

PIIRVÄÄRTUSTEOREEMID

Kui $x(t) \xleftrightarrow{L} X(s)$

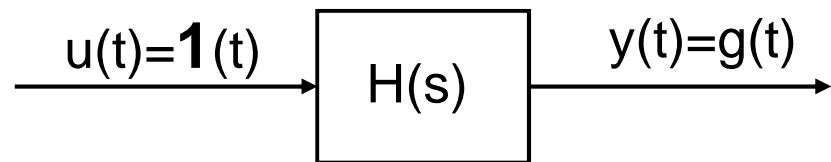
Siis

$$1) \quad \lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

$$2) \quad \lim_{t \rightarrow 0} x(t) = \lim_{s \rightarrow \infty} sX(s)$$

Ülekandekarakteristikud

1) Hüppekaja $g(t)$



$$\delta(t) \xleftrightarrow[L]{S} \frac{1}{s}$$

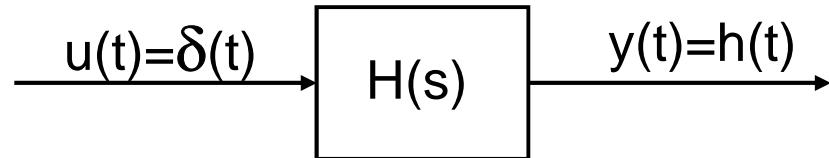
$$g(t) = L^{-1} \left[\frac{H(s)}{s} \right] \xleftrightarrow[L]{S} \frac{H(s)}{s}$$

- $\lim_{t \rightarrow 0} g(t) = \lim_{s \rightarrow \infty} H(s) \Rightarrow$ kui $m < n$, siis $g(0) = 0$
kui $m = n$, siis $g(0) \neq 0$

- $\lim_{t \rightarrow \infty} g(t) = \lim_{s \rightarrow 0} H(s) = \frac{b_0}{a_0}$

$$H(0) = \frac{b_0}{a_0} \quad \text{staatiline ülekandetegur}$$

2) Impulsskaja



$$\delta(t) \xleftrightarrow{L} 1$$

$$h(t) = L^{-1}[H(s)] \xleftrightarrow{L} H(s)$$

- $\lim_{t \rightarrow 0} h(t) = \lim_{s \rightarrow \infty} sH(s) \Rightarrow$

kui $m=n-1$, siis $h(t)$ on hetkel $t=0$ hüpe

kui $m=n$, siis $h(t)$ sisaldab $\delta(t)$ impulsiga komponenti

- $\lim_{t \rightarrow \infty} g(t) = \lim_{s \rightarrow 0} H(s) = \frac{b_0}{a_0}$

$t \rightarrow \infty$ saab $h(t)$ jäada nullist erinevaks,

kui $H(s)$ sisaldab poolust $s=0$.

Kokkuvõte:

