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Lecture 3: The Synchronous Machine

This lecture presents a dynamic model of the synchronous machine. We demonstrate how to use this model in power system simulations, and explain the relations between the machine's $dq0$ model and time-varying phasor model.

Synchronous machines are often operated as generators, and are a major source of energy in electric power systems. In several applications synchronous machines are also operated as motors. A basic diagram of the machine is shown in Fig. 1. The key mechanical components of the machine are the rotor and stator. There are also two key electric parts: the field winding on the rotor, and the three-phase winding on the stator. The field winding is typically connected to a DC source, which creates a magnetic field with alternating north and south polarities, as illustrated in Fig. 1. As the rotor rotates AC voltages are induced in the stator windings. In addition, the interaction between magnetic fields and currents in the machine produces a torque that acts to decelerate the rotor. These two processes result in energy conversion, from mechanical energy to electromagnetic energy as a generator, or vice-versa as a motor.

The machine is named "synchronous" since at steady-state the rotor speed is proportional to the frequency of currents and voltages in the stator. This is not necessarily the case in other machines. For instance in induction motors the frequency of the rotor must be slightly lower than the frequency of the AC current.

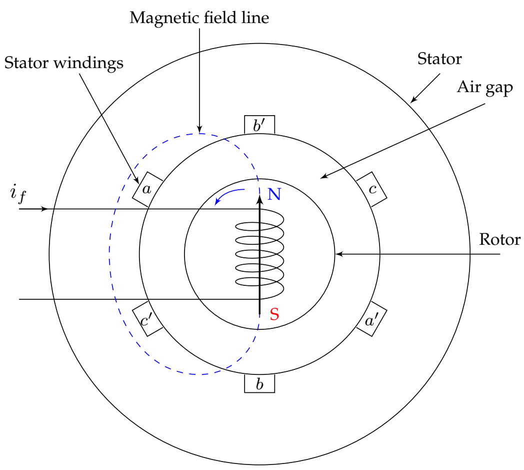

Figure 1: Schematic diagram of a three-phase synchronous machine with two poles.

Remark 1: Synchronous machines have complex dynamics and control mechanisms that cannot be fully addressed in a short lecture. Therefore several topics such as the accurate prime mover dynamics or the structure of excitation systems are not fully explained. Additional information may be found in [1,2].

Remark 2: For the convenience of the reader a list of symbols is provided in Appendix A.

Series of lectures on power system dynamics. The lectures are freely available on
https://a-lab.ee/projects/dq0-dynamics

Mechanical model

Synchronous machines often have more than two magnetic poles on the rotor. Since quantities associated with any pair of poles are identical, it is usually convenient to define *electrical* quantities which are related to a single pair of poles. For this reason the rotor angle is often expressed in *electrical degrees* or *electrical radians*, rather than in mechanical units.

One pair of poles is mapped to 360 electrical degrees or 2π electrical radians. Since there are poles/2 electrical cycles in one mechanical cycle it follows that

$$\theta = \frac{p_f}{2}\theta_m, \tag{1}$$

where

- ✓ θ denotes the rotor electrical angle;
- ✓ θ_m denotes the rotor mechanical angle, with respect to a fixed point on the stator;
- ✓ p_f denotes the number of magnetic poles on the rotor (an even integer).

In addition frequencies are defined as follows:

- ✓ $\omega = \mathrm{d}\theta/\mathrm{d}t$ denotes the rotor electrical frequency;
- ✓ $\omega_m = \mathrm{d}\theta_m/\mathrm{d}t$ denotes the rotor mechanical frequency;
- ✓ ω_s denotes the nominal grid frequency, for instance $\omega_s = 2\pi 50$ or $2\pi 60$ rad/s.

Following (1) we have

$$\omega = \frac{p_f}{2}\omega_m. \tag{2}$$

The frequencies are typically measured in rad/s.

The angular acceleration of the rotor is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_m = \frac{1}{J}(T_m - T_e), \tag{3}$$

where

- ✓ J denotes the rotor moment of inertia.
- ✓ T_m denotes the mechanical torque, accelerating the rotor.
- ✓ T_e denotes the electromagnetic torque, decelerating the rotor.

In addition powers are defined as

$$\begin{aligned} p_m &= T_m\omega_m = \frac{2}{p_f}T_m\omega, \\ p_e &= T_e\omega_m = \frac{2}{p_f}T_e\omega, \end{aligned} \tag{4}$$

where

- ✓ p_m denotes the mechanical power accelerating the rotor,
- ✓ p_e denotes the electromagnetic power decelerating the rotor.

Equations (2) and (3) yield

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = \frac{p_f}{2J}(T_m - T_e), \tag{5}$$

and following (4) and (5) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = \left(\frac{p_f}{2}\right)^2 \frac{1}{J\omega}(p_m - p_e) = K\frac{\omega_s}{\omega}(p_m - p_e), \tag{6}$$

where $K = \left(\frac{p_f}{2}\right)^2 \frac{1}{J\omega_s}$.
Under the approximation $\omega \approx \omega_s$ this is the classic swing equation. However in this lecture we do not use this approximation, and work directly with the exact expressions in (5) and (6). It is also assumed that the mechanical torque is governed by a droop mechanism which is described as

$$T_m = \frac{p_f}{2\omega_s}\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s)\right), \tag{7}$$

where

- ✓ D denotes the damping coefficient.
- ✓ P_{ref} denotes the reference power.

As we have seen in Lecture 1 this type of control is crucial for regulating the frequency and maintaining stability.

Remark 1: The linear relationship in (7) ignores the complex dynamics of the prime-mover, and holds only for slow transients. A more accurate model may be found in [2].

Remark 2: The droop mechanism in (7) is defined with respect to torque instead of with respect to power, as done in Lecture 1. This form is chosen to simplify the dynamic equations without using the approximation $\omega \approx \omega_s$.

Substituting (7) into (5) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = K\left(3P_{ref} - \frac{2}{p_f}\omega_s T_e - \frac{1}{D}(\omega - \omega_s)\right). \tag{8}$$

This equation defines the dynamics of the frequency ω with respect to the electromagnetic torque T_e. Recall that in Lecture 1 we have used a time-varying phasor model and defined the swing equation as

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega = K\left(3P_{ref} - 3P - \frac{1}{D}(\omega - \omega_s)\right). \tag{9}$$

Equations (8) and (9) are similar except for the more accurate term $\frac{2}{p_f}\omega_s T_e$ that replaces the active power $3P$.

Electrical model

This section presents an electrical model of the machine which is based on $dq0$ quantities. The model is an extension of the time-varying phasor model presented in Lecture 1, and may be used to describe fast transients. However, as all models, the model presented here is an approximation. Specifically, it is assumed that

- ✓ The machine is a magneto-quasi-static device,

- ✓ Saturation of the magnetic materials and other sources of imbalance and harmonic distortion are ignored,
- ✓ Self- and mutual inductances are composed of a constant term, in addition to a sinusoidal term varying with 2θ.

