

## OLEKUMUDEL

Lineaarse, mittestatsionaarnse, pidevaja süsteemi olekumudel

$$\frac{dX(t)}{dt} = A(t)X(t) + B(t)U(t)$$

$$Y(t) = C(t)X(t) + D(t)U(t)$$

$A(n \times n)$  - olekumaatriks

$B(n \times r)$  - sisendmaatriks

$C(m \times n)$  - väljundmaatriks

$D(m \times r)$  - otse(edasi)sidemaatriks

Lineaarse, statsionaarse, diskreetaja süsteemi olekumudel

$$X(k+1) = A_d X(k) + B_d U(k)$$

$$Y(k) = C X(k) + D U(k)$$

# Olekumudeli näide 1

## Antenni mudel

Antenni keerab mootor (juhtsignaal sisendpinge [V]), nurga anduri järgi saab leida ka nurga muutumise kiirus[rad/s].

$\theta$  - antenni nurk [rad],

$\dot{\theta}$ - antenni nurga muutumise kiirus,

$J$  - kõikide keerlevate osade inertsmoment [ $\text{kg m}^2$ ],

$B_s$  - igasuguste sumbumiste summaarne koefitsient [ $\text{kg m}^2/\text{s}$ ]

$M$  - mootori poolt arendatav moment [ $\text{kg m}^2/\text{s}^2$ ],  $M = \mathbf{k} \cdot U(t)$ ,

$U(t)$  - mootori sisendpinge [V],

Pöördliikumist kirjeldav pöördemomentide tasakaaluvõrrand (diferentsiaalvõrrandina):

$$J \cdot \ddot{\theta}(t) + B_s \cdot \dot{\theta}(t) = M(t)$$

Sellest võrrandist saab tuletadaolekumudeli valides X<sub>1</sub>-ks  $\theta$  ja X<sub>2</sub>-ks  $\dot{\theta}$

# Antenni mudeli kirjeldus olekumudelina

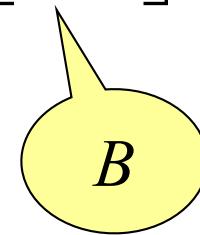
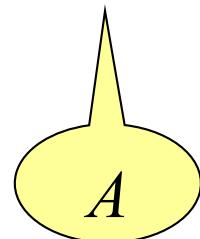
Üldkujul maatriksesituses:

$$\frac{dX(t)}{dt} = A \cdot X(t) + B \cdot U(t)$$

$$Y(t) = C \cdot X(t) + D \cdot U(t)$$

Valides olekumodelis  $X_1$ -ks  $\theta$  ja  $X_2$ -ks  $\dot{\theta}$  saame:

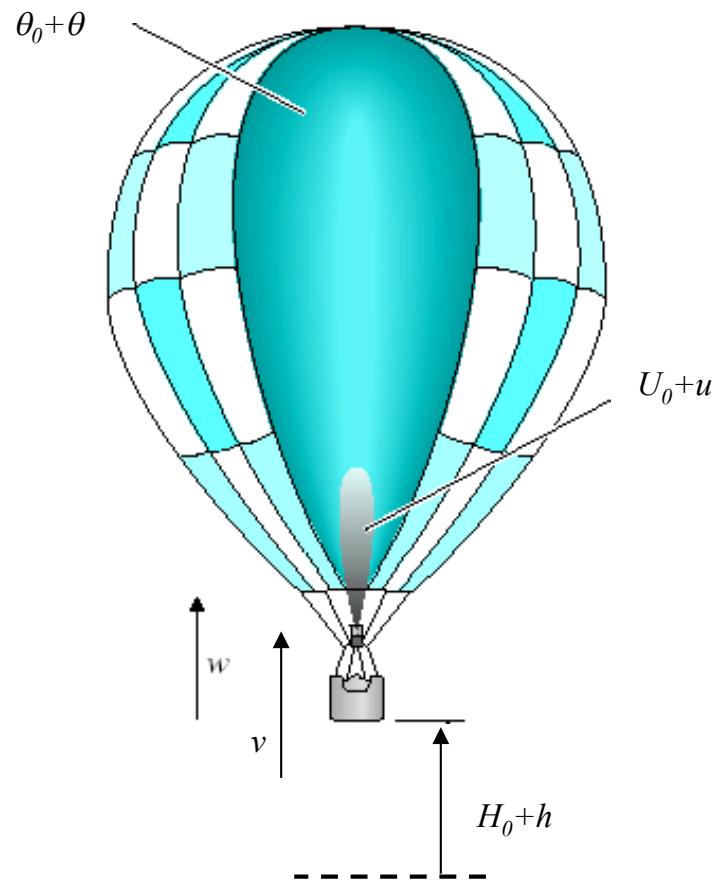
$$X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix},$$
$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -B_s / J \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ k / J \end{bmatrix} U(t)$$