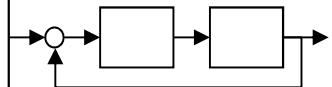
1) Ülekandefunksioon

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} = \frac{B(s)}{A(s)}$$

2) Hüppekaja $g(t) \xleftrightarrow[s]{L} H(s)$

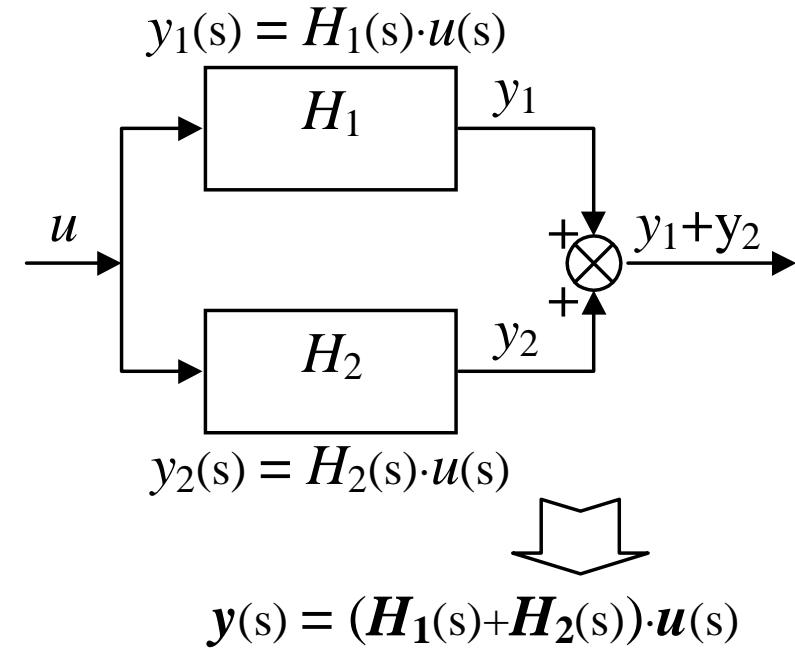
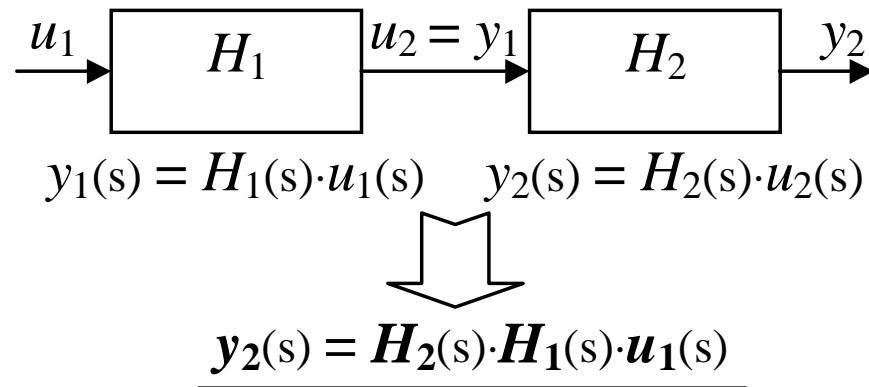
3) Impulsskaja $h(t) \xleftrightarrow[s]{L} H(s)$

Iseloomustavad süsteemi nullistel algtingimustel!



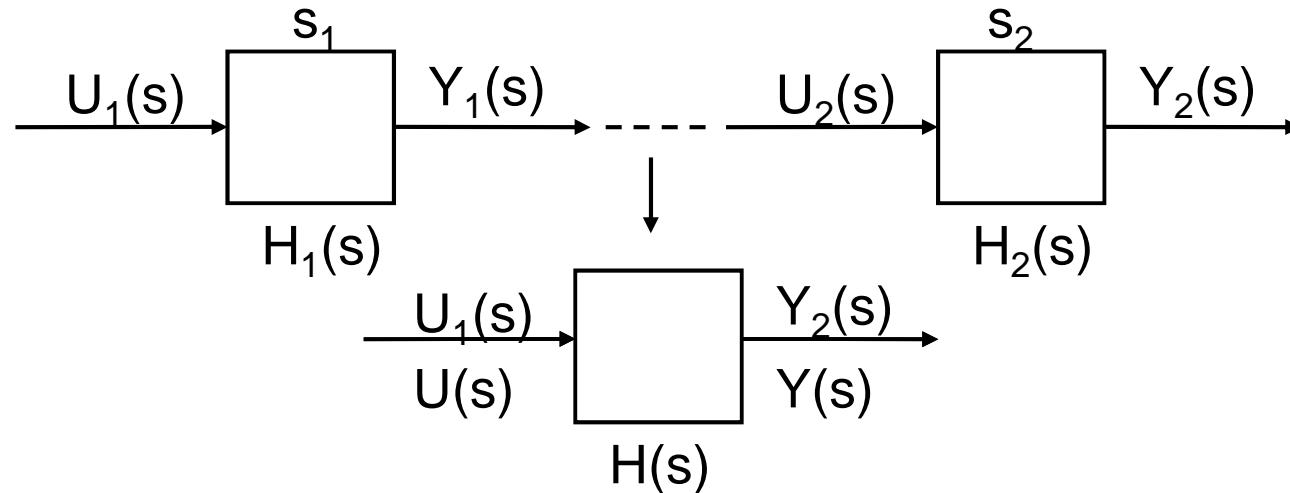
Ülekandefunksioon

Omadused / süsteemide kompositsioon



Süsteemide kompositsioon:

1) Järjestikühendus



$$\underbrace{Y_1(s) = H_1(s) \cdot U_1(s)}_{\text{System } S_1}$$

$$\leftarrow U_2(s) = Y_1(s)$$

$$Y_2(s) = H_2(s) \cdot U_2(s)$$

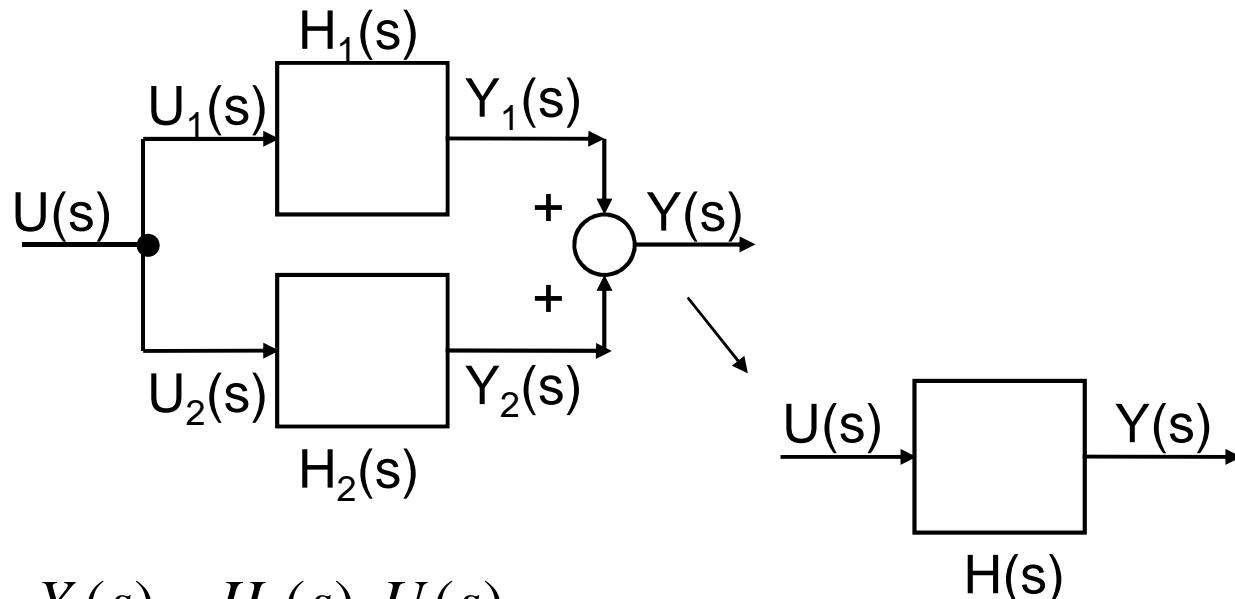
$$Y(s) = H_1(s)H_2(s) \cdot U_1(s)$$

$$H(s) = H_1(s)H_2(s) \quad 2 \text{ järjestikku}$$

$$H(s) = H_1(s) \cdots H_n(s) \quad n \text{ järjestikku}$$

$$Y(s) = H(s)U(s)$$

2) Paralleelühendus



$$Y_1(s) = H_1(s) \cdot U(s)$$

$$Y_2(s) = H_2(s) \cdot U(s)$$

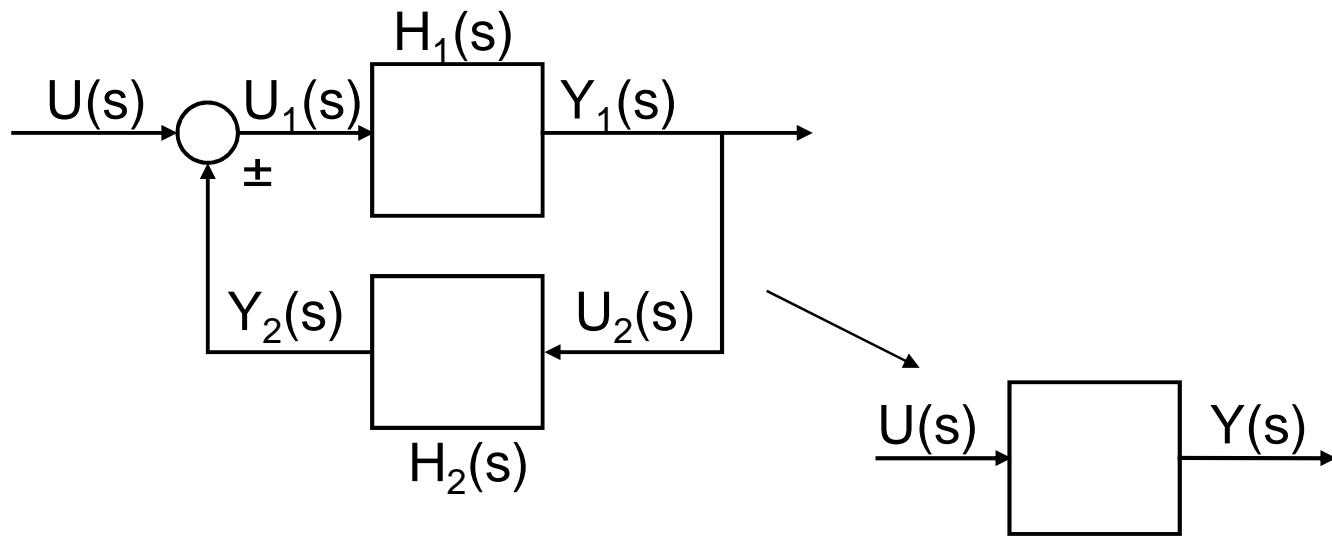
