generator has an output power $P_g(t)$ that can be controlled, and is characterized by a cost function $F(P_g)$ that is twice differentiable, monotonically increasing and strictly convex. This function may represent fuel consumption or another cost. The generated energy is defined as

$$E_g(t) = \int_0^t P_g(\tau)\mathrm{d}\tau, \tag{9}$$

and the total cost is

$$F_{tot} = \int_0^T F(P_g(\tau))\mathrm{d}\tau. \tag{10}$$

Furthermore we define the load energy as

$$E_L(t) = \int_0^t P_L(\tau)\mathrm{d}\tau. \tag{11}$$

The storage device is assumed to be ideal with $\eta_c = \eta_d = 1$. The power flowing into the device is $P_s(t) = P_g(t) - P_L(t)$, and the stored energy is given by

$$E(t) = \int_0^t P_s(\tau)\mathrm{d}\tau = \int_0^t \left(P_g(\tau) - P_L(\tau)\right)\mathrm{d}\tau. \tag{12}$$

Following (9), (11) and (12) we have

$$E_g(t) = E(t) + E_L(t). \tag{13}$$

In addition, the stored energy is limited by the device capacity, such that

$$0 \le E(t) \le E_{\max}. \tag{14}$$

The challenge is to determine the generated power $P_g(t)$ that minimizes the total cost:

$$\begin{aligned} \text{minimize} \quad & F_{tot} = \int_0^T F(P_g(t))\mathrm{d}t, \\ \text{subject to} \quad & E(t) = \int_0^t \left(P_g(\tau) - P_L(\tau)\right)\mathrm{d}\tau, \quad 0 \le E(t) \le E_{\max}, \\ & E(0) = E_i, \quad E(T) = E_f. \end{aligned} \tag{15}$$

where the constants E_i and E_f denote the energy stored at the initial and final times. Using (13), this problem may be reformulated as follows:

$$\begin{aligned} \text{minimize} \quad & F_{tot} = \int_0^T F(P_g(\tau))\mathrm{d}\tau, \\ \text{subject to} \quad & E_L(t) \leq E_g(t) \leq E_L(t) + E_{\max}, \\ & E_g(0) = E_i, \quad E_g(T) = E_f + E_L(T), \\ & P_g(t) = \frac{\mathrm{d}}{\mathrm{d}t} E_g. \end{aligned} \tag{16}$$

In this form we search for a bounded function $E_g(t)$ that minimizes the total cost. This problem is solved in [4].

To understand the properties of the optimal solution, consider first a time interval $[t_1, t_2]$ in which $E_g(t)$ is between the bounds, that is

$$E_L(t) < E_g(t) < E_L(t) + E_{\max}.$$

In this interval the original problem (16) may be written as

$$\begin{aligned} \text{minimize} \quad & \int_{t_1}^{t_2} F(P_g(\tau))\mathrm{d}\tau, \\ \text{subject to} \quad & P_g(t) = \frac{\mathrm{d}}{\mathrm{d}t} E_g, \qquad E_g(t_1) = E_1, \quad E_g(t_2) = E_2, \end{aligned} \tag{17}$$

where E_1 and E_2 are given constants. It can be shown[2] that the solution to this subproblem is characterized by a constant generated power $P_g(t)$. As a result the optimal generated energy $E_g(t)$ must be a straight line, as illustrated in Fig. 6.

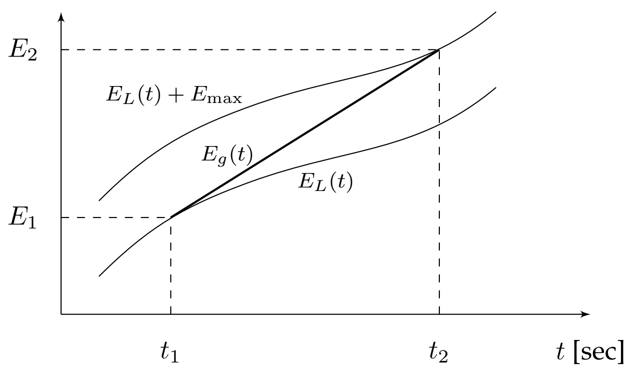


Figure 6: When between bounds, the optimal generated energy $E_g(t)$ must follow a straight line.

Another feature of the optimal solution is that $E_g(t)$ must be tangent to the bounds, as illustrated in Fig. 7. This property is proven in [4].

