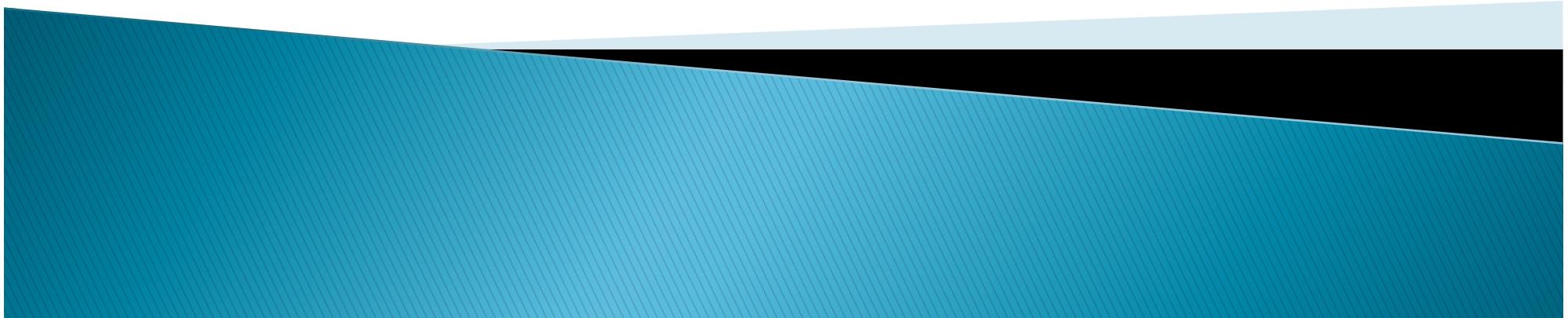


NN-based model structures for identification and control of nonlinear systems.

Non-fully connected neural networks

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Model Based Control

Model structure independent

Model structure dependent

Neural Networks (NN) based Models



- Choosing proper structure of the model may increase accuracy of the model
- Adaptivity by using network's ability to learn



- Model may become
 - Nonanalytical or hardly analytical
 - Overparameterized

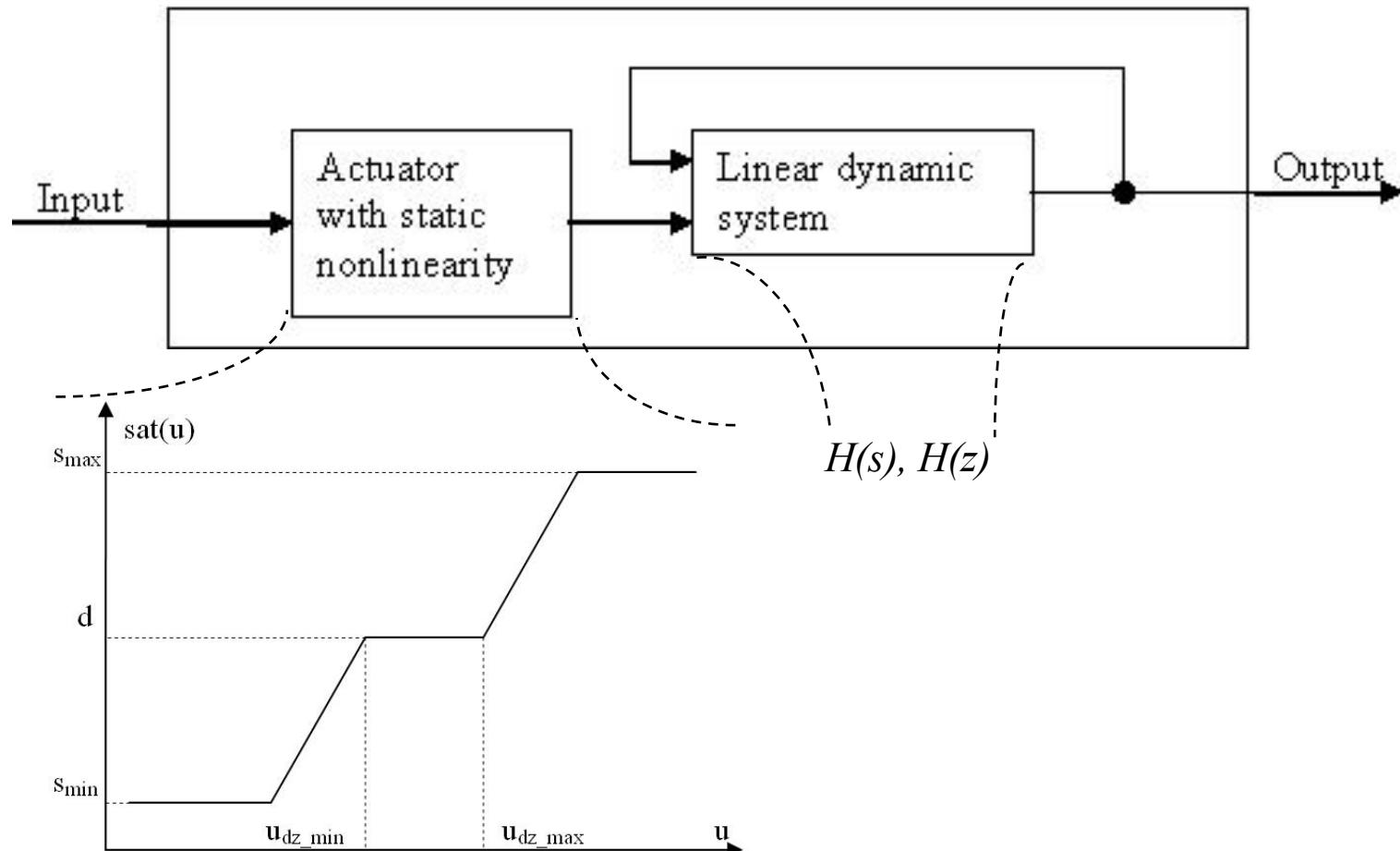


- Easy realization of the model's structure relevant to the control algorithm
- Adaptivity by using network's ability to learn
- Analytical models (for example, state-space representation of nonlinear models)