$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y(s) = [H_1(s) + H_2(s)]U(s)$$

$$H(s) = H_1(s) + H_2(s) \quad \text{2 paralleelselt}$$

$$H(s) = H_1(s) + \dots + H_n(s) \quad n \text{ paralleelselt}$$

3) Tagasisideühendus



$$Y_1(s) = H_1(s) \cdot U_1(s)$$

$$Y_2(s) = H_2(s) \cdot U_2(s)$$

$$U_1(s) = U(s) \pm Y_2(s)$$

$$Y_1(s) = H_1(s)[U(s) \pm Y_2(s)]$$

$$Y_1(s) = H_1(s)[U(s) \pm H_2(s)U_2(s)]$$

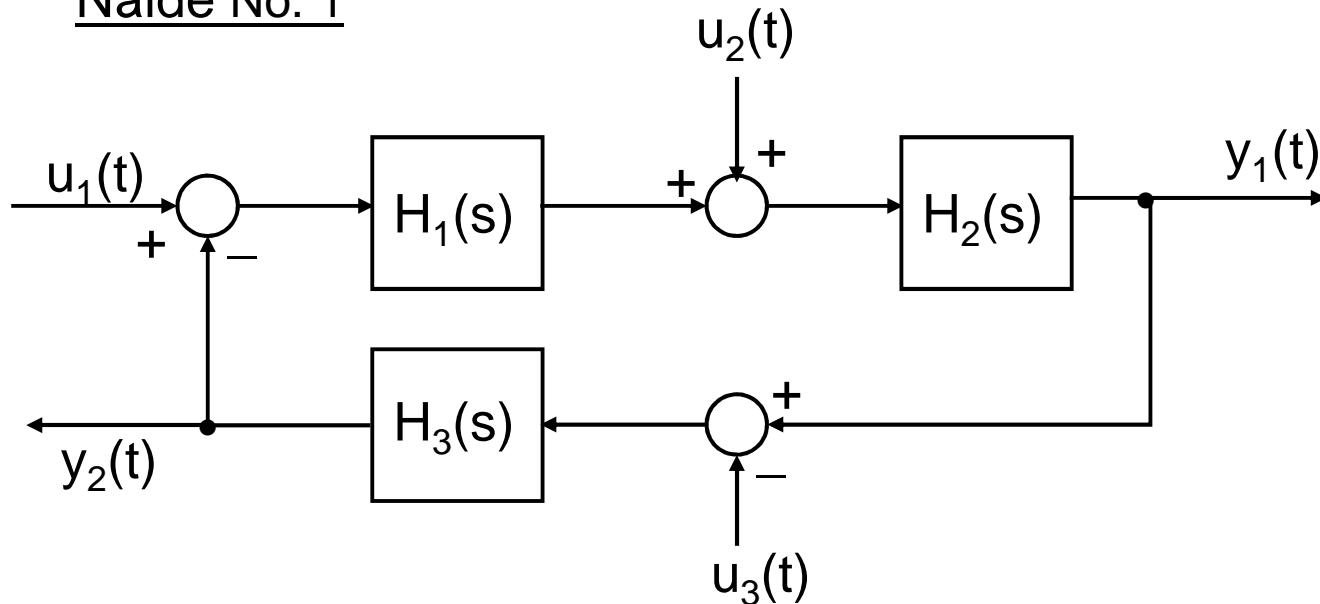
$$[1 \pm H_1(s)H_2(s)]Y_1(s) = H_1(s)U(s)$$