Typically these two features completely define the optimal solution, and hold regardless of the cost function $F(P_g)$, as long as this function is twice differentiable, monotonically increasing and strictly convex. If this is true we may choose any cost function that obeys these conditions, and the optimal solution will be the same. One possible choice is

$$F(P_g) = \sqrt{1 + P_g^2}, \tag{18}$$

[2]This can be proven based on Pontryagin's minimum principle, which is not discussed in this text.

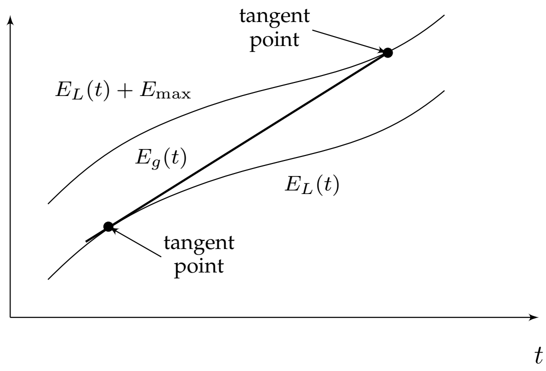


Figure 7: The optimal energy $E_g(t)$ must be tangent to the bounds.

and the resulting total cost is

$$F_{tot} = \int_0^T F(P_g(\tau))\mathrm{d}\tau = \int_0^T \sqrt{1 + P_g^2(\tau)}\mathrm{d}\tau = \int_0^T \sqrt{1 + \left(\frac{\mathrm{d}E_g}{\mathrm{d}t}\bigg|_{t=\tau}\right)^2}\mathrm{d}\tau, \tag{19}$$

which is the *total length* of the curve $E_g(t)$.

Based on the arguments above we can conclude that

- ✓ The optimal generated energy $E_g(t)$ does not depend on the cost function $F(\cdot)$,
- ✓ The optimal generated energy minimizes the total length of the curve $E_g(t)$,

and therefore

the optimal generated energy $E_g(t)$ follows the *shortest path* between the bounds $E_L(t)$ and $E_L(t) + E_{\max}$.

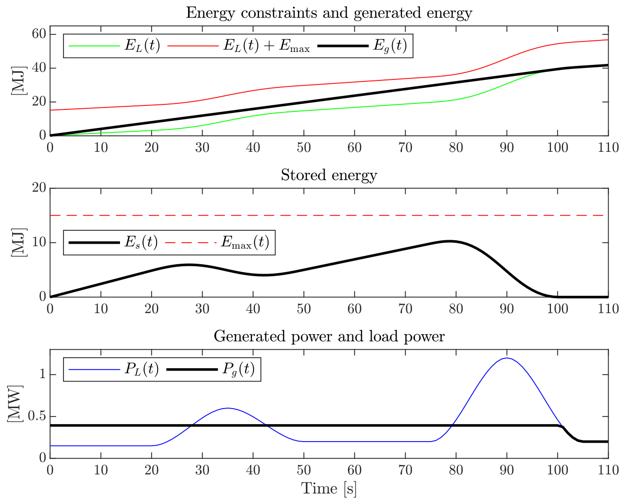
This result provides a graphical design procedure that may be used to calculate the optimal solution:

- ✓ Plot the lower bound $E_L(t)$ and the upper bound $E_L(t) + E_{\max}$.
- ✓ Choose the initial and final values of the generated energy $E_g(t)$. A typical choice is $E_g(0) = E_L(0)$ and $E_g(T) = E_L(T)$, which is equivalent to $E(0) = E(T) = 0$.
- ✓ Plot the shortest path that connects $E_g(0)$ and $E_g(T)$ and is between bounds. This is the optimal generated energy $E_g(t)$.
- ✓ All the other functions may be computed directly. For instance, the stored energy is given by $E(t) = E_g(t) - E_L(t)$.

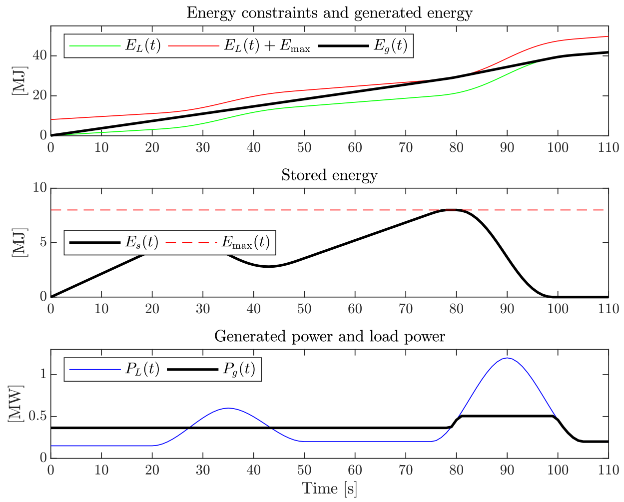
Several examples are shown in Figs. 8 and 9.