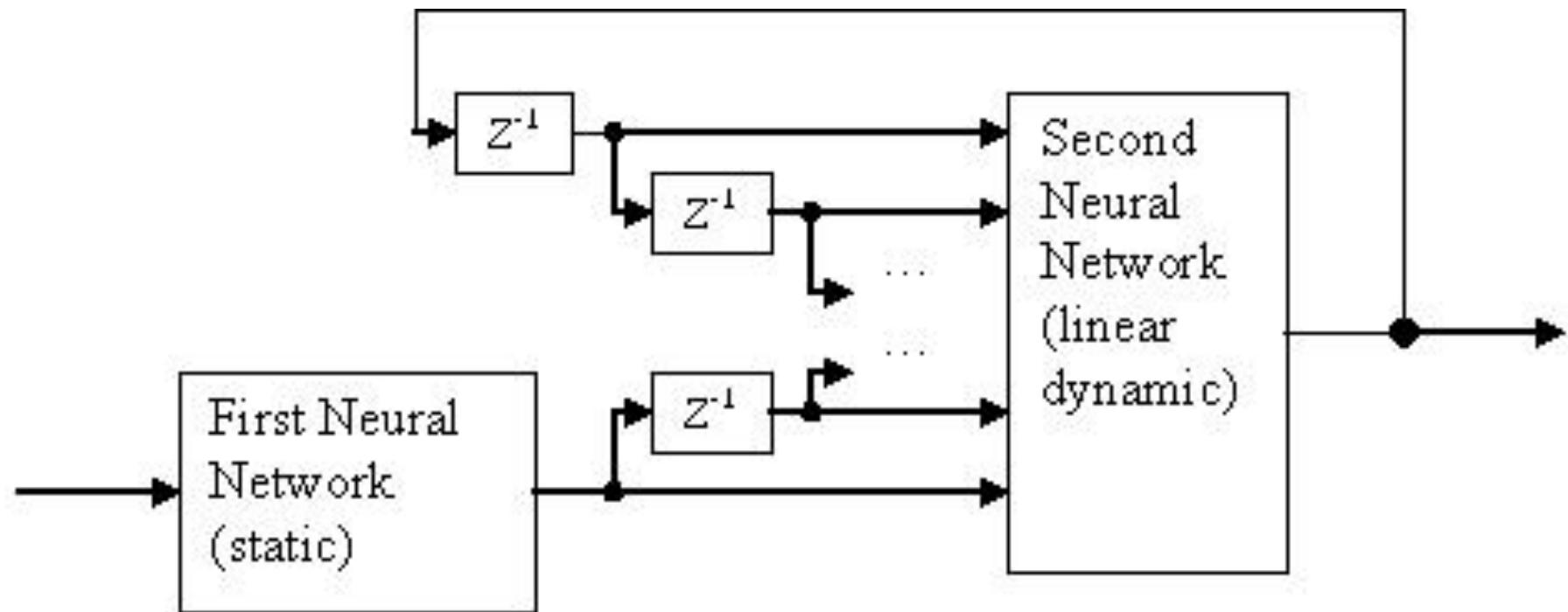


- Restricted class of systems to which the control technique can be applied

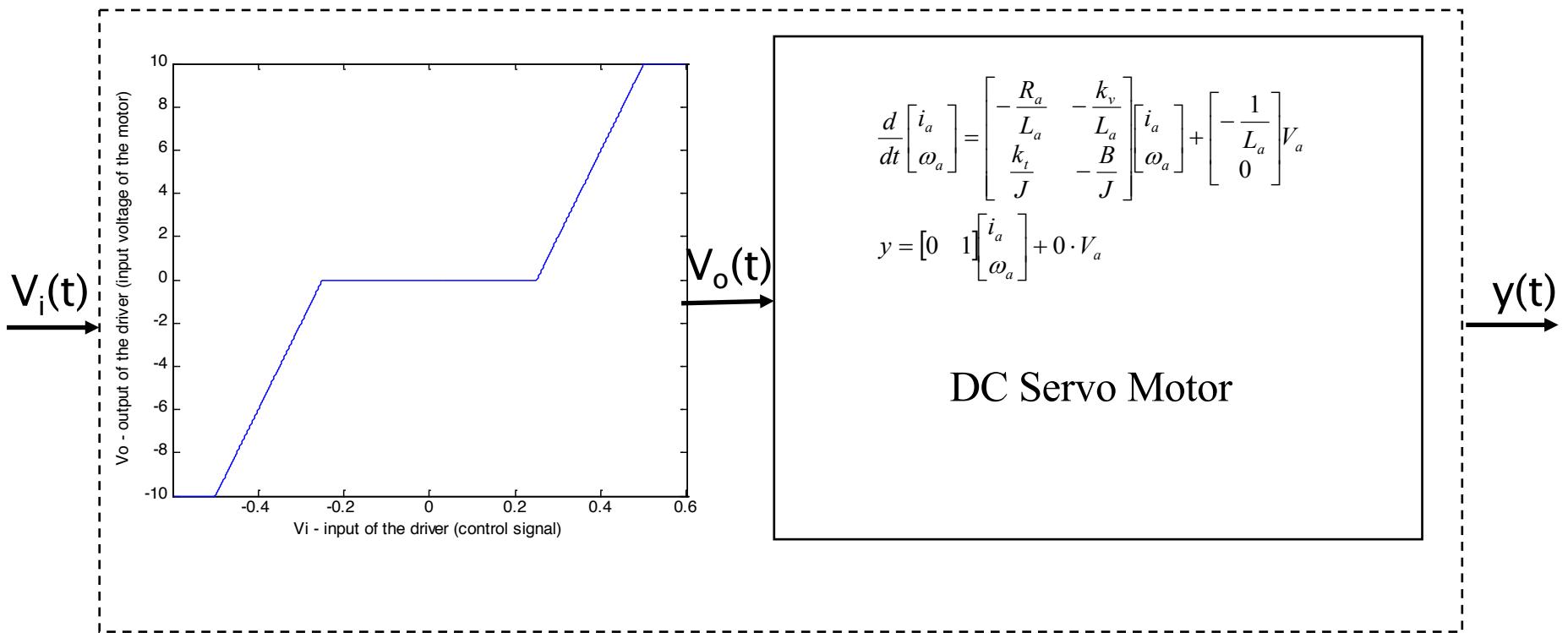
Linear dynamic system with static actuator nonlinearity