$$J=10, B_s=46, k=7.78$$

# Olekumudeli näide 2

## Õhupalli mudel



Olekuvõrandid:

$$\left\{ \begin{array}{l} \dot{\theta} = -\frac{1}{\tau_1}\theta + u \\ \dot{v} = -\frac{1}{\tau_2}v + \sigma\theta + \frac{1}{\tau_2}w \\ \dot{h} = v \end{array} \right.$$

Olekuvõrrandi karakteristlik polünoom  $\det(sE - A)$ ;

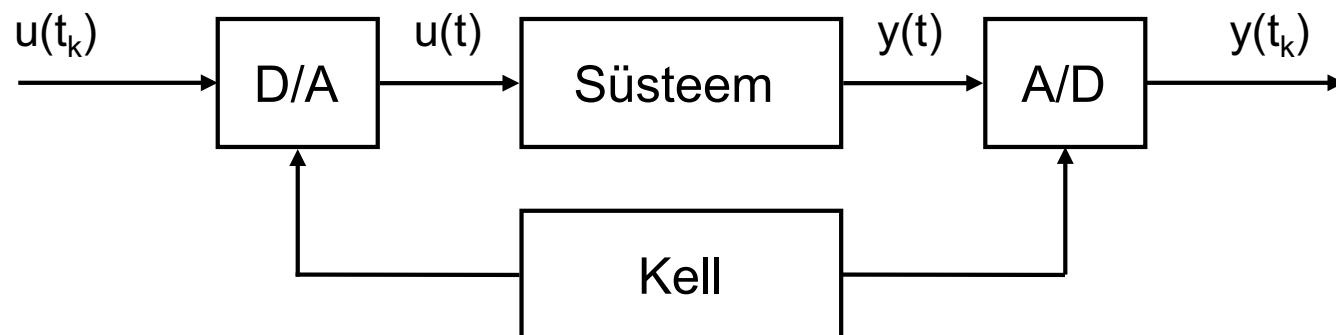
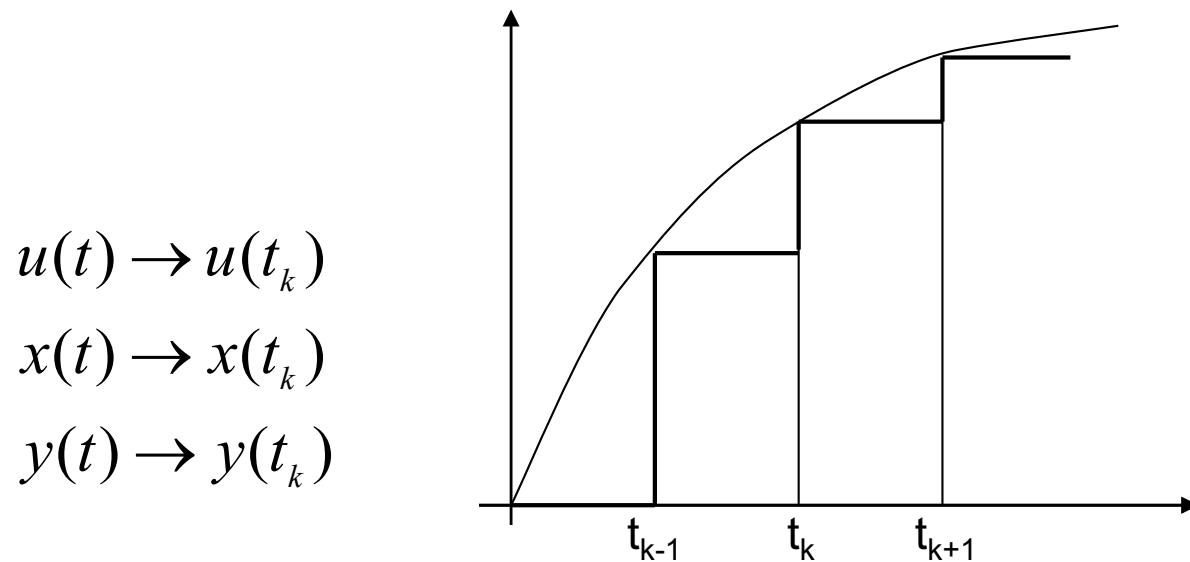
Karakteristliku võrrandi  $\det(sE - A) = 0$  juured on A omaväärused.