Although these assumptions may seem restrictive, they form the basis of the classic $dq0$ model, and have been found to give excellent results in a wide variety of applications [1].

The machine is described as a system of coupled inductors

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_f \end{bmatrix} = \begin{bmatrix} \ell_{aa} & \ell_{ab} & \ell_{ac} & \ell_{af} \\ \ell_{ba} & \ell_{bb} & \ell_{bc} & \ell_{bf} \\ \ell_{ca} & \ell_{cb} & \ell_{cc} & \ell_{cf} \\ \ell_{fa} & \ell_{fb} & \ell_{fc} & \ell_{ff} \end{bmatrix} \begin{bmatrix} -i_a \\ -i_b \\ -i_c \\ i_f \end{bmatrix}, \tag{10}$$

where

- ✓ λ_a, λ_b, λ_c denote the stator flux linkages;
- ✓ i_a, i_b, i_c denote the stator currents (generator output currents). The negative signs have been included since currents are positive when flowing out of the generator;
- ✓ λ_f denotes the field winding flux linkage;
- ✓ i_f denotes the field winding current.

Following these definitions we have

$$\begin{aligned} v_a &= -R_a i_a + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_a, \\ v_b &= -R_a i_b + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_b, \\ v_c &= -R_a i_c + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_c, \\ v_f &= R_f i_f + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_f, \end{aligned} \tag{11}$$

where

- ✓ v_a, v_b, v_c denote the stator terminal voltages (generator output voltages);
- ✓ v_f denotes the field winding voltage;
- ✓ R_a denotes the resistance of each winding on the stator;
- ✓ R_f denotes the field winding resistance.

The self- and mutual inductances in (10) depend on the rotor position. The stator self-inductances are given by

$$\begin{aligned} \ell_{aa} &= L_{aa} + L_{g2}\cos(2\theta), \\ \ell_{bb} &= L_{aa} + L_{g2}\cos(2\theta + 120^\circ), \\ \ell_{cc} &= L_{aa} + L_{g2}\cos(2\theta - 120^\circ), \end{aligned} \tag{12}$$

and the stator-to-stator mutual inductances are

$$\begin{aligned} \ell_{ab} = \ell_{ba} &= L_{ab} + L_{g2}\cos(2\theta - 120^\circ), \\ \ell_{bc} = \ell_{cb} &= L_{ab} + L_{g2}\cos(2\theta), \\ \ell_{ac} = \ell_{ca} &= L_{ab} + L_{g2}\cos(2\theta + 120^\circ), \end{aligned} \tag{13}$$

where L_{aa} is a positive constant and L_{ab} is a negative constant. These inductances are composed of a constant term, in addition to a sinusoidal term varying with 2θ. This additional term is required in case the rotor is not perfectly round (which causes "saliency effects").

In addition, the stator-to-rotor mutual inductances vary according to the rotor position and are given by

$$\begin{aligned} \ell_{af} &= \ell_{fa} = L_{af}\cos(\theta), \\ \ell_{bf} &= \ell_{fb} = L_{af}\cos(\theta - 120°), \\ \ell_{cf} &= \ell_{fc} = L_{af}\cos(\theta + 120°), \end{aligned} \tag{14}$$

and the field winding self-inductance is

$$\ell_{ff} = L_{ff} = \text{constant}. \tag{15}$$

The model defined by the above equations typically does not have an equilibrium point since the inductances depend on the rotor angle θ, and thus vary with time. For this reason we will now develop an equivalent model which is based on $dq0$ quantities. Such a model may be obtained by applying the $dq0$ transformation to the equations above. Omitting the algebraic details, the resulting $dq0$-based model is given by

$$\begin{aligned} \lambda_d &= -L_d i_d + L_{af} i_f, \\ \lambda_q &= -L_q i_q, \\ \lambda_0 &= -L_0 i_0, \\ \lambda_f &= -\frac{3}{2} L_{af} i_d + L_{ff} i_f, \end{aligned} \tag{16}$$

and

$$\begin{aligned} v_d &= -R_a i_d + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_d - \omega\lambda_q, \\ v_q &= -R_a i_q + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_q + \omega\lambda_d, \\ v_0 &= -R_a i_0 + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_0, \\ v_f &= R_f i_f + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_f, \end{aligned} \tag{17}$$

where

- ✓ v_d, v_q, v_0 are the $dq0$ transformation of v_a, v_b, v_c (stator terminal voltages).
- ✓ λ_d, λ_q, λ_0 are the $dq0$ transformation of λ_a, λ_b, λ_c (stator flux linkages).
- ✓ L_d is the *direct-axis synchronous inductance*.
- ✓ L_q is the *quadrature-axis synchronous inductance*.
- ✓ L_0 is the *zero-sequence inductance*.

Notes:

1. The reference angle for the $dq0$ transformation is the rotor electrical angle θ.
2. The $dq0$ variables do not depend directly on θ.

The inductances in (16) are given by

$$
\begin{aligned}
L_d &= L_{aa} - L_{ab} + \frac{3}{2}L_{g2},\\
L_q &= L_{aa} - L_{ab} - \frac{3}{2}L_{g2},\\
L_0 &= L_{aa} + 2L_{ab}.
\end{aligned}
\tag{18}
$$

We will usually work directly with these inductances.

We also denote

$$L_s = \frac{1}{2}(L_d + L_q). \tag{19}$$

For a perfectly round rotor with $L_{g2} = 0$ (no "saliency effects") the synchronous inductances are equal:

$$L_s = L_d = L_q. \tag{20}$$

In this case L_s is the *synchronous inductance* of the machine, as in Lecture 1.

To complete the basic model, the machine's output power (flowing from the stator terminals into the grid) is

$$p_s = \frac{3}{2}(v_d i_d + v_q i_q + 2v_0 i_0), \tag{21}$$

and the electromagnetic power decelerating the rotor is

$$p_e = \frac{3}{2}\omega(\lambda_d i_q - \lambda_q i_d). \tag{22}$$

Following (22) and using (2) the electromagnetic torque is given by

$$T_e = \frac{p_e}{\omega_m} = \frac{3}{2}\frac{p_f}{2}(\lambda_d i_q - \lambda_q i_d). \tag{23}$$

A complete state-space model of the synchronous machine is obtained by merging the equations above. Using equations (8), (16), (17) and (23) and again omitting the algebraic details we obtain

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\theta = \omega,\\
&\frac{\mathrm{d}}{\mathrm{d}t}\omega = K\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s) + \left(\frac{3\beta\omega_s - 6L_{ff}L_q\omega_s}{2\beta L_q}\right)\lambda_d\lambda_q + \frac{3L_{af}\omega_s}{\beta}\lambda_q\lambda_f\right),\\
&\frac{\mathrm{d}}{\mathrm{d}t}\lambda_d = -\frac{2R_aL_{ff}}{\beta}\lambda_d + \omega\lambda_q + \frac{2R_aL_{af}}{\beta}\lambda_f + v_d,\\
&\frac{\mathrm{d}}{\mathrm{d}t}\lambda_q = -\omega\lambda_d - \frac{R_a}{L_q}\lambda_q + v_q,\\
&\frac{\mathrm{d}}{\mathrm{d}t}\lambda_0 = -\frac{R_a}{L_0}\lambda_0 + v_0,\\
&\frac{\mathrm{d}}{\mathrm{d}t}\lambda_f = \frac{3R_fL_{af}}{\beta}\lambda_d - \frac{2R_fL_d}{\beta}\lambda_f + v_f,
\end{aligned}
\tag{24}
$$

