



Evolutionary Design of the Closed Loop Control on the Basis of NN-ANARX Model Using Genetic Algoritm



Principles of Genetic Algorithms

*Initial **Population** – a set of strings called **Chromosomes***

[0 1 0 1 1 0 1 0 ... 0 1 1 1 0 1]

[0 0 0 0 1 1 1 0 ... 1 1 0 1 0 1]

...

[0 1 0 0 0 0 1 0 ... 1 1 1 1 0 1]

*Calculation of **fitness function**,*

sort Chromosomes and choose the best ones



Principles of Genetic Algorithms

Formation of new generation:

1. Crossover

$[0\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ \dots\ 0\ 1\ 0\ 1\ 0\ 1]$
 $[1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ \dots\ 1\ 0\ 1\ 1\ 1\ 1]$
at random places

$[0\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ \dots\ 0\ 0\ 1\ 1\ 1\ 1]$
 $[1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ \dots\ 1\ 1\ 0\ 1\ 0\ 1]$

2. Mutation

$[0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ \dots\ 0\ 1\ 1\ 1\ 0\ 1]$
at random places (~1%)

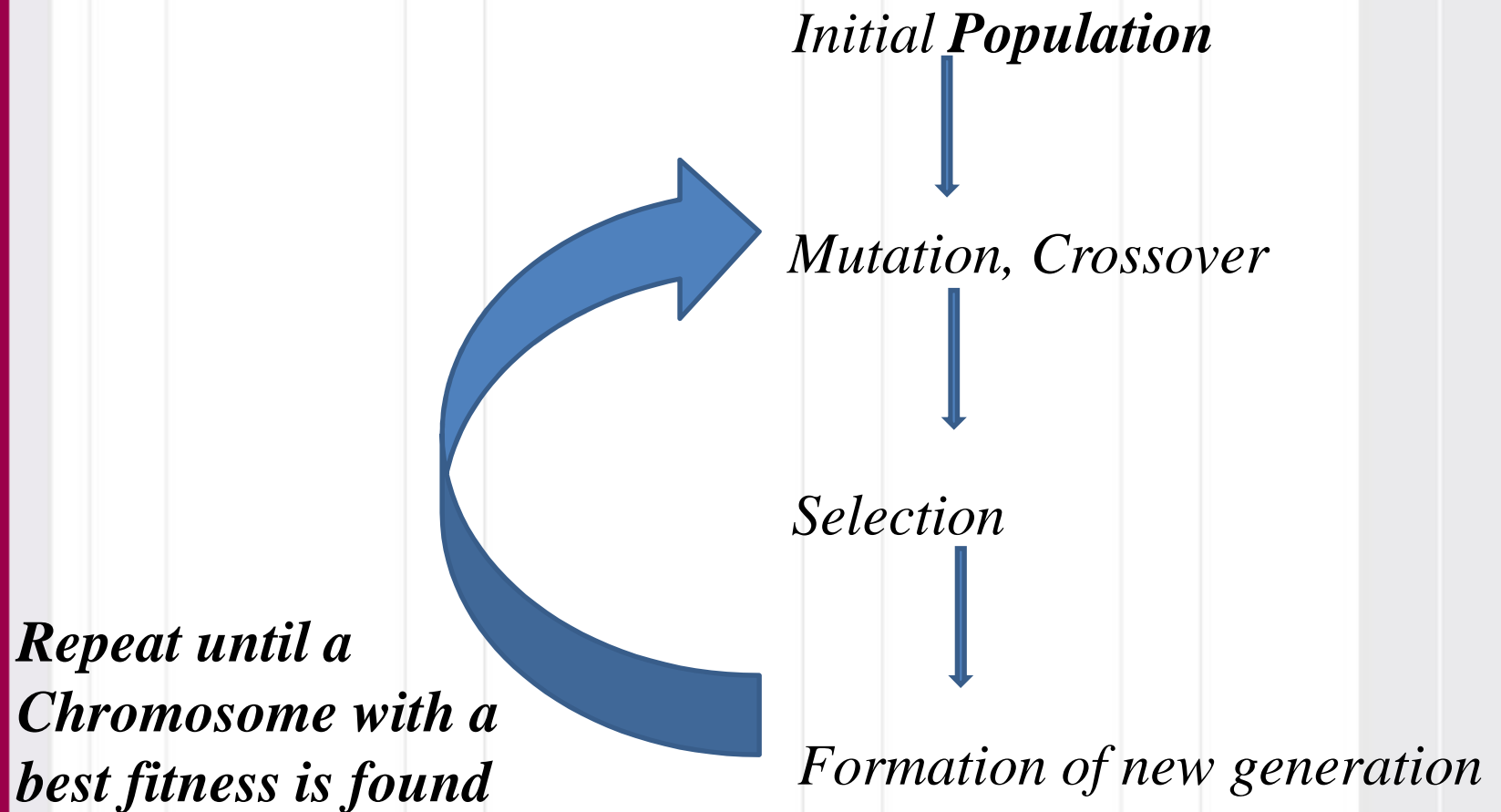
$[0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ \dots\ 0\ 1\ 1\ 1\ 0\ 1]$