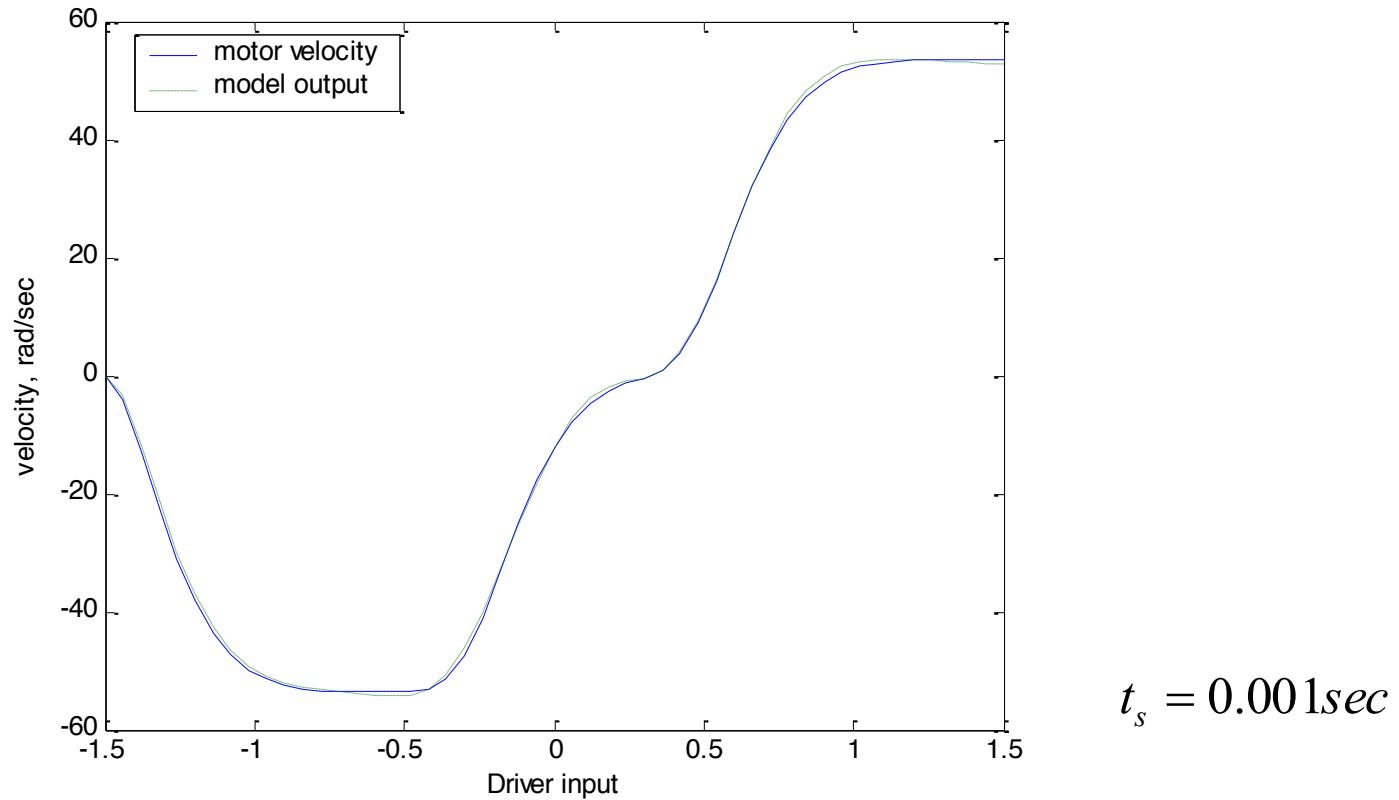
Neural Network Structure for dynamic systems with static actuator nonlinearity



DC Servo Motor with Nonlinear Driver



Identification of the DC Servo Motor



Nonlinear Discrete-Time Input-Output Models

NARX (Nonlinear Autoregressive Exogenous) model:

$$y(t+n) = f(y(t), y(t+1), \dots, y(t+n-1), u(t), u(t+1), \dots, u(t+n-1))$$

ANARX (Additive Nonlinear Autoregressive Exogenous) model:

$$y(t+n) = f_1(y(t), u(t)) + f_2(y(t+1), u(t+1)) + \dots + f_n(y(t+n-1), u(t+n-1))$$

or

$$y(t+n) = \sum_{i=1}^n f_i(y(t+i-1), u(t+i-1))$$

Advantages of ANARX Structure over NARX

- Easy to change the order of the model
- Always realizable in the classical state-space form

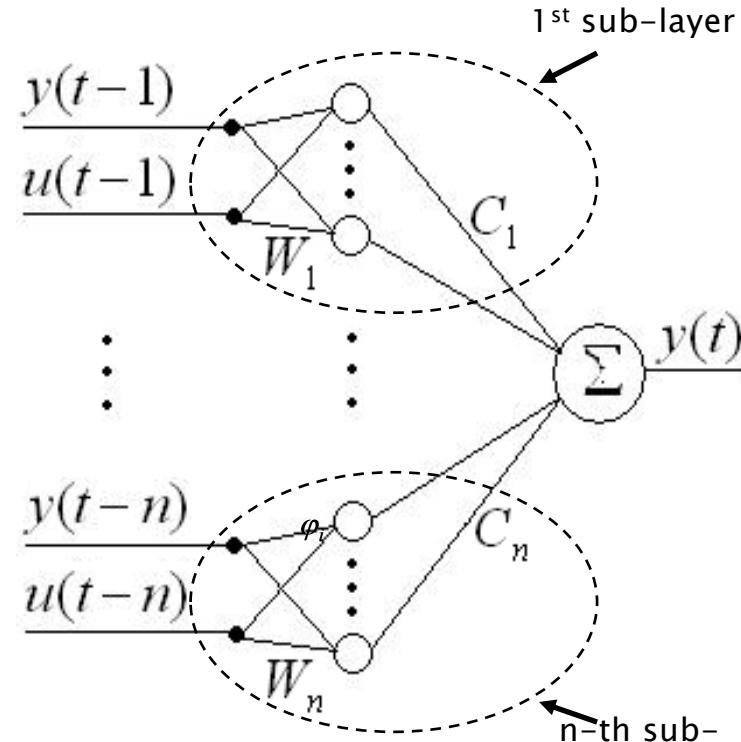
$$y(t+n) = \sum_{i=1}^n f_i(y(t+i-1), u(t+i-1))$$



$$\begin{cases} x_1(t+1) = x_2(t) + f_1(x_1(t), u(t)) \\ x_2(t+1) = x_3(t) + f_2(x_1(t), u(t)) \\ \vdots \\ x_{n-1}(t+1) = x_n(t) + f_{n-1}(x_1(t), u(t)) \\ x_n(t+1) = f_n(x_1(t), u(t)) \\ y(t) = x_1(t) \end{cases}$$