The shortest path method allows simple planning of the optimal generated energy, and enables designers to estimate the size of the storage device for a specific application. Several properties of the optimal solution are:

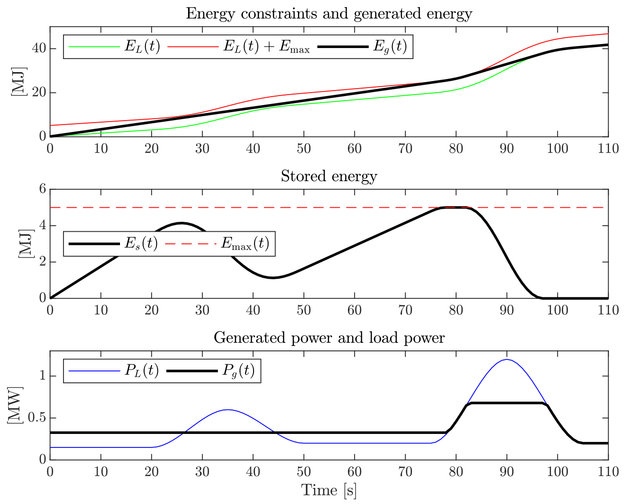
- ✓ If the capacity $E_{\max}$ is very low then the generated power is approximately equal to the load. In this case the storage device has little effect.
- ✓ If the capacity $E_{\max}$ is high enough then the generated energy approximately follows a straight line. The generated power is approximately constant and is equal to the *average load*.



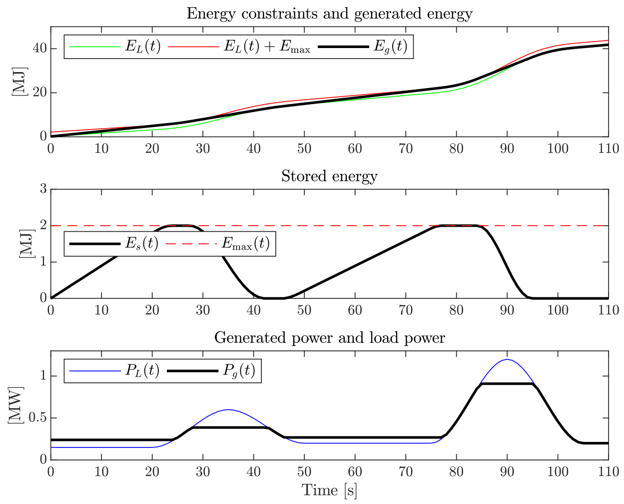
(a) $E_{\max} = 15$ MJ



(b) $E_{\max} = 8$ MJ



(c) $E_{\max} = 5$ MJ



(d) $E_{\max} = 2$ MJ

Figure 8: The optimal generated energy $E_g(t)$ is the **shortest path** between $E_L(t)$ and $E_L(t) + E_{\max}$, and the generated power $P_g(t)$ is "as constant as possible". For high capacity values $P_g(t)$ is approximately equal to the *average* load, and for low capacity values $P_g(t) \approx P_L(t)$.

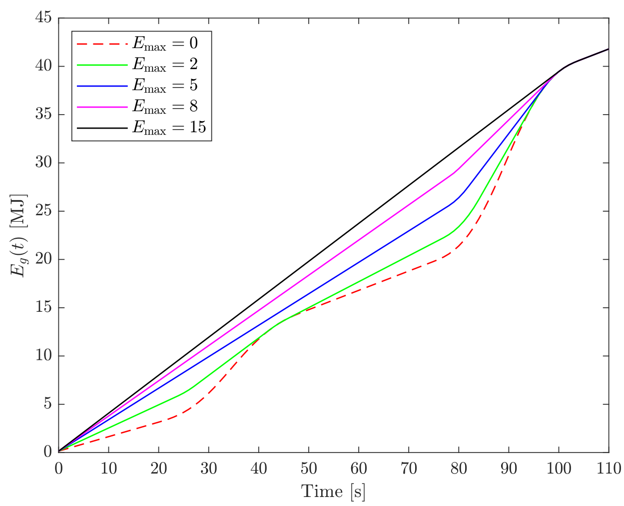


Figure 9: The generated energy for various capacities. In each case the optimal generated energy follows the shortest path.