The inputs of this model are v_d, v_q, v_0, v_f, P_{ref}, and the constants are defined as

$$
\begin{aligned}
K &= \left(\frac{p_f}{2}\right)^2\frac{1}{J\omega_s},\\
\beta &= 2L_dL_{ff} - 3L_{af}^2.
\end{aligned}
\tag{25}
$$

Several optional outputs (in addition to the state variables) are

$$
\begin{aligned}
i_d &= \frac{2L_{af}}{\beta}\lambda_f - \frac{2L_{ff}}{\beta}\lambda_d, & i_q &= -\frac{1}{L_q}\lambda_q, \qquad i_0 = -\frac{1}{L_0}\lambda_0, \\
i_f &= \frac{2L_d}{\beta}\lambda_f - \frac{3L_{af}}{\beta}\lambda_d, \\
T_e &= \frac{3}{2}\frac{p_f}{2}(\lambda_d i_q - \lambda_q i_d), & p_e &= \frac{3}{2}\omega(\lambda_d i_q - \lambda_q i_d), \\
T_m &= \frac{p_f}{2\omega_s}\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s)\right), & p_m &= \frac{\omega}{\omega_s}\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s)\right), \\
p_s &= \frac{3}{2}(v_d i_d + v_q i_q + 2v_0 i_0).
\end{aligned}
\tag{26}
$$

Following are several examples demonstrating how to use the above model in simulation. Consider first a synchronous generator feeding a symmetrically configured resistive load R_L. The objective here is to simulate the transient that follows a sudden 3-phase short circuit at the stator terminals. A possible signal flow diagram is shown in Fig. 2. Under normal operating conditions the ratio between the voltages and currents is R_L. When the short occurs the voltages v_d, v_q, v_0 are zeroed, and a transient takes place.

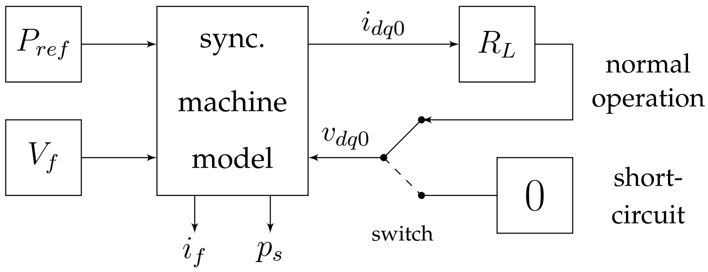

Figure 2: Simulating a sudden 3-phase short circuit at the stator terminals.

As a second example, consider the system shown in Fig. 3. A signal flow diagram is shown in Fig. 4.

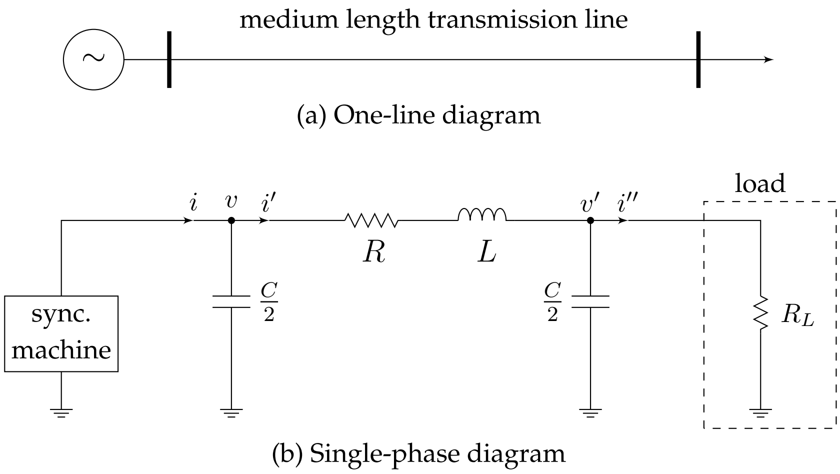

Figure 3: Example: synchronous machine connected to a medium length transmission line and load.

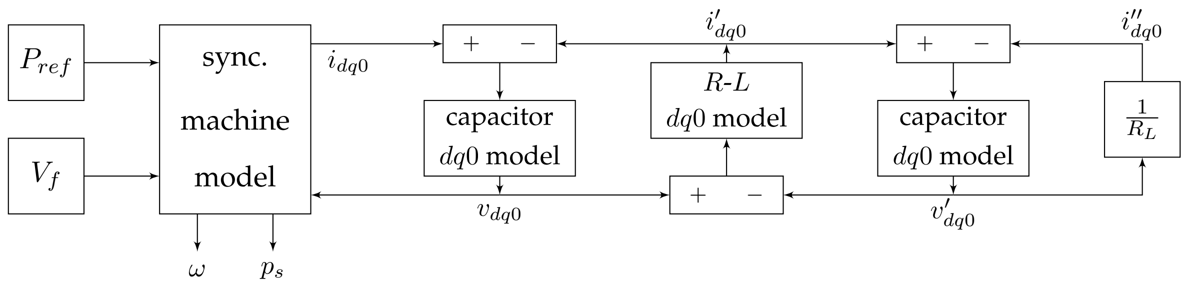

Figure 4: Signal flow diagram for a synchronous machine connected to a medium length transmission line and load.

The capacitors, inductor and resistor are modeled as in Lecture 3. The left capacitor is modeled as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}v_d &= \omega v_q + \frac{1}{C/2}(i_d - i'_d),\\ \frac{\mathrm{d}}{\mathrm{d}t}v_q &= -\omega v_d + \frac{1}{C/2}(i_q - i'_q),\\ \frac{\mathrm{d}}{\mathrm{d}t}v_0 &= \frac{1}{C/2}(i_0 - i'_0),\end{aligned} \tag{27}$$

the right capacitor is modeled as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}v'_d &= \omega v'_q + \frac{1}{C/2}(i'_d - i''_d),\\ \frac{\mathrm{d}}{\mathrm{d}t}v'_q &= -\omega v'_d + \frac{1}{C/2}(i'_q - i''_q),\\ \frac{\mathrm{d}}{\mathrm{d}t}v'_0 &= \frac{1}{C/2}(i'_0 - i''_0),\end{aligned} \tag{28}$$

and the series resistor and inductor are modeled as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i'_d \\ i'_q \\ i'_0 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & 0 \\ -\omega & -\frac{R}{L} & 0 \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} i'_d \\ i'_q \\ i'_0 \end{bmatrix} + \frac{1}{L}\begin{bmatrix} v_d - v'_d \\ v_q - v'_q \\ v_0 - v'_0 \end{bmatrix}. \tag{29}$$

Note that these equations depend on ω, which is an output of the machine's model. It is also possible to use the approximation $\omega \approx \omega_s$ to eliminate this dependency.