- Linearizable by dynamic feedback

Neural Network based ANARX (NN-ANARX) Model



$$y(t+n) = \sum_{i=1}^n C_i \varphi_i \left(W_i \cdot [y(t+i-1), u(t+i-1)]^T \right)$$

φ_i is a sigmoid function

ANARX Model based Dynamic Output Feedback Linearization Algorithm

ANARX model

$$y(t+n) = \sum_{i=1}^n f_i(y(t+i-1), u(t+i-1))$$

NN-ANARX model

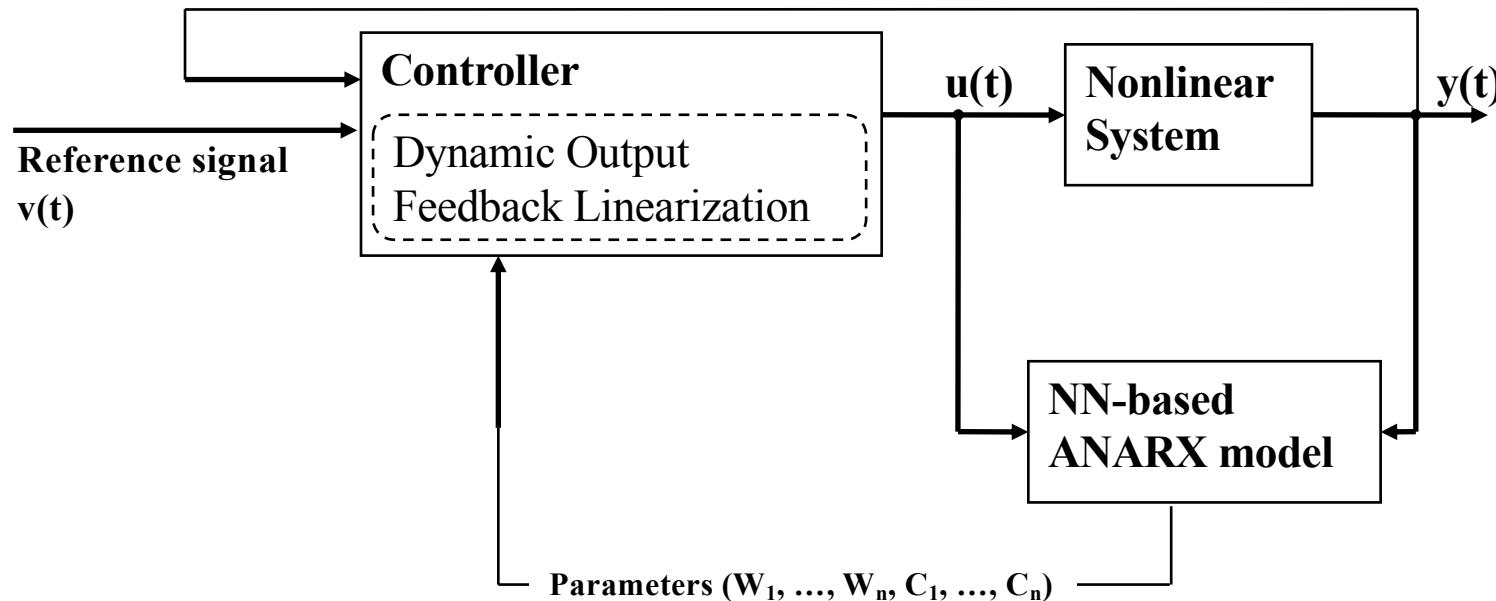
$$y(t+n) = \sum_{i=1}^n C_i \varphi_i \left(W_i \cdot [y(t+i-1), u(t+i-1)]^T \right)$$

$$\begin{cases} F = f_1(y(t), u(t)) = \eta_1(t) \\ \eta_1(t+1) = \eta_2(t) - f_2(y(t), u(t)) \\ \vdots \\ \eta_{n-2}(t+1) = \eta_{n-1}(t) - f_{n-1}(y(t), u(t)) \\ \eta_{n-1}(t+1) = v(t) - f_n(y(t), u(t)) \end{cases} \quad \Rightarrow \quad \text{NN}$$

$$\begin{cases} F = C_1 \varphi_1 \left(W_1 \cdot [y(t), u(t)]^T \right) = \eta_1(t) \\ \eta_1(t+1) = \eta_2(t) - C_2 \varphi_2 \left(W_2 \cdot [y(t), u(t)]^T \right) \\ \vdots \\ \eta_{n-2}(t+1) = \eta_{n-1}(t) - C_{n-1} \varphi_{n-1} \left(W_{n-1} \cdot [y(t), u(t)]^T \right) \\ \eta_{n-1}(t+1) = v(t) - C_n \varphi_n \left(W_n \cdot [y(t), u(t)]^T \right) \end{cases}$$