$$Y_1(s) = Y(s) = \frac{H_1(s)}{1 \mp H_1(s)H_2(s)} U(s)$$

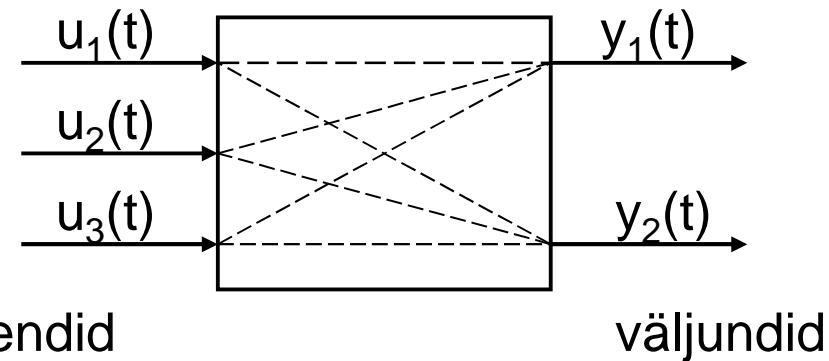
- (-) - positiivne tagasiside
- (+) - negatiivne tagasiside

- Lihtsatest süsteemidest on võimalik moodustada (soovitud omadustega) keerukaid süsteeme.
- Moodustuvad mitmemõõtmelised süsteemid (mitu sisendit või mitu väljundit).

Näide No. 1



$$H_1(s) = \frac{1}{s+2}; \quad H_2(s) = 10; \quad H_3(s) = \frac{s}{s+3}$$



6 ülesannet

Üritame matemaatiliselt kirjeldada moodustunud süsteemi

Ülekanne: $u_1(t) \rightarrow y_1(t)$

$$H_{u_1y_1}(s) = \frac{H_1(s)H_2(s)}{1 + H_1(s)H_2(s)H_3(s)} = \frac{\frac{10}{s+2}}{1 + \frac{10}{s+2} \cdot \frac{s}{s+3}} = \frac{10(s+3)}{s^2 + 15s + 6}$$

Ülekanne: $u_1(t) \rightarrow y_2(t)$

$$H_{u_1 y_2}(s) = \frac{H_1(s)H_2(s)H_3(s)}{1+H_1(s)H_2(s)H_3(s)} = \frac{\frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}}{1 + \frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}} = \frac{10s}{s^2 + 15s + 6}$$

Ülekanne: $u_2(t) \rightarrow y_1(t)$

$$H_{u_2 y_1}(s) = \frac{H_2(s)}{1 + H_1(s)H_2(s)H_3(s)} = \frac{10}{1 + \frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}} = \frac{10(s+2)(s+3)}{s^2 + 15s + 6}$$