Simplified machine model

This section presents a simplified dynamic model of the machine. The model is based on the following assumptions:

- ✓ Round rotor: $L_{g2} = 0$, or equivalently $L_d = L_q = L_s$,
- ✓ Constant field current: $i_f = I_f = \text{const}$,
- ✓ Balanced voltages and currents: $v_0 = 0$, $i_0 = 0$.

We will now see that under these assumptions

- ✓ The machine may be described as an internal voltage source behind a series impedance;
- ✓ The magnitude of the internal voltage is proportional to the frequency ω;

✓ The electromagnetic torque is proportional to the current i_q.

Based on (16) and (17) and using $L_d = L_q = L_s$, $i_f = I_f$, $v_0 = 0$, $i_0 = 0$ we have

$$\begin{aligned}\lambda_d &= -L_s i_d + L_{af} I_f,\\ \lambda_q &= -L_s i_q,\\ \lambda_f &= -\frac{3}{2} L_{af} i_d + L_{ff} I_f,\end{aligned} \tag{30}$$

and

$$\begin{aligned}v_d &= -R_a i_d + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_d - \omega\lambda_q,\\ v_q &= -R_a i_q + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_q + \omega\lambda_d,\\ v_f &= R_f I_f + \frac{\mathrm{d}}{\mathrm{d}t}\lambda_f.\end{aligned} \tag{31}$$

In addition, substituting (30) into (31) yields

$$\begin{aligned}v_d &= -R_a i_d - L_s \frac{\mathrm{d}}{\mathrm{d}t} i_d + \omega L_s i_q,\\ v_q &= -R_a i_q - L_s \frac{\mathrm{d}}{\mathrm{d}t} i_q - \omega L_s i_d + \omega L_{af} I_f,\\ v_f &= R_f I_f - \frac{3}{2} L_{af} \frac{\mathrm{d}}{\mathrm{d}t} i_d.\end{aligned} \tag{32}$$

Note that v_f does not affect the other quantities, and may be considered an output of the model. Using the definition

$$V_E = \omega_s L_{af} I_f = \text{constant} \tag{33}$$

These equations may be written as

$$\begin{aligned}v_d &= -R_a i_d - L_s \frac{\mathrm{d}}{\mathrm{d}t} i_d + \omega L_s i_q,\\ v_q &= -R_a i_q - L_s \frac{\mathrm{d}}{\mathrm{d}t} i_q - \omega L_s i_d + \frac{\omega}{\omega_s} V_E,\end{aligned} \tag{34}$$

or

$$\begin{aligned}v_d &= 0 & -R_a i_d & -\left(L_s \frac{\mathrm{d}}{\mathrm{d}t} i_d - \omega L_s i_q\right),\\ \underbrace{v_q}_{\text{stator voltages}} &= \underbrace{\frac{\omega}{\omega_s} V_E}_{\text{internal voltage}} & \underbrace{-R_a i_q}_{\text{voltage drop on series resistance}} & \underbrace{-\left(L_s \frac{\mathrm{d}}{\mathrm{d}t} i_q + \omega L_s i_d\right)}_{\text{voltage drop on series inductance}}.\end{aligned} \tag{35}$$

Based on these expressions the machine's simplified model may be compactly described as an internal voltage source (induced EMF) behind a series impedance, as shown in Fig. 5.

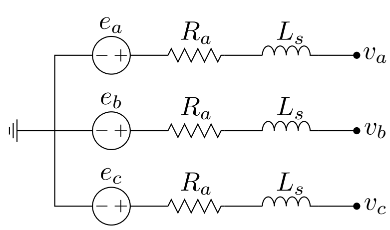

Figure 5: The simplified machine model: an internal voltage source behind a series impedance.

In this equivalent circuit the internal voltage source is given by

$$\begin{bmatrix} e_d \\ e_q \\ e_0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\omega}{\omega_s} V_E \\ 0 \end{bmatrix}, \tag{36}$$

where $[e_d, e_q, e_0]^\mathsf{T}$ is the $dq0$ transformation of $[e_a, e_b, e_c]^\mathsf{T}$. It can be seen that the internal voltage is proportional to the frequency, where at steady-state and at nominal frequency the peak voltage is $V_E = \omega_s L_{af} I_f$.

In addition, using (23), (30) and (33), the electromagnetic torque acting on the rotor (in the simplified model) is

$$T_e = \frac{3}{2}\frac{p_f}{2}(\lambda_d i_q - \lambda_q i_d) = \frac{3}{2}\frac{p_f}{2} L_{af} I_f i_q = \frac{3}{2}\frac{p_f}{2\omega_s} V_E i_q. \tag{37}$$

The electromagnetic torque is proportional to the stator current i_q.

Based on these results, a simplified model of the machine may be obtained by separating the internal voltage source from the series impedance (R_a and L_s). This series impedance, although physically a part of the machine, can be modeled as if it is a part of the transmission network. The dynamic equations of the internal voltage source are

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\omega &= K\left(3P_{ref} - \frac{2\omega_s}{p_f} T_e - \frac{1}{D}(\omega - \omega_s)\right), \\ T_e &= \frac{3}{2}\frac{p_f}{2\omega_s} V_E i_q, \end{aligned} \tag{38}$$

and direct substitution of T_e provides the simplified model

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\theta &= \omega, \\ \frac{\mathrm{d}}{\mathrm{d}t}\omega &= K\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s) - \frac{3}{2} V_E i_q\right). \end{aligned} \tag{39}$$

Here the inputs are P_{ref} and i_q, and several outputs (in addition to the state variables) are

$$\begin{aligned} e_d &= 0, & e_q &= \frac{\omega}{\omega_s} V_E, \qquad e_0 = 0, \\ T_e &= \frac{3}{2}\frac{p_f}{2\omega_s} V_E i_q, & p_e &= \frac{3}{2}\frac{\omega}{\omega_s} V_E i_q, \\ T_m &= \frac{p_f}{2\omega_s}\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s)\right), & p_m &= \frac{\omega}{\omega_s}\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s)\right). \end{aligned} \tag{40}$$

All the $dq0$ quantities are defined here with respect to the rotor electrical angle θ.

Energy conversion in the machine

We will now use the simplified machine model to explain the flow of energy in the machine. The total energy stored in the machine is composed of two parts:

- ✓ The kinetic energy of the rotor: $E_{rot} = \frac{1}{2} J \omega_m^2 = \frac{1}{2} J \left(\frac{2}{p_f}\right)^2 \omega^2$,
- ✓ The magnetic energy, represented by the energy stored in the synchronous inductance: $E_L = \frac{3}{4} L_s \left(i_d^2 + i_q^2\right)$.