$$u(t) = F^{-1}(y(t), \eta_1(t))$$

NN-ANARX Model based Control of Nonlinear Systems



* HSA=History-Stack Adaptation

NN-ANARX Model based Control of Nonlinear MIMO Systems: Problem Statement

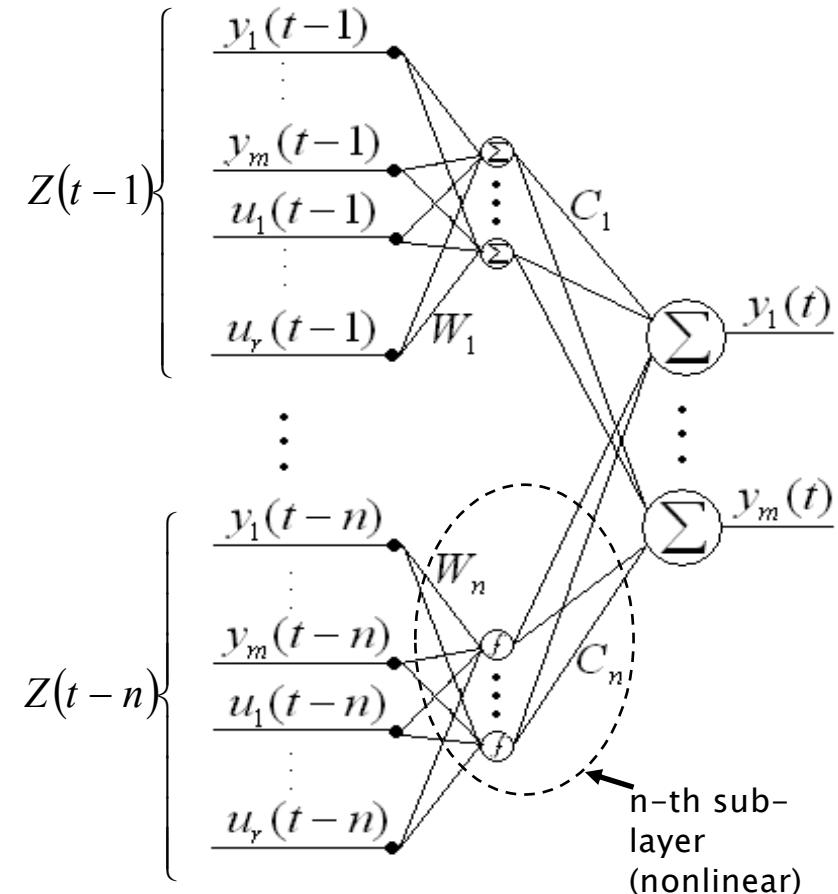
$$F = C_1 \varphi_1 \left(W_1 \cdot [Y(t), U(t)]^T \right) = \eta_1(t) \quad U(t) = ?$$

SISO systems: Numerical calculation of $u(t)$ by Newton's Method, which needs several iterations to converge

MIMO systems: Numerical algorithms do not converge or calculation takes too much time



NN-based Simplified ANARX Model (NN-SANARX)



$$y(t) = C_1 \cdot W_1 \cdot Z(t-1) + \sum_{i=2}^n C_i \varphi_i(W_i \cdot Z(t-i))$$

NN-SANARX Model based Control (1)

$$C_1 \varphi_1 \left(W_1 \cdot [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]^T \right) = [\eta_{11}(t), \dots, \eta_{1m}(t)]$$



$$C_1 \cdot \boxed{W_1 \cdot [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]^T} = [\eta_{11}(t), \dots, \eta_{1m}(t)] \quad (*)$$

$$\boxed{W_{11} \cdot [y_1(t), \dots, y_m(t)]^T + W_{12} \cdot [u_1(t), \dots, u_r(t)]^T}$$

Let's define: $T_1 = C_1 \cdot W_{11}$ $T_2 = C_1 \cdot W_{12}$

$$T_1 \cdot [y_1(t), \dots, y_m(t)]^T + T_2 \cdot [u_1(t), \dots, u_r(t)]^T = [\eta_{11}(t), \dots, \eta_{1m}(t)]$$

where $T_1 \in \Re^{m \times m}$ $T_2 \in \Re^{m \times r}$

If $m=r$ and T_2 is not singular or close to singular then

$$[u_1(t), \dots, u_r(t)]^T = T_2^{-1} \left([\eta_{1,1}(t), \dots, \eta_{1,m}(t)]^T - T_1 \cdot [y_1(t), \dots, y_m(t)]^T \right)$$



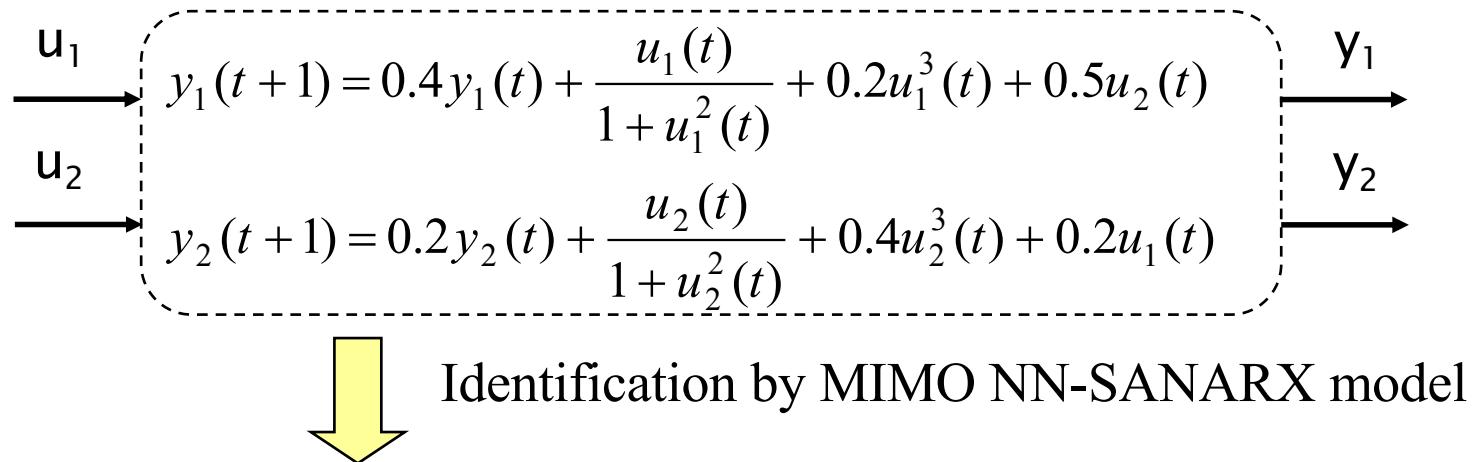
NN-SANARX Model based Control (2)

$$[u_1(t), \dots, u_r(t)]^T = T_2^{-1} \left([\eta_{1,1}(t), \dots, \eta_{1,m}(t)]^T - T_1 \cdot [y_1(t), \dots, y_m(t)]^T \right)$$

$$\begin{cases} [\eta_{1,1}(t+1), \dots, \eta_{1,m}(t+1)]^T = [\eta_{2,1}(t), \dots, \eta_{2,m}(t)]^T - \\ \quad - C_2 \varphi_2 (W_2 \cdot [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]^T) \\ \vdots \\ [\eta_{n-2,1}(t+1), \dots, \eta_{n-2,m}(t+1)]^T = [\eta_{n-1,1}(t), \dots, \eta_{n-1,m}(t)]^T - \\ \quad - C_{n-1} \varphi_{n-1} (W_{n-1} \cdot [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]^{T^T}) \\ [\eta_{n-1,1}(t+1), \dots, \eta_{n-1,m}(t+1)]^T = [v_1(t), \dots, v_m(t)] - \\ \quad - C_n \varphi_n (W_n \cdot [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]^T) \end{cases}$$