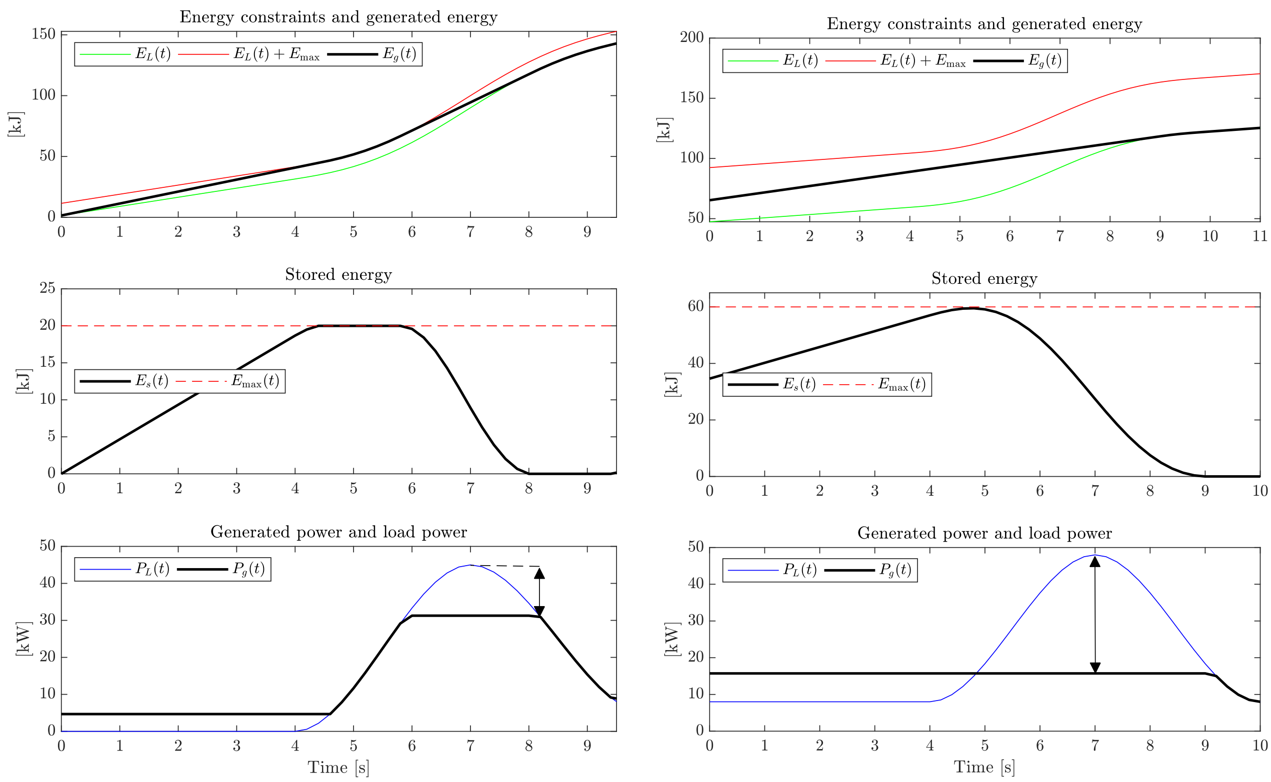
As shown in [5], another property of the "shortest path" method is that it minimizes the peak of the generated power $P_g(t)$. To see this, consider the cost function

$$F(P_g(t)) = |P_g(t)|^m. \tag{20}$$

When $m \to \infty$ we have

$$\lim_{m\to\infty} F_{tot}^{\frac{1}{m}} = \lim_{m\to\infty} \left(\int_0^T |P_g(t)|^m \mathrm{d}t \right)^{\frac{1}{m}} = \max_t \{|P_g(t)|\}. \tag{21}$$

Since typically the optimal solution is the same for every value of m, the shortest path method minimizes the peak power $\max\{|P_g(t)|\}$. This result is demonstrated in Fig. 10.



(a) Low capacity, high peak power

(b) High capacity, low peak power

Figure 10: The shortest path method minimizes the peak of the generated power $P_g(t)$.

Energy trading

Storage devices may be also used for energy trading [6]. Energy is bought when the price is low (usually at night), and sold when the price is high (usually during the day). In addition to generating profits, this also helps to match the energy supply to the energy demand.