The kinetic energy derivative is

$$\frac{\mathrm{d}}{\mathrm{d}t}E_{rot} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}J\omega_m^2\right) = J\omega_m\frac{\mathrm{d}\omega_m}{\mathrm{d}t}, \tag{41}$$

and $\frac{\mathrm{d}\omega_m}{\mathrm{d}t} = \frac{1}{J}(T_m - T_e)$ yields

$$\frac{\mathrm{d}}{\mathrm{d}t}E_{rot} = \omega_m T_m - \omega_m T_e = p_m - p_e. \tag{42}$$

This is an intuitive result: the change in kinetic energy is equal to the difference between the mechanical and electromagnetic powers. In addition, the magnetic energy derivative is

$$\frac{\mathrm{d}}{\mathrm{d}t}E_L = \frac{3}{2}L_s\left(i_d\frac{\mathrm{d}i_d}{\mathrm{d}t} + i_q\frac{\mathrm{d}i_q}{\mathrm{d}t}\right), \tag{43}$$

and using (35) we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}E_L &= \frac{3}{2}\left(i_d\left(-R_a i_d + \omega L_s i_q - v_d\right) + i_q\left(-R_a i_q - \omega L_s i_d + \frac{\omega}{\omega_s}V_E - v_q\right)\right)\\ &= \underbrace{\frac{3}{2}\frac{\omega}{\omega_s}V_E i_q}_{p_e} - \underbrace{\frac{3}{2}(v_d i_d + v_q i_q)}_{p_s} - \underbrace{\frac{3}{2}R_a\left(i_d^2 + i_q^2\right)}_{\text{ohmic loss}}\\ &= p_e - p_s - \frac{3}{2}R_a\left(i_d^2 + i_q^2\right).\end{aligned} \tag{44}$$

The flow of power in the machine is summarized in Fig. 6.

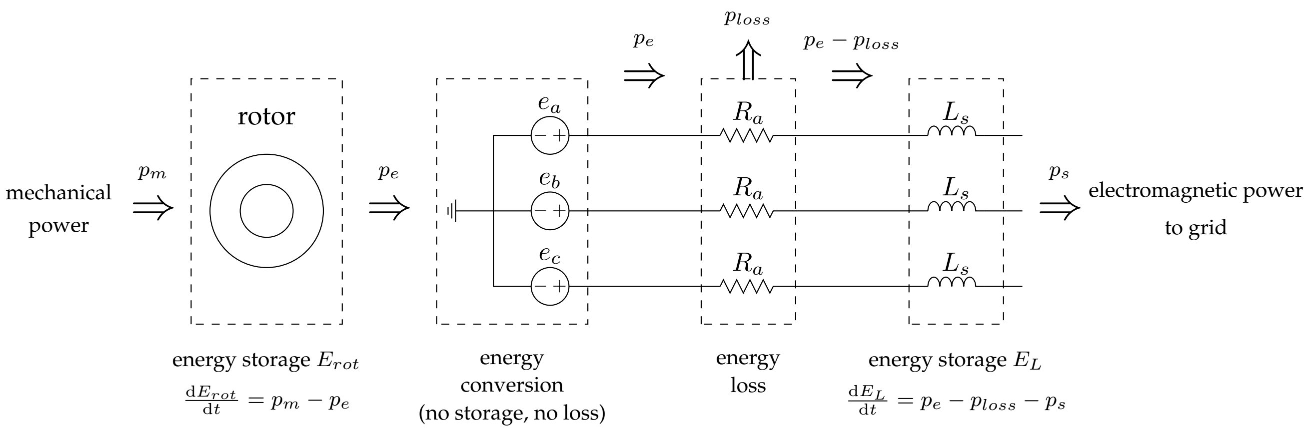

Figure 6: Energy conversion in the machine, based on the simplified model.

The central energy conversion process is described by the internal voltage source, which converts mechanical energy to electrical energy (or vice-versa).

From a mechanical perspective, based on (4): $p_e = T_e\omega_m$.

From an electrical perspective, based on (40):

$$\begin{aligned}p_e &= \frac{3}{2}\left(e_d i_d + e_q i_q + 2e_0 i_0\right)\\ &= \frac{3}{2}\left(0 \cdot i_d + \frac{\omega}{\omega_s} V_E \cdot i_q + 2 \cdot 0 \cdot 0\right)\\ &= \frac{3}{2}\frac{\omega}{\omega_s} V_E i_q\\ &= T_e \frac{2}{p_f}\omega\\ &= T_e \omega_m.\end{aligned} \tag{45}$$

Note that both these forms of p_e are identical, and therefore the internal voltage source is an ideal converter of mechanical energy to electromagnetic energy.

Transformation from one reference frame to another

The dynamic models presented above are defined in the rotor reference frame, with respect to the angle θ. A typical question is how to construct a dynamic model of a system with more than one natural reference frame. Such a need arises when the system includes more than one machine, or when a machine is connected to an infinite bus.

A typical solution is to transform the $dq0$ variables from one reference frame to another. Assume a system with two sets of signals:

- ✓ x_{dq0} is defined with respect to a reference angle θ_r.
- ✓ $\tilde{x}_{dq0}$ is defined with respect to a reference angle θ.

The relations between these signals can be found by transforming the signals to the abc reference frame and back, which may be written as $x_{dq0} = T_{\theta_r} T_\theta^{-1} \tilde{x}_{dq0}$ or $\tilde{x}_{dq0} = T_\theta T_{\theta_r}^{-1} x_{dq0}$. Using the $dq0$ identities in Lecture 2 this yields

$$\theta \to \theta_r : \qquad x_{dq0} = \begin{bmatrix}\cos(\theta-\theta_r) & -\sin(\theta-\theta_r) & 0\\ \sin(\theta-\theta_r) & \cos(\theta-\theta_r) & 0\\ 0 & 0 & 1\end{bmatrix}\tilde{x}_{dq0}, \tag{46}$$

$$\theta \leftarrow \theta_r : \qquad \tilde{x}_{dq0} = \begin{bmatrix}\cos(\theta-\theta_r) & \sin(\theta-\theta_r) & 0\\ -\sin(\theta-\theta_r) & \cos(\theta-\theta_r) & 0\\ 0 & 0 & 1\end{bmatrix}x_{dq0}. \tag{47}$$

Consider now a general unit, which $dq0$ model is described with respect to the angle θ. This model can be linked to a system with a reference angle θ_r as described in Fig. 7.

