Homework Assignment 2

General guidelines:

- ✓ You may work alone or in pairs.
- ✓ Submit the results as a PDF file to yoashl@ee.technion.ac.il.
- ✓ Clearly state your name(s) and ID(s) at the top of the first page and in the Email.
- ✓ If you need help with Matlab or Simulink please contact me.

This assignment focuses on the basics of the $dq0$ transformation.

Question 1: Calculate the $dq0$ transformation of the following signals. The reference angle is $\theta = \omega t$ and ω is a given constant.

1. $\left(-5\sin(\omega t), -5\sin\left(\omega t - \frac{2\pi}{3}\right), -5\sin\left(\omega t + \frac{2\pi}{3}\right)\right)^{\mathsf{T}}$
2. $\left(\sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right), \sin\left(\omega t + \frac{\pi}{3}\right) - \cos\left(\omega t + \frac{\pi}{3}\right), \cos\left(\omega t + \frac{2\pi}{3}\right) - \sin\left(\omega t + \frac{2\pi}{3}\right)\right)^{\mathsf{T}}$
3. $(1, 1, 1)^{\mathsf{T}}$
4. $(1, 1, 2)^{\mathsf{T}}$
5. $\begin{pmatrix} \cos(\omega t)\left(1 + \frac{1}{10}\cos(\omega t)\right) \\ -\cos\left(\omega t + \frac{\pi}{3}\right)\left(1 + \frac{1}{10}\cos(\omega t)\right) \\ \cos\left(\omega t + \frac{2\pi}{3}\right)\left(1 + \frac{1}{10}\cos(\omega t)\right) \end{pmatrix}$

Hints:

- ✓ Use the definition of the dq0 transformation shown in class.
- ✓ You may use Matlab's Symbolic Math Toolbox to do complex algebraic calculations.

Question 2: Prove the following identities:

1. $T_\theta T_\theta^{\mathsf{T}} = \frac{2}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$

   Hint: use the expressions for T_θ and T_θ^{-1} shown in class, and find a matrix U such that $T_\theta^{\mathsf{T}} = T_\theta^{-1} U$.
2. $\frac{\mathrm{d}}{\mathrm{d}t} T_\theta = \mathcal{W} T_\theta$ with $\mathcal{W} = \begin{pmatrix} 0 & \frac{\mathrm{d}\theta}{\mathrm{d}t} & 0 \\ -\frac{\mathrm{d}\theta}{\mathrm{d}t} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
3. $T_{\theta_a} T_{\theta_b}^{-1} = \begin{pmatrix} \cos(\theta_a - \theta_b) & \sin(\theta_a - \theta_b) & 0 \\ -\sin(\theta_a - \theta_b) & \cos(\theta_a - \theta_b) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   Hint: it is easier to prove that $T_{\theta_a} = \begin{pmatrix} \cos(\theta_a - \theta_b) & \sin(\theta_a - \theta_b) & 0 \\ -\sin(\theta_a - \theta_b) & \cos(\theta_a - \theta_b) & 0 \\ 0 & 0 & 1 \end{pmatrix} T_{\theta_b}$.

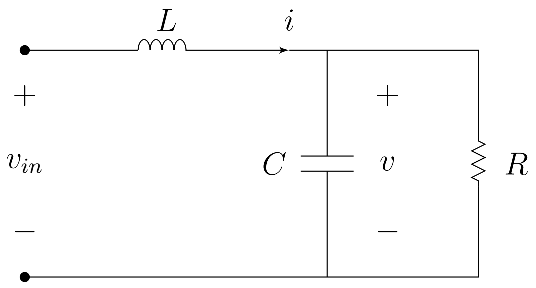

Figure 1: Linear circuit, single-phase diagram.

Question 3: Consider the linear three-phase circuit shown in Figure 1.

1. Write the circuit's state-space model in the abc reference frame. Use the state vector $x_{abc} = (i_a, i_b, i_c, v_a, v_b, v_c)^\mathsf{T}$ and the input vector $u = (v_{in,a}, v_{in,b}, v_{in,c})^\mathsf{T}$.
2. Convert the state-space model to the $dq0$ reference frame. Use the reference angle $\theta = \omega t$.
   Hint: use the method shown in class, and show that in this case $\Lambda_\theta A = A\Lambda_\theta$.
3. Express the total energy $E(t)$ stored in the circuit as a function of the state variables in the $dq0$ reference frame.
4. Show that $\mathrm{d}E/\mathrm{d}t =$ (input power) $-$ (power loss).

Question 4: Consider a hypothetical three-phase device with terminal voltages v_a, v_b, v_c and terminal currents i_a, i_b, i_c. The device is described by the state-space model

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\theta &= \omega, \\
\frac{\mathrm{d}}{\mathrm{d}t}\omega &= K\left(P_{in} - I_g\cos(\theta)v_a - I_g\cos\left(\theta - \frac{2\pi}{3}\right)v_b - I_g\cos\left(\theta + \frac{2\pi}{3}\right)v_c\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}v_a &= \frac{1}{C}\left(\frac{\omega}{\omega_s}I_g\cos(\theta) - i_a\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}v_b &= \frac{1}{C}\left(\frac{\omega}{\omega_s}I_g\cos\left(\theta - \frac{2\pi}{3}\right) - i_b\right), \\
\frac{\mathrm{d}}{\mathrm{d}t}v_c &= \frac{1}{C}\left(\frac{\omega}{\omega_s}I_g\cos\left(\theta + \frac{2\pi}{3}\right) - i_c\right),
\end{aligned}
$$

where

- ✓ The inputs are i_a, i_b, i_c, P_{in}.
- ✓ The parameters K, C, I_g are given constants.

Find an equivalent state-space model in the $dq0$ reference frame with inputs i_d, i_q, i_0, P_{in}, and outputs v_d, v_q, v_0. Use the reference angle θ.
Hints:

- ✓ Express the second equation as $\frac{\mathrm{d}}{\mathrm{d}t}\omega = K\left(P_{in} - \frac{3}{2}(I_g\ \ 0\ \ 0)T_\theta v_{abc}\right)$.
- ✓ Write the last three equations in matrix form using the transformation T_θ^{-1} and the vectors $v_{abc} = (v_a, v_b, v_c)^\mathsf{T}$, $i_{abc} = (i_a, i_b, i_c)^\mathsf{T}$. Then express these vectors as $v_{abc} = T_\theta^{-1}v_{dq0}$ and $i_{abc} = T_\theta^{-1}i_{dq0}$ and use the $dq0$ identities shown in class.