Ülekandemaatriks  $H(s) = C(sE - A)^{-1}B$

## Diskreetaja süsteemid

Olgu meil antud pidevaja olekumudel

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t), \quad x(0) \end{cases} \quad \text{kus } A - n \times n; \quad B - n \times r; \quad C - m \times n.$$



Meil on olemas olekumudel

$$\begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) \\ y(k) = Cx(k), \quad x(0) \end{cases}$$

Soovime leida sisend-väljund mudeli.

Võtame kasutusele operaatori  $z$

$$y(k) - zy(k) = y(k+1)$$

$$z^{-1}y(k) = y(k-1)$$

$$x(k+1) = zx(k) = \Phi x(k) + \Gamma u(k)$$

$$zx(k) - \Phi x(k) = \Gamma u(k)$$

$$(zE - \Phi)x(k) = \Gamma u(k)$$

$$x(k) = \underbrace{(zE - \Phi)^{-1}}_{Hux(z)} \Gamma u(k)$$

$$y(k) = Cx(k) = \underbrace{C(zE - \Phi)^{-1} \Gamma u(k)}_{H(z), Huy(z)}$$

$H(z)$  – ülekandemaatriks

–  $m \times r$ .

Eeldame, et  $m=r=1 \rightarrow$  ühemõõtmeline süsteem.

$$H(z) = \frac{B(z)}{A(z)} \quad \text{ülekandefunktsioon}$$
$$= \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 z^{-1} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}}$$

$$H(z) = \frac{y(k)}{u(k)}$$

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_1 u(k-1) + \cdots + b_n u(k-n)$$

diferentsvõrrand.

Kui  $u(k), \dots$ , siis  $y(k), \dots$  on leitav

$$y(k) = -a_1 y(k-1) - \cdots - a_n y(k-n) + b_1 u(k-1) + \cdots + b_n u(k-n)$$

## Näide

$$\begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) & h! \\ y(k) = Cx(k), & \text{kus} \end{cases} \quad \Phi = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0].$$

$$H(z) = C(zE - \Phi)^{-1} \Gamma$$

Siis

$$zE - \Phi = \begin{bmatrix} z-1 & -1 \\ 0 & z-1 \end{bmatrix}, \quad (zE - \Phi)^{-1} = \frac{1}{(z-1)^2} \begin{bmatrix} z-1 & 1 \\ 0 & z-1 \end{bmatrix}$$

$$H(z) = [1 \quad 0] \frac{1}{(z-1)^2} \begin{bmatrix} z-1 & 1 \\ 0 & z-1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{bmatrix} = \frac{\frac{1}{2}(z+1)}{(z-1)^2} =$$