“Best Parents” are used in crossover and mutation more frequently

3. “New Blood” – some absolutely new chromosomes

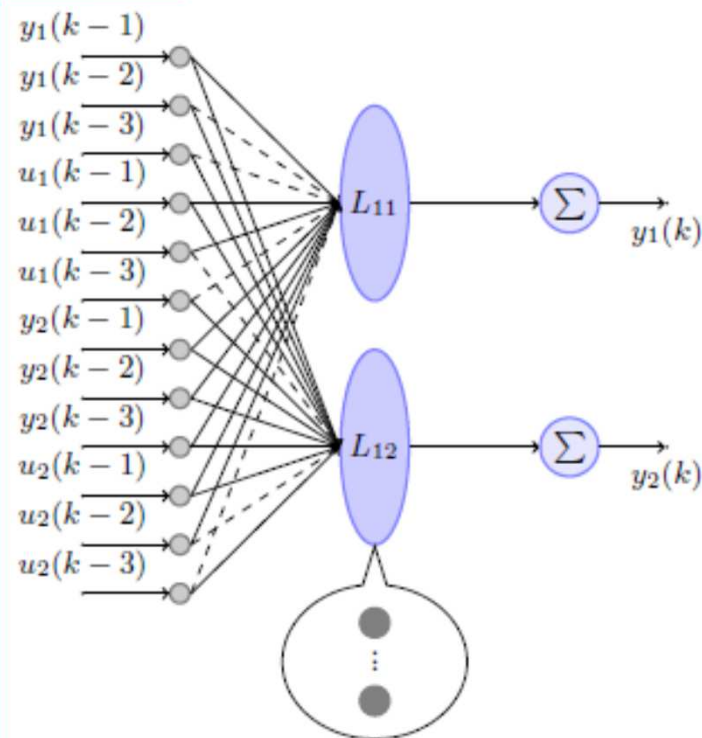


Principles of Genetic Algorithms





Principles of Genetic Algorithms for selection of Neural Network's Structure



For fully connected NN-NARX

gene = [111111111111 111111111111]

For custom structure NN

gene = [110101111001 111111111001]

Fitness function – for example, MSE



ANARX model

NARX (Nonlinear Autoregressive Exogenous) model:

$$y(t+n) = f(y(t), y(t+1), \dots, y(t+n-1), u(t), u(t+1), \dots, u(t+n-1))$$

ANARX (Additive Nonlinear Autoregressive Exogenous) model:

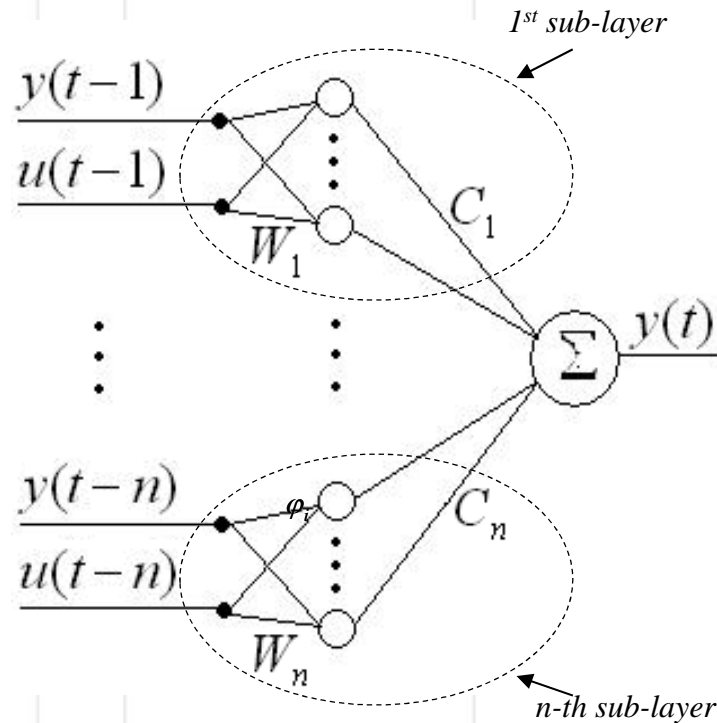
$$y(t+n) = f_1(y(t), u(t)) + f_2(y(t+1), u(t+1)) + \dots + f_n(y(t+n-1), u(t+n-1))$$

or

$$y(t+n) = \sum_{i=1}^n f_i(y(t+i-1), u(t+i-1))$$



NN-based ANARX model (NN-ANARX)



$$y(t+n) = \sum_{i=1}^n C_i \varphi_i \left(W_i \cdot [y(t+i-1), u(t+i-1)]^T \right)$$

φ_i is a sigmoid function



ANARX Model based Dynamic Output Feedback Linearization Algorithm

ANARX model

$$y(t+n) = \sum_{i=1}^n f_i(y(t+i-1), u(t+i-1))$$

NN-ANARX model

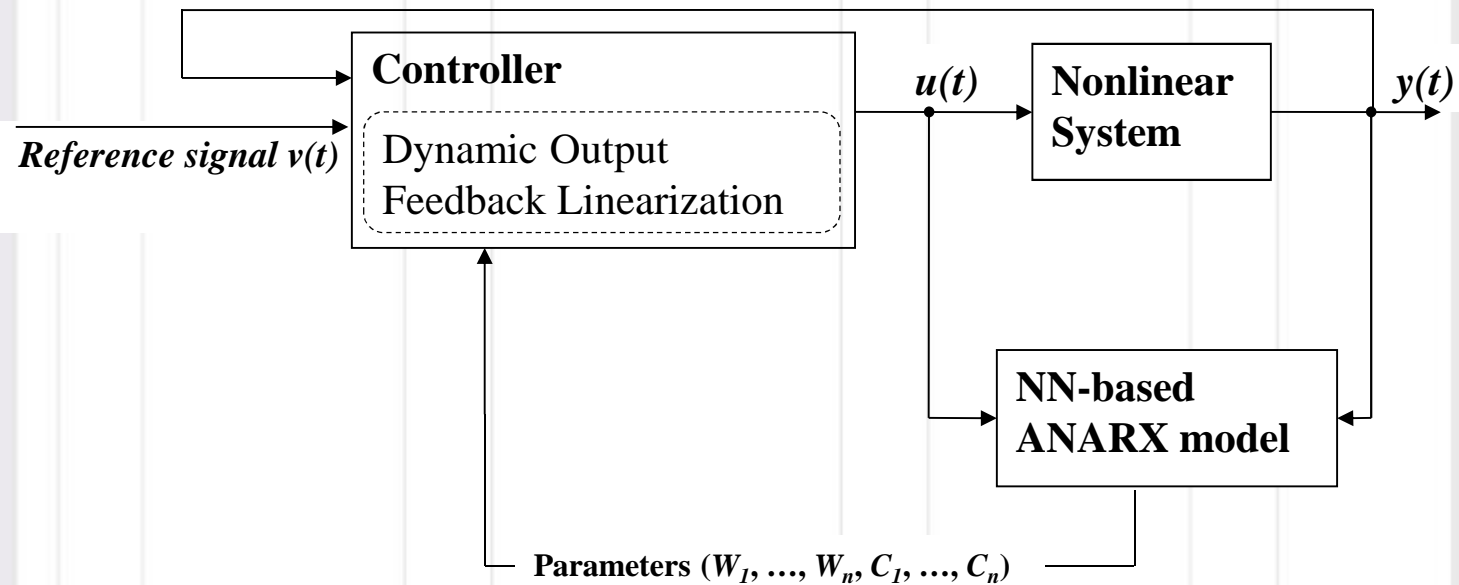
$$y(t+n) = \sum_{i=1}^n C_i \varphi_i(W_i \cdot [y(t+i-1), u(t+i-1)]^T)$$