Numerical Example



$$\begin{aligned}[y_1(t), y_2(t)]^T &= C_1 \cdot W_1 \cdot [y_1(t-1), y_2(t-1), u_1(t-1), u_2(t-1)]^T + \\ &\quad + C_2 \cdot \varphi_2 \left(W_2 \cdot [y_1(t-2), y_2(t-2), u_1(t-2), u_2(t-2)]^T \right)\end{aligned}$$

$$l_1 = 2 \quad l_2 = 5$$

$$\varphi_2(I) = \frac{1 - e^{-\alpha \cdot I}}{1 + e^{-\alpha \cdot I}}$$



Numerical Example: NN-based SANARX model

$$W_1 = \begin{bmatrix} 0.9239 & -1.2909 & 0.3075 & -0.7308 \\ -1.2669 & -0.2253 & -0.7675 & -0.6805 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0.0358 & -0.0214 & -0.0046 & -0.0424 \\ 0.1837 & -0.2258 & -0.0553 & -0.0473 \\ 0.0259 & -0.0167 & -0.0094 & -0.0373 \\ 0.2637 & 0.1549 & 0.0501 & 0.1054 \\ 0.2699 & 0.1615 & 0.0513 & 0.1096 \end{bmatrix}$$

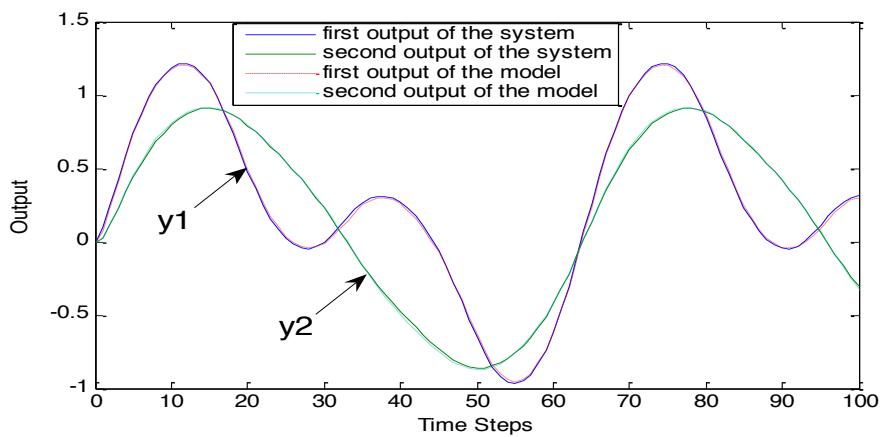
$$C_1 = \begin{bmatrix} 0.1414 & -0.8987 \\ -0.7948 & -0.5827 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -84.6868 & -0.1466 & 107.9394 & -7.3793 & 6.8476 \\ 1.9867 & -1.6542 & 22.1403 & -27.8546 & 25.9052 \end{bmatrix}$$

$$u_{v1}(t) = 0.4 \sin(0.2 \cdot t) + 0.1$$

$$u_{v2}(t) = 0.8 \sin(0.1 \cdot t)$$

$$MSE \approx 1.36 \cdot 10^{-4}$$



Numerical Example: Control (1 / 2)

$$W_1 = \begin{bmatrix} 0.9239 & -1.2909 \\ -1.2669 & -0.2253 \end{bmatrix}$$

$$W_{11} = \begin{bmatrix} 0.9239 & -1.2909 \\ -1.2669 & -0.2253 \end{bmatrix} \quad W_{12} = \begin{bmatrix} 0.3075 & -0.730 \\ -0.7675 & -0.6805 \end{bmatrix}$$

$$T_1 = W_{11} \cdot C_1 = \begin{bmatrix} 1.2720 & 0.0161 \\ 0.0039 & 1.1573 \end{bmatrix} \quad T_2 = W_{12} \cdot C_1 = \begin{bmatrix} 0.7341 & 0.5061 \\ 0.2028 & 0.9773 \end{bmatrix}$$

$$\det(T_2) \approx 0.6 \rightarrow \text{rank}(T_2) = 2$$

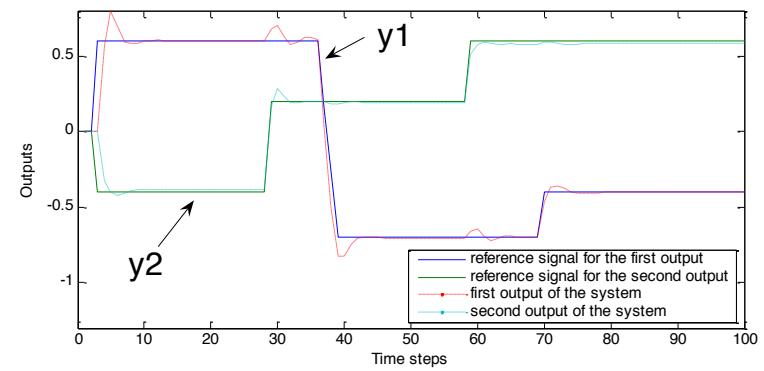
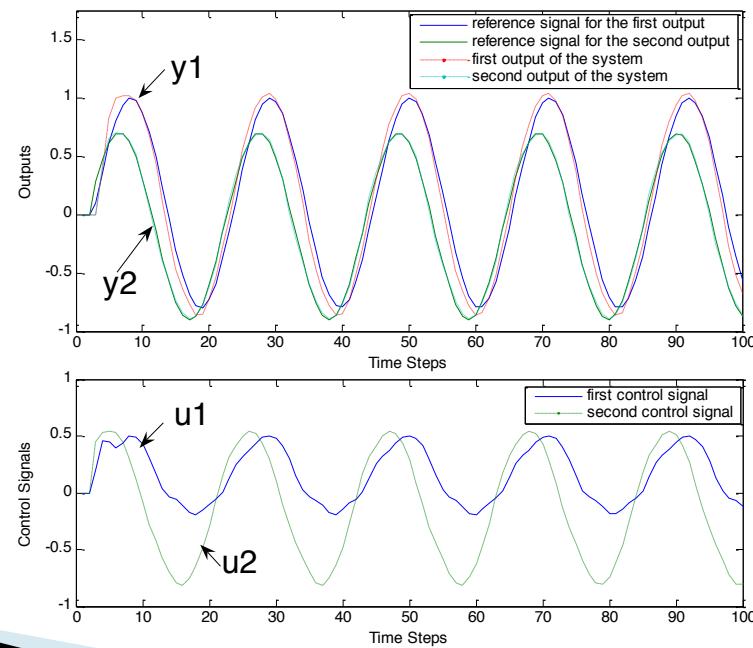
$$T_2^{-1} = \begin{bmatrix} 1.5896 & -0.8231 \\ -0.3299 & 1.1940 \end{bmatrix}$$

$$[u_1(t), u_2(t)]^T = T_2^{-1} ([\eta_{1,1}(t), \eta_{1,2}(t)]^T - T_1 \cdot [y_1(t), y_2(t)]^T)$$