As an example, consider the following energy trading optimization problem:

$$\begin{aligned} \text{maximize} \quad & -\int_0^T C(t)P(t)\mathrm{d}t, \\ \text{subject to} \quad & 0 \le E(t) \le E_{\max}, \\ & \frac{\mathrm{d}E}{\mathrm{d}t} = \begin{cases} \eta_c(E,P)P, & P \ge 0, \\ \eta_d^{-1}(E,P)P, & P < 0, \end{cases} \end{aligned} \tag{22}$$

where

- ✓ $C(t)$ is the cost of energy,
- ✓ $P(t)$ is the power flowing into the storage device,
- ✓ $E(t)$ is the stored energy,
- ✓ $\eta_c(E, P)$ is the charging efficiency,
- ✓ $\eta_d(E, P)$ is the discharging efficiency.

For simplicity it is assumed that

$$\begin{aligned} C(t) &= \begin{cases} C_1, & 0 \le t < \alpha T, \\ C_2, & \alpha T \le t < T, \end{cases} \\ \eta_c(E,P) &= \eta_c = \text{constant}, \\ \eta_d(E,P) &= \eta_d = \text{constant}, \end{aligned} \tag{23}$$

with $0 < C_1 < C_2$ and $0 < \alpha < 1$. The integral $-\int_0^T C(t)P(t)\mathrm{d}t$ appearing in the objective represents the profit over a cycle.

The optimal energy management strategy is to buy energy when the price is low, and to sell energy when the price is high. One optimal solution to the problem above is

$$P(t) = \begin{cases} \frac{E_{\max}}{\eta_c \alpha T}, & 0 \le t < \alpha T, \\ -\frac{\eta_d E_{\max}}{(1-\alpha)T}, & \alpha T \le t < T, \end{cases} \tag{24}$$

which is illustrated in Fig. 11.

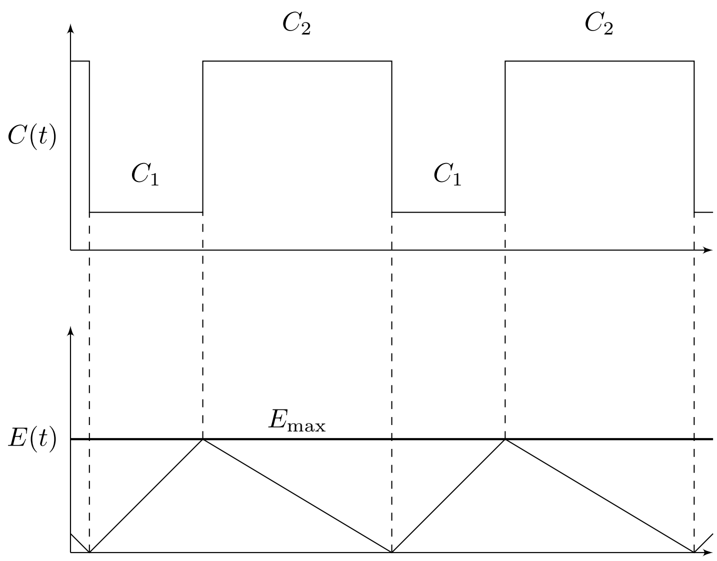


Figure 11: Energy is bought when the price is low, and sold when the price is high.

The profit over one cycle is

$$-\int_0^T C(t)P(t)\mathrm{d}t = -C_1\frac{E_{\max}}{\eta_c} + C_2\eta_d E_{\max} = \left(C_2\eta_d - \frac{C_1}{\eta_c}\right)E_{\max}, \tag{25}$$

and it is positive if

$$C_2\eta_d - \frac{C_1}{\eta_c} > 0 \qquad \Longrightarrow \qquad \eta_c\eta_d > \frac{C_1}{C_2}. \tag{26}$$

We can conclude that in order to make a profit **the round-trip efficiency must be higher than the price ratio**. For instance:

✓ If the storage device is lossless ($\eta_c = \eta_d = 1$) then the profit over a cycle is $(C_2 - C_1)E_{\max}$. In this case the profit is proportional to the price difference, and to the device capacity.

✓ If there is no price difference ($C_2 = C_1$) then the profit is zero or negative.

✓ If $C_1 = 0$ a positive profit is guaranteed, regardless of the device efficiency.

In practice, most energy storage control problems are more complex than the ones mentioned above, and are generally solved by numeric techniques such as linear programming, dynamic programming, or Pontryagin's minimum principle. Several examples are shown in the reference list below, and a summary of the mostly used methods is presented in [7].

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