$$\left\{ \begin{array}{l} F = f_1(y(t), u(t)) = \eta_1(t) \\ \eta_1(t+1) = \eta_2(t) - f_2(y(t), u(t)) \\ \vdots \\ \eta_{n-2}(t+1) = \eta_{n-1}(t) - f_{n-1}(y(t), u(t)) \\ \eta_{n-1}(t+1) = v(t) - f_n(y(t), u(t)) \end{array} \right\} \xrightarrow{\text{NN}} \left\{ \begin{array}{l} F = C_1 \varphi_1(W_1 \cdot [y(t), u(t)]^T) = \eta_1(t) \\ \eta_1(t+1) = \eta_2(t) - C_2 \varphi_2(W_2 \cdot [y(t), u(t)]^T) \\ \vdots \\ \eta_{n-2}(t+1) = \eta_{n-1}(t) - C_{n-1} \varphi_{n-1}(W_{n-1} \cdot [y(t), u(t)]^T) \\ \eta_{n-1}(t+1) = v(t) - C_n \varphi_n(W_n \cdot [y(t), u(t)]^T) \end{array} \right.$$

$$y(t+n) = v(t)$$



NN-ANARX Model based Control of Nonlinear Systems





Problems to be solved

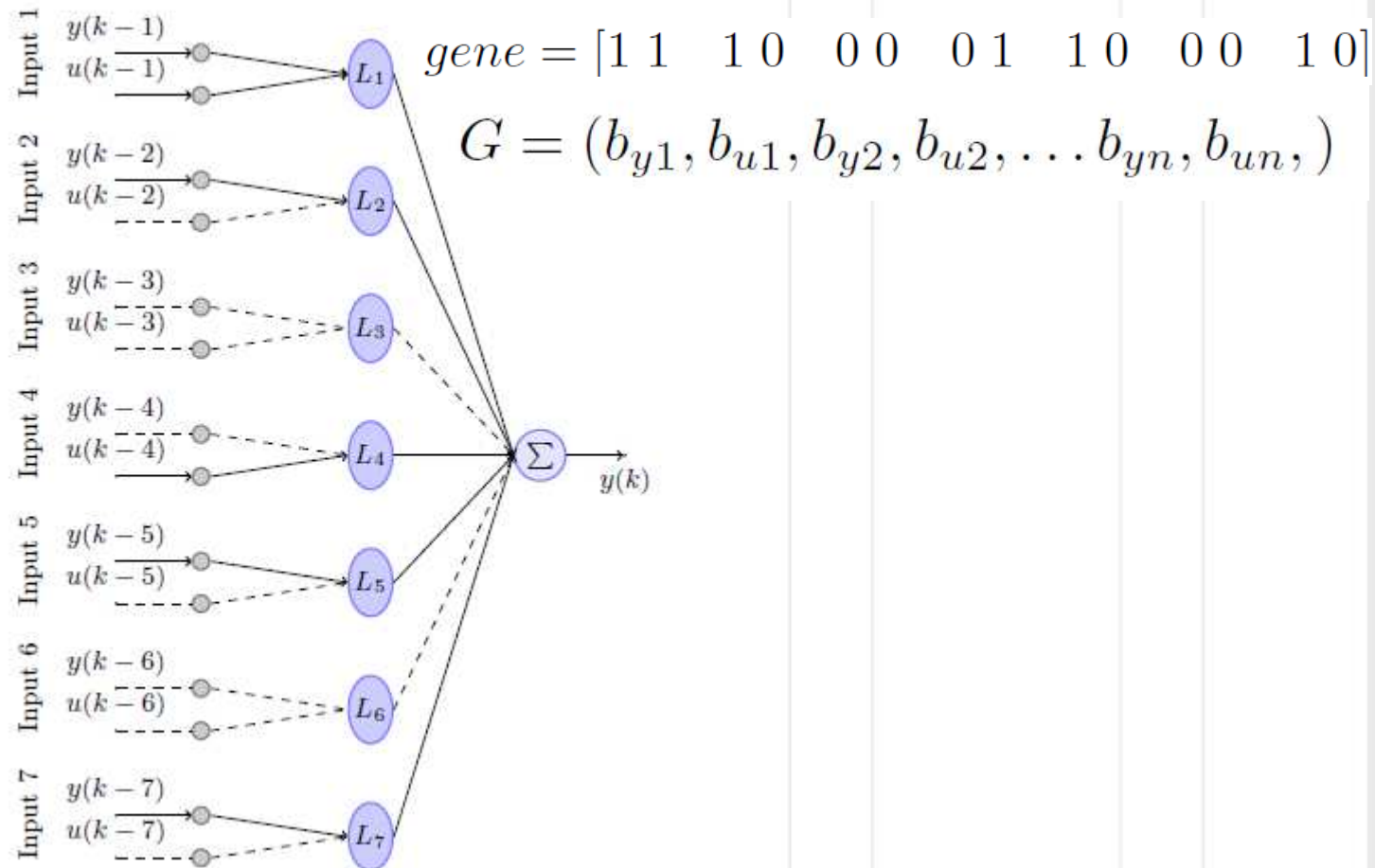
- *A little or no knowledge about structure of the system is given a priori*
- *A set of neural networks must be trained to find an optimal structure*
- *Quality of the model depends on the choice of initial parameters*
- *Quality of the model should be evaluated in the closed loop*