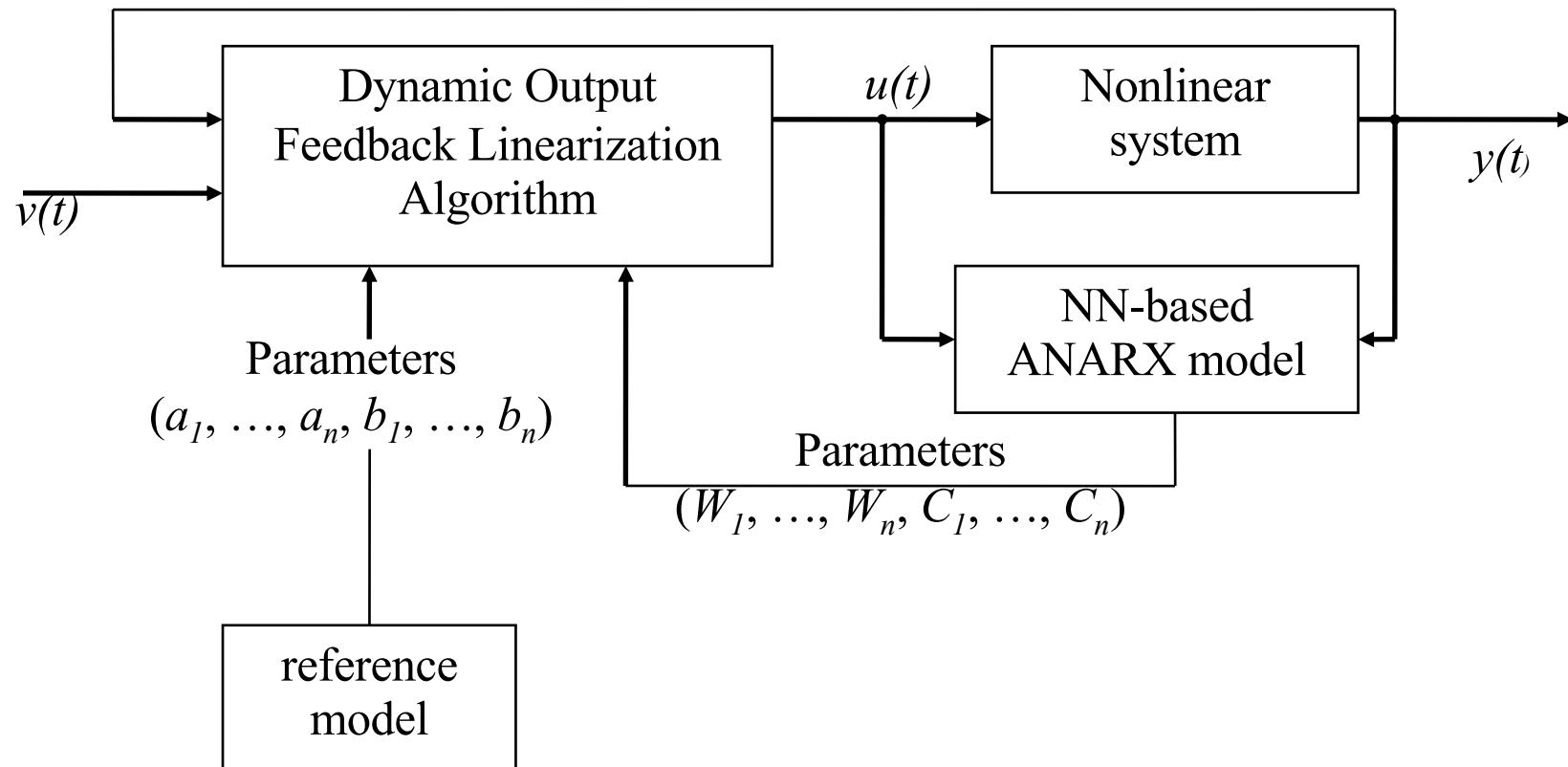
$$[\eta_{1,1}(t+1), \eta_{1,2}(t+1)]^T = [v_1(t), v_2(t)]^T - C_2 \varphi_2 (W_2 \cdot [y_1(t), y_2(t), u_1(t), u_2(t)]^T)$$

Numerical Example: Control (2 / 2)

$$\begin{aligned}
 [u_1(t), u_2(t)]^T &= T_2^{-1} \left([\eta_{1,1}(t), \eta_{1,2}(t)]^T - T_1 \cdot [y_1(t), y_2(t)]^T \right) \\
 \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} &= T_2^{-1} \left(\begin{bmatrix} v_1(t-1) \\ v_2(t-1) \end{bmatrix} - C_2 \varphi_2 \begin{bmatrix} W_2 \cdot y_2(t-1) \\ C_2 \varphi_2 (W_2^{-1} y_1(t-1), y_2(t-1), u_1(t-1), u_2(t-1))^T \end{bmatrix} \right)
 \end{aligned}$$