As an important special case, consider the internal voltage source $\tilde{e}_d = 0$, $\tilde{e}_q = \frac{\omega}{\omega_s}V_E$, $\tilde{e}_0 = 0$, with a reference angle θ. Direct transformation from θ to θ_r yields

$$\begin{bmatrix}e_d\\ e_q\\ e_0\end{bmatrix} = \begin{bmatrix}\cos(\theta-\theta_r) & -\sin(\theta-\theta_r) & 0\\ \sin(\theta-\theta_r) & \cos(\theta-\theta_r) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ \frac{\omega}{\omega_s}V_E\\ 0\end{bmatrix} = \frac{\omega}{\omega_s}V_E\begin{bmatrix}-\sin(\theta-\theta_r)\\ \cos(\theta-\theta_r)\\ 0\end{bmatrix} \tag{48}$$

or

$$\begin{bmatrix}e_d\\ e_q\\ e_0\end{bmatrix} = \frac{\omega}{\omega_s}V_E\begin{bmatrix}\cos(\delta)\\ \sin(\delta)\\ 0\end{bmatrix} \qquad \text{with} \qquad \delta = \theta - \theta_r + \frac{\pi}{2}, \tag{49}$$

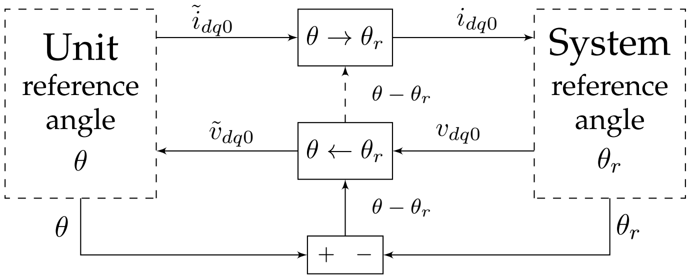

Figure 7: Transformation from one reference frame to another.

where e_d, e_q, e_0 are defined with respect to θ_r. The reader may recognize δ as the **power angle** of the machine, as discussed in Lecture 1.

We will now use this result to show that the simplified machine model of Section 3 leads to the time-varying phasor model of Lecture 1. Consider a slow transient for which *time-varying phasors* may be used instead of $dq0$ quantities. In this case the internal voltage source may be described by a phasor:

$$E = \frac{1}{\sqrt{2}}\left(e_d + je_q\right) = \frac{\omega V_E}{\sqrt{2}\omega_s}\left(\cos(\delta) + j\sin(\delta)\right). \tag{50}$$

Let $|E|$ denote the amplitude of this phasor,

$$|E| = \frac{\omega V_E}{\sqrt{2}\omega_s} = \omega L_{af} I_f/\sqrt{2}, \tag{51}$$

and substitute (51) in (50) to obtain

$$E = |E|\angle\delta. \tag{52}$$

Note that $|E|$ depends on ω, however in a time-varying phasor model it is typically assumed that $\omega \approx \omega_s$, and therefore $|E|$ is constant.

Also recall that in a time-varying phasor model $p_e = 3P$. This relation holds since

$$\begin{aligned} 3P &= 3\,\mathrm{Re}\{EI_a^*\} = 3\,\mathrm{Re}\left\{\frac{1}{\sqrt{2}}\left(e_d + je_q\right)\frac{1}{\sqrt{2}}\left(i_d + ji_q\right)^*\right\} \\ &= \frac{3}{2}\,\mathrm{Re}\{e_d i_d + e_q i_q\} = p_e. \end{aligned} \tag{53}$$

An equivalent circuit of the machine which is based on time-varying phasors is shown in Fig. 8. Recall that this is the same circuit presented in Lecture 1. It is valid only under the assumptions mentioned above, which are

- ✓ Round rotor: $L_{g2} = 0$, or equivalently $L_d = L_q = L_s$;
- ✓ Constant field current: $i_f = I_f = \text{const}$;
- ✓ Balanced voltages and currents: $v_0 = 0$, $i_0 = 0$;
- ✓ Slow transients: voltages and currents are nearly sinusoidal over a single line cycle, so time-varying phasors may be used instead of $dq0$ quantities.

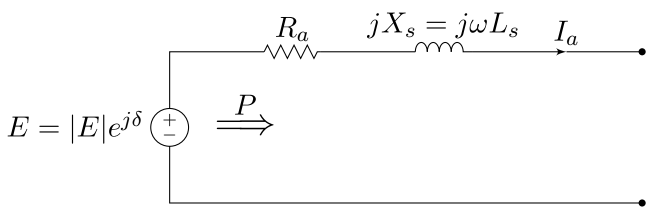

Figure 8: A time-varying phasor model of the synchronous machine, single-phase diagram.

Example: machine connected to an infinite bus

Consider a synchronous machine connected to an infinite bus. The machine is described by the simplified model presented in Section 3, and the infinite bus is modeled as

$$\begin{aligned} v_{d,\infty} &= \sqrt{2}V_g = \text{const}, \\ v_{q,\infty} &= 0, \\ v_{0,\infty} &= 0, \end{aligned} \tag{54}$$

with respect to a reference angle $\theta_r = \omega_s t$. A signal flow diagram is shown in Fig. 9.

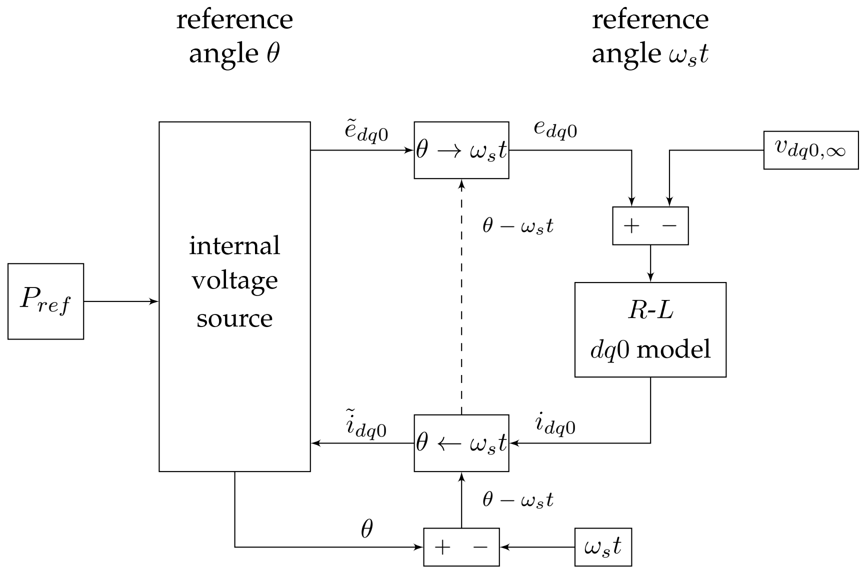

Figure 9: Signal-flow diagram: synchronous machine connected to an infinite bus.