These problems can be solved using GA.



GA for structural identification

NN-ANARX structure may be easily coded as a gene.
Consider an example (custom structure model):





Dynamic controller based on custom structure model

$$\begin{aligned} F &= C_1 W_1 \left[b_{y1} y(t), b_{u1} u(t) \right]^T = \eta_1(t) \\ \eta_1(t+1) &= \eta_2(t) - C_2 \phi_2 \left(W_2 (b_{y2} x_1(t), b_{u1} u(t)) \right)^T \\ &\dots \\ \eta_{n-2}(t+1) &= \eta_{n-1}(t) - C_{n-1} \phi_{n-1} \left(W_{n-1} (b_{y,n-1} x_1(t), b_{u,n-1} u(t)) \right)^T \\ \eta_{n-1}(t+1) &= \nu(t) - C_n \phi_n \left(W_n (b_{y,n} x_1(t), b_{u,n} u(t)) \right)^T \end{aligned}$$

Gene:

$$G = (b_{y1}, b_{u1}, b_{y2}, b_{u2}, \dots, b_{yn}, b_{un},)$$



Fitness function

Model structure optimization is based on fitness function consisting of 3 parameters:

- Error of the closed-loop control system
- Order of the model
- Correlation test

All of the criteria are normalized



Numerical evaluation of the criteria

- $e = 1 - e^{-k \cdot mse}$
- $\hat{o}_i = \frac{o_i}{\|o\|}$ where $\|.\| := \max(|o_1|, \dots, |o_n|)$
- Correlation between the following three parameters is checked:
 - residuals $y(t)$ and delayed outputs $y(t - \tau)$
 - residuals $u(t)$ and delayed inputs $u(t - \tau)$
 - residuals $\epsilon(t)$ and delayed residuals $\epsilon(t - \tau)$

Further the parameter Q is the mean of the means of the means of cross correlation coefficients computed for three parameters mentioned above.

Evaluation function: $f = k_1 \cdot e + k_2 \cdot \hat{o} + k_3 \cdot Q$

with $k_1 + k_2 + k_3 = 1$