Model Reference Control



Reference model

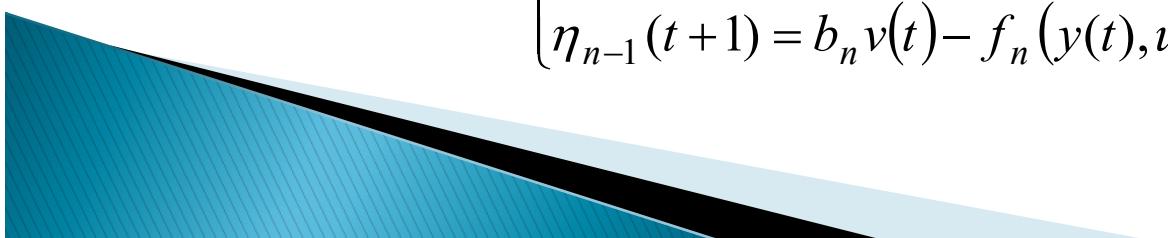
$$Y(z) = H_m(z) \cdot V(z)$$

$$y(t) + a_1 y(t-1) + \dots + a_n y(t-n) = b_1 v(t-1) + \dots + b_n v(t-n)$$

$$H_m(z) = \frac{b_1 z^{n-1} + b_2 z^{n-2} + \dots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_{n-1} z + a_n}$$

 *Dynamic output feedback linearization
of ANARX model*

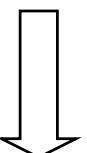
$$\begin{cases} F = f_1(y(t), u(t)) + a_1 y(t) - b_1 v(t) = \eta_1(t) \\ \eta_1(t+1) = \eta_2(t) + b_2 v(t) - f_2(y(t), u(t)) - a_2 y(t) \\ \vdots \\ \eta_{n-2}(t+1) = \eta_{n-1}(t) + b_{n-1} v(t) - f_{n-1}(y(t), u(t)) \\ \qquad \qquad \qquad - a_{n-1} y(t) \\ \eta_{n-1}(t+1) = b_n v(t) - f_n(y(t), u(t)) - a_n y(t) \end{cases}$$



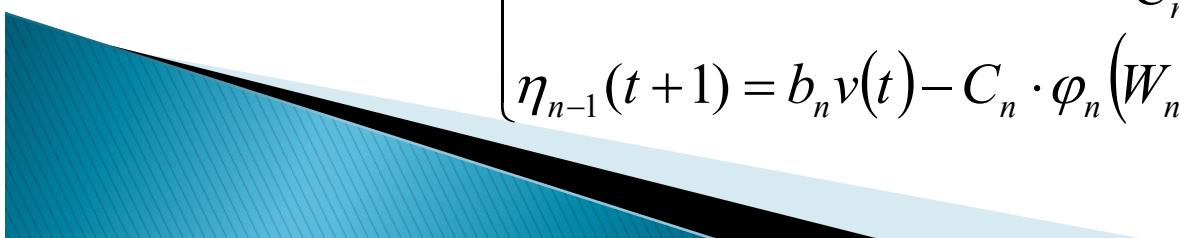
NN-ANARX Model based Reference Model Control

NN-ANARX model

$$y(t+n) = \sum_{i=1}^n C_i \varphi_i \left(W_i \cdot [y(t+i-1), u(t+i-1)]^T \right)$$

 *Dynamic output feedback linearization
of NN-ANARX model*

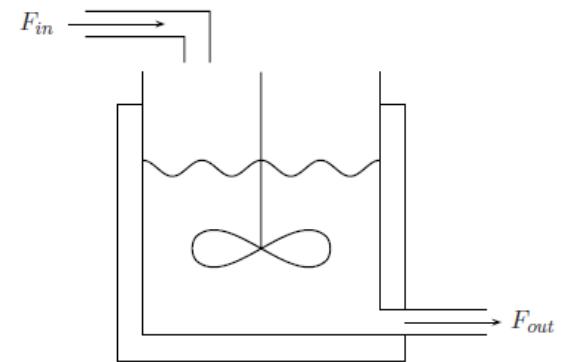
$$\begin{cases} F = C_1 \cdot \varphi_1 \left(W_1 \cdot [y(t), u(t)]^T \right) + a_1 y(t) - b_1 v(t) = \eta_1(t) \\ \eta_1(t+1) = \eta_2(t) + b_2 v(t) - a_2 y(t) - C_2 \cdot \varphi_2 \left(W_2 \cdot [y(t), u(t)]^T \right) \\ \vdots \\ \eta_{n-2}(t+1) = \eta_{n-1}(t) + b_{n-1} v(t) - a_{n-1} y(t) - C_{n-1} \cdot \varphi_{n-1} \left(W_{n-1} \cdot [y(t), u(t)]^T \right) \\ \eta_{n-1}(t+1) = b_n v(t) - C_n \cdot \varphi_n \left(W_n \cdot [y(t), u(t)]^T \right) - a_n y(t) \end{cases}$$



Numerical Example: JCSTR model

Input-Output equation:

$$\begin{aligned}y(t+2) = & 0.7653y(t+1) - 0.231y(t) + \\& + 0.4801u(t+1) - 0.6407y^2(t+1) + \\& + 1.014y(t)y(t+1) - 0.3921y^2(t+1) + \\& + 0.592y(t+1)u(t+1) - 0.5611y(t)u(t+1)\end{aligned}$$



Example: Control Task

Linear second order discrete time reference model with poles $p_1=-0.9$, $p_2=0.8$, zero $n_1=-0.5$, steady-state gain $K_{ss}=1$

$$H(z) = \frac{19}{75} \frac{z + 0.5}{(z + 0.9)(z - 0.8)} = \frac{19/75 z + 19/150}{z^2 + 0.1z - 0.72}$$

Parameters of the reference model for control algorithm are as follows:

$$b_1 = 19/75 \quad b_2 = 19/150 \quad a_1 = 0.1 \quad a_2 = -0.72$$

It defines the desired behavior of the closed loop system (=control system)



Example: NN-based model and control

First sub-layer is linear

$$y(t) = D \cdot [y(t-1), u(t-1)]^T + C_2 \varphi_2 \left(W_2 \cdot [y(t-2), u(t-2)]^T \right)$$

Here $D = C_1 \cdot W_1$ and $D = [d_1 \ d_2]$

We know that $D \cdot [y(t), u(t)]^T + a_1 y(t) - b_1 v(t) = \eta_1(t)$

$$\eta_1(t+1) = b_2 v(t) - a_2 y(t) - C_2 \cdot \varphi_2 \left(W_2 \cdot [y(t), u(t)]^T \right)$$

Thus,

$$u(t) = \frac{1}{d_2} \left(b_2 v(t-1) - a_2 y(t-1) - C_2 \cdot \varphi_2 \left(W_2 \cdot [y(t-1), u(t-1)]^T \right) + b_1 v(t) - d_1 \cdot y(t) - a_1 y(t) \right)$$

Example: NN-based model and control