The internal voltage source is modeled with respect to the reference angle $\omega_s t$ as

$$\begin{bmatrix} e_d \\ e_q \\ e_0 \end{bmatrix} = \frac{\omega}{\omega_s} V_E \begin{bmatrix} \cos(\delta) \\ \sin(\delta) \\ 0 \end{bmatrix} \tag{55}$$

with $\delta = \theta - \omega_s t + \frac{\pi}{2}$. In addition, based on (39) we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\theta &= \omega, \\ \frac{\mathrm{d}}{\mathrm{d}t}\omega &= K\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s) - \frac{3}{2}V_E\tilde{i}_q\right), \end{aligned} \tag{56}$$

where $\tilde{i}_q$ is referenced to θ. Using (47) this variable may be expressed in terms of the reference angle $\omega_s t$ as

$$\begin{bmatrix} \tilde{i}_d \\ \tilde{i}_q \\ \tilde{i}_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta - \omega_s t) & \sin(\theta - \omega_s t) & 0 \\ -\sin(\theta - \omega_s t) & \cos(\theta - \omega_s t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}, \tag{57}$$

and since $\delta = \theta - \omega_s t + \frac{\pi}{2}$ we have

$$\tilde{i}_q = \cos(\delta) i_d + \sin(\delta) i_q. \tag{58}$$

Moreover, the machine's series impedance is modeled (with respect to $\omega_s t$) as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_s} & \omega_s & 0 \\ -\omega_s & -\frac{R_a}{L_s} & 0 \\ 0 & 0 & -\frac{R_a}{L_s} \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \frac{1}{L_s}\left[\frac{\omega}{\omega_s} V_E \begin{bmatrix} \cos(\delta) \\ \sin(\delta) \\ 0 \end{bmatrix} - \begin{bmatrix} \sqrt{2}V_g \\ 0 \\ 0 \end{bmatrix}\right]. \tag{59}$$

Combining these equations, the resulting state-space model is

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\delta &= \omega - \omega_s, \\
\frac{\mathrm{d}}{\mathrm{d}t}\omega &= K\left(3P_{ref} - \frac{1}{D}(\omega - \omega_s) - \frac{3}{2}V_E i_d \cos(\delta) - \frac{3}{2}V_E i_q \sin(\delta)\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}i_d &= -\frac{R_a}{L_s}i_d + \omega_s i_q + \frac{1}{L_s}\left(\frac{\omega}{\omega_s}V_E\cos(\delta) - \sqrt{2}V_g\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}i_q &= -\omega_s i_d - \frac{R_a}{L_s}i_q + \frac{1}{L_s}\frac{\omega}{\omega_s}V_E\sin(\delta), \\
\frac{\mathrm{d}}{\mathrm{d}t}i_0 &= -\frac{R_a}{L_s}i_0.
\end{aligned} \tag{60}$$

Here δ is used instead of θ as a state variable.

Example: two machines connected to each other, and feeding a resistive load

Consider two synchronous machines connected to each other, and feeding a resistive load R_L. The machines are described using the detailed model in (24). The reference angle of the first machine is θ_1, and the reference angle of the second machine is θ_2. A signal flow diagram is shown in Fig. 10.

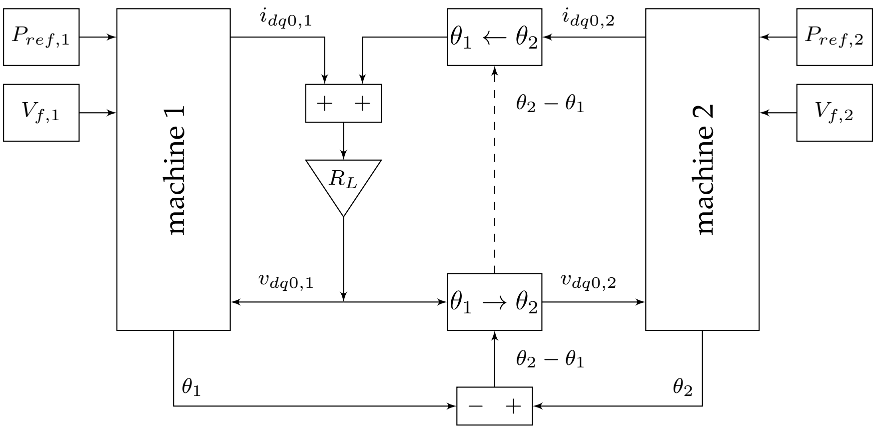

Figure 10: Signal-flow diagram: two machines connected to each other, and feeding a resistive load.

Permanent Magnet Synchronous Motor (PMSM)

The Permanent Magnet Synchronous Motor (PMSM) is a synchronous machine in which permanent magnets are embedded in the rotor to create a constant magnetic field. As in all synchronous machines, at steady state the rotor speed is proportional to the frequency of currents and voltages in the stator. For this reason such motors are especially useful in applications that require precise speed or position control. This section presents a $dq0$ model of a PMSM with sinusoidal EMF (as opposed to motors with trapezoidal EMF).

PMSMs may be modeled similar to synchronous generators, with three key modifications:

- ✓ The term $L_{af}i_f$ is replaced with λ, which is the amplitude of the flux induced in the stator by the permanent magnets on the rotor.
- ✓ The stator currents are defined positive when flowing into the machine.
- ✓ The electromagnetic torque accelerates the rotor, and the mechanical torque decelerates the rotor. The angular acceleration is defined as $\frac{\mathrm{d}}{\mathrm{d}t}\omega_m = \frac{1}{J}(T_e - T_m)$.

Based on these assumptions the resulting model is

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\theta &= \frac{p_f}{2}\omega_m, \\
\frac{\mathrm{d}}{\mathrm{d}t}\omega_m &= \frac{1}{J}(T_e - T_m), \\
\frac{\mathrm{d}}{\mathrm{d}t}i_d &= \frac{1}{L_d}v_d - \frac{R_a}{L_d}i_d + \frac{L_q}{L_d}\frac{p_f}{2}\omega_m i_q, \\
\frac{\mathrm{d}}{\mathrm{d}t}i_q &= \frac{1}{L_q}v_q - \frac{R_a}{L_q}i_q - \frac{L_d}{L_q}\frac{p_f}{2}\omega_m i_d - \frac{\lambda p_f \omega_m}{2L_q}, \\
\frac{\mathrm{d}}{\mathrm{d}t}i_0 &= \frac{1}{L_0}v_0 - \frac{R_a}{L_0}i_0.
\end{aligned} \tag{61}$$

Here the reference angle for the $dq0$ transformation is the electrical angle θ.
The symbols are defined as follows:

- ✓ θ denotes the rotor electrical angle;
- ✓ p_f denotes the number of magnetic poles on the rotor (even integer);
- ✓ L_d, L_q, L_0 denote the direct axis, quadrature axis, and zero sequence inductances;
- ✓ R_a denotes the resistance of the stator windings;
- ✓ i_d, i_q, i_0 denote the stator currents (positive when flowing into the machine);
- ✓ v_d, v_q, v_0 denote the stator terminal voltages;
- ✓ ω_m denotes the angular velocity of the rotor;
- ✓ λ denotes the amplitude of the flux induced in the stator phases by the permanent magnets on the rotor;
- ✓ J denotes the rotor moment of inertia;
- ✓ T_m, T_e denote the mechanical and electromagnetic torques;
- ✓ p_m, p_e denote the mechanical and electromagnetic powers.