$$= \frac{0.5z + 0.5}{z^2 - 2z + 1} = \frac{0.5z^{-1} + 0.5z^{-2}}{1 - 2z^{-1} + z^{-2}}$$

$$H(z) = \frac{0.5z^{-1} + 0.5z^{-2}}{1 - 2z^{-1} + z^{-2}} \leftrightarrow \frac{y(k)}{u(k)}$$

$$y(k) - 2y(k-1) + y(k-2) = 0.5u(k-1) + 0.5u(k-2)$$

$$y(k) = 2y(k-1) - y(k-2) + 0.5u(k-1) + 0.5u(k-2)$$

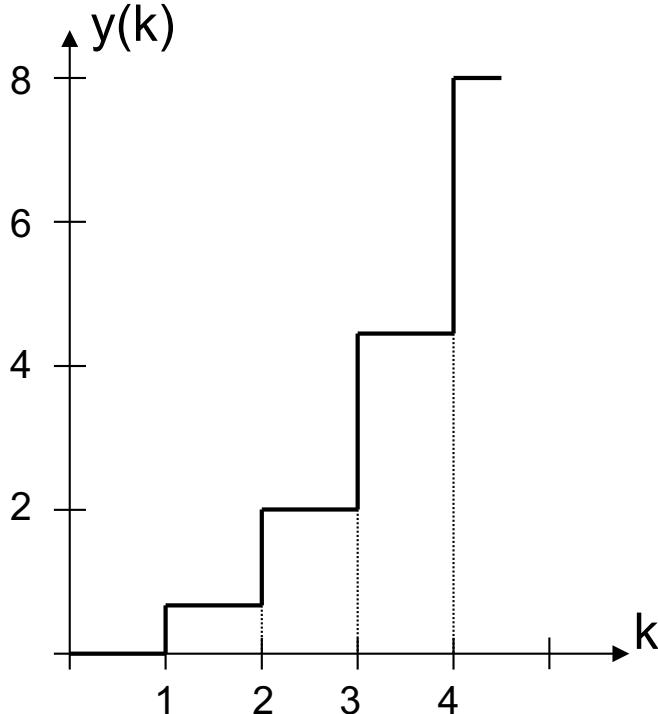
$$y(k) = f[y(k-1), y(k-2), u(k-1), u(k-2)].$$

Süsteemi järk on 2.

$k$	$y(k-2)$	$y(k-1)$	$u(k-2)$	$u(k-1)$	$y(k)$
0	0	0	0	0	0
1	0	0	0	1	0.5
2	0	0.5	1	1	2
3	0.5	2	1	1	4.5
4	2	4.5	1	1	8

$$u(k)=1, k \geq 0$$





## z-teisendus

$\{x(k); k = 0,1,2,\dots\}$  jada

$$X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

↓                      ↑  
 kujutis              originaal

1952 – 1958  
Jury  
Barker  
Tsõpkin

$$\begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) \\ y(k) = Cx(k), \quad x(0) \end{cases}$$

$$\begin{aligned} \sum_{k=0}^{\infty} z^{-k} x(k+1) &= z \left[ \sum_{k=0}^{\infty} z^{-k} x(k) - x(0) \right] = \\ &= \sum_{k=0}^{\infty} \Phi z^{-k} x(k) + \sum_{k=0}^{\infty} \Gamma z^{-k} u(k) \end{aligned}$$

$$\sum_{k=0}^{\infty} x(k) z^{-k} = X(z)$$

$$\sum_{k=0}^{\infty} u(k) z^{-k} = U(z)$$

$$z[X(z) - x(0)] = \Phi X(z) + \Gamma U(z)$$

$$X(z) = \underbrace{(zE - \Phi)^{-1} z X(0)}_{vabaliikumine} + \underbrace{(zE - \Phi)^{-1} \Gamma U(z)}_{sundliikumine}$$

$$H(z) = C(zE - \Phi)^{-1} \Gamma$$