The inputs are v_d, v_q, v_0, T_m, and several outputs are

$$\begin{aligned}
T_e &= \frac{3p_f}{4}\left(\lambda i_q + (L_d - L_q)\, i_d i_q\right), \\
p_m &= T_m\omega_m, \quad p_e = T_e\omega_m.
\end{aligned} \tag{62}$$

Similar to synchronous generators this model may be simplified by assuming a round rotor, so that $L_d = L_q = L_s$. In this case the motor may be described by the equivalent circuit shown in Fig. 11. In this simplified model the internal voltage source is described as

$$\begin{bmatrix} e_d \\ e_q \\ e_0 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda\frac{p_f}{2}\omega_m \\ 0 \end{bmatrix}, \tag{63}$$

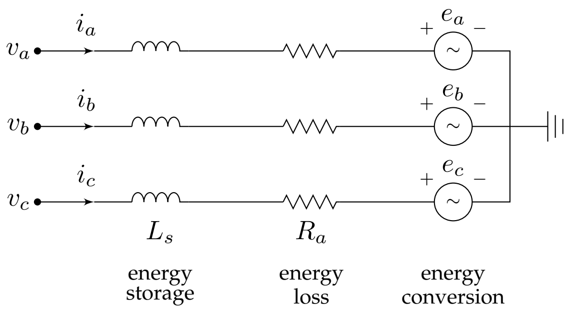

Figure 11: Equivalent circuit for a permanent magnet synchronous motor with a round rotor (assuming $L_d = L_q = L_s$).

where $\begin{bmatrix} e_d & e_q & e_0 \end{bmatrix}^\mathsf{T}$ is the $dq0$ transformation of $\begin{bmatrix} e_a & e_b & e_c \end{bmatrix}^\mathsf{T}$. In addition, for $L_d = L_q = L_s$ the electromagnetic torque is given by

$$T_e = \frac{3}{4} p_f \lambda i_q. \tag{64}$$

As we have seen before, the magnitude of the internal voltage is proportional to the angular velocity, and the electromagnetic torque is proportional to the quadrature axis current.

The heart of the energy conversion process is described by the internal voltage source, which converts electrical energy to mechanical energy. From a mechanical perspective, the electromagnetic power is

$$p_e = T_e \omega_m = \frac{3}{4} p_f \lambda \omega_m i_q, \tag{65}$$

and from an electrical perspective,

$$\begin{aligned} p_e &= \frac{3}{2} \left(e_d i_d + e_q i_q + 2 e_0 i_0 \right) \\ &= \frac{3}{2} \left(0 \cdot i_d + \lambda \frac{p_f}{2} \omega_m \cdot i_q + 2 \cdot 0 \cdot 0 \right) \\ &= \frac{3}{4} p_f \lambda \omega_m i_q. \end{aligned} \tag{66}$$

Both expressions are identical.

A basic control scheme for the PMSM is shown in Fig. 12. The design consists of two loops: an inner *current loop* and an outer *speed loop*. The inner loop regulates the currents such that $i_d \approx i_d^*$ and $i_q \approx i_q^*$, by adjusting the inverter duty cycles. The objective of the outer loop is to regulate the speed, such that in steady state $\omega_m \approx \omega_m^*$. This is implemented by controlling i_q^* based on the approximated relation between torque and current $T_e = \frac{3}{4} p_f \lambda i_q$. If the speed ω_m is too low then i_q^* increases to produce more torque, and if ω_m is too high then i_q^* decreases to produce less torque. Two sensors are placed on the motor to measure the speed ω_m and the electrical angle θ. The latter is used as a reference angle for the $dq0$ transformation.

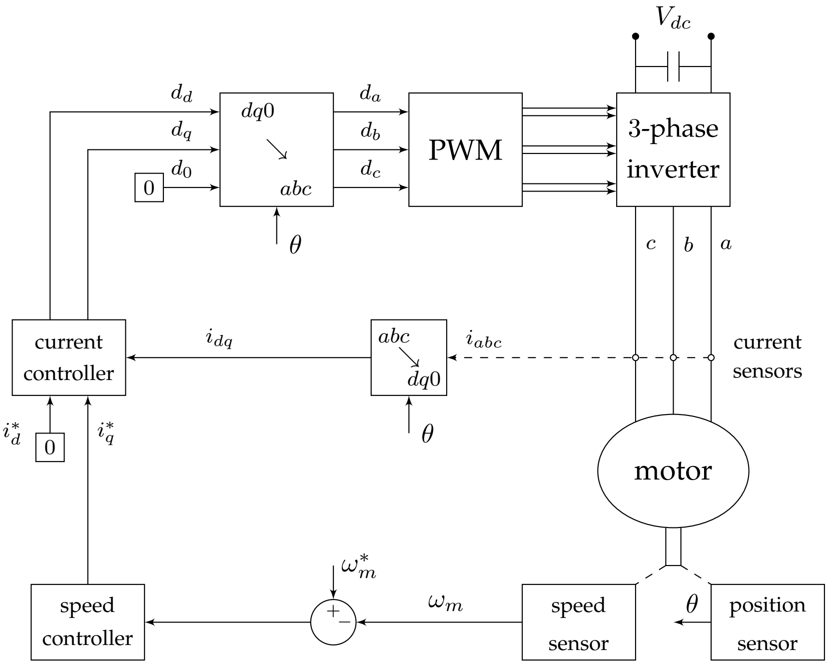

Figure 12: A basic control scheme for a permanent magnet synchronous motor (PMSM).

Appendix A: summary of symbols

p_f - number of magnetic poles on the rotor (even integer)	θ - rotor electrical angle	$V_E = \omega_s L_{af} I_f$ - internal voltage at nominal frequency, peak value
J - rotor moment of inertia	$\omega = \mathrm{d}\theta/\mathrm{d}t$ - rotor electrical frequency	$\lvert E\rvert = \frac{\omega V_E}{\sqrt{2}\omega_s} = \omega L_{af} I_f/\sqrt{2}$ - internal voltage at rotor frequency, RMS value
D - damping coefficient	$\omega_m = \frac{2}{p_f}\omega$ - rotor mechanical frequency	i_d, i_q, i_0 - stator currents
L_d, L_q - direct axis and quadrature axis synchronous inductances	ω_s - nominal frequency, $2\pi 50$ or $2\pi 60$ rad/sec	v_d, v_q, v_0 - stator terminal voltages
L_0 - zero sequence inductance	T_m - mechanical torque (accelerating the rotor for generator)	$K = \left(\frac{p_f}{2}\right)^2 \frac{1}{J\omega_s}$ - swing equation constant
$L_s = \frac{1}{2}(L_d + L_q)$ synchronous inductance in the simplified model	T_e - electromagnetic torque (decelerating the rotor for generator)	e_d, e_q, e_0 - internal voltages in the simplified machine model
L_{af} - stator to rotor mutual inductance (maximum value)	P_{ref} - reference power for the droop controller	v_f - field winding voltage
L_{ff} - field winding self-inductance	$p_m = T_m \omega_m$ - mechanical power	i_f - field winding current
R_a - stator winding resistance	$p_e = T_e \omega_m$ - electromagnetic power	λ_d, λ_q, λ_0 - stator flux linkages
R_f - field winding resistance	p_s - output power	λ_f - field winding flux linkage

All the $dq0$ quantities in this table are defined in the rotor reference frame (with respect to θ).

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