Ülekanne: $u_2(t) \rightarrow y_2(t)$

$$H_{u_2 y_2}(s) = \frac{H_2(s)H_3(s)}{1 + H_1(s)H_2(s)H_3(s)} = \frac{10 \cdot \frac{s}{s+3}}{1 + \frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}} = \frac{10s(s+2)}{s^2 + 15s + 6}$$

Ülekanne: $u_3(t) \rightarrow y_1(t)$

$$H_{u_3 y_1}(s) = \frac{-H_1(s)H_2(s)H_3(s)}{1 + H_1(s)H_2(s)H_3(s)} = \frac{-\frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}}{1 + \frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}} = \frac{-10s}{s^2 + 15s + 6}$$

Ülekanne: $u_3(t) \rightarrow y_2(t)$

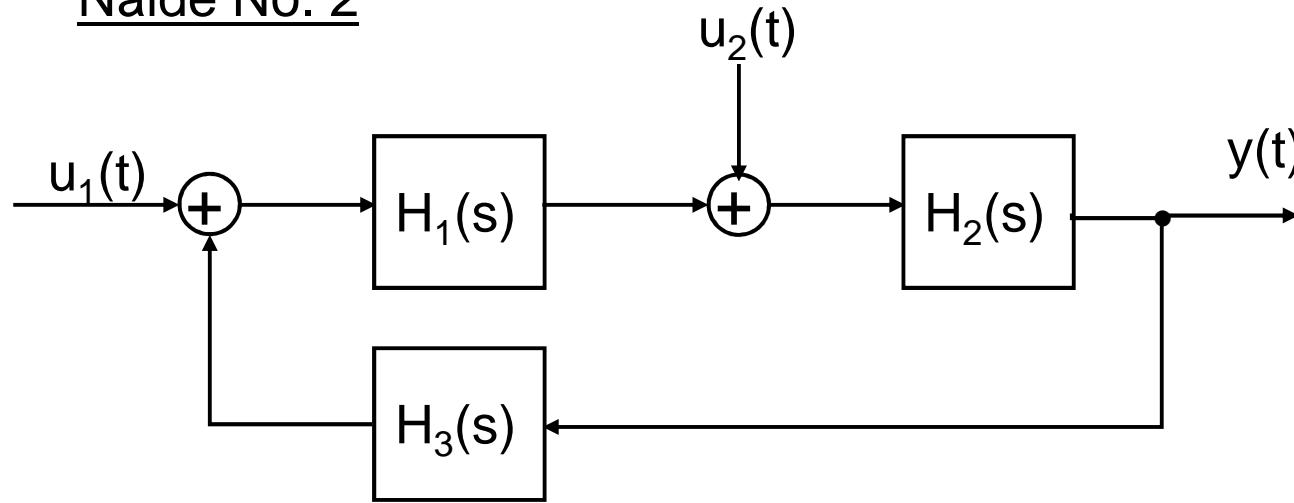
$$H_{u_3y_2}(s) = \frac{-H_3(s)}{1 + H_1(s)H_2(s)H_3(s)} = \frac{-\frac{s}{s+3}}{1 + \frac{1}{s+2} \cdot 10 \cdot \frac{s}{s+3}} = \frac{-s(s+2)}{s^2 + 15s + 6}$$

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} H_{u_1y_1} & H_{u_2y_1} & H_{u_3y_1} \\ H_{u_1y_2} & H_{u_2y_2} & H_{u_3y_2} \end{bmatrix} \cdot \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \end{bmatrix}$$

$$\underbrace{Y(s)}_{2 \times 1} = \underbrace{\mathcal{H}(s)}_{2 \times 3} \cdot \underbrace{\mathcal{U}(s)}_{3 \times 1}$$

$\mathcal{H}(s)$ – ülekandemaatriks (koosneb ülekandefunktsioonidest)

Näide No. 2



$$H_1(s) = \frac{3}{s+3}; \quad H_2(s) = \frac{1}{s+1}; \quad H_3(s) = 1;$$

$$u_1(t) = 3e^{-t}; \quad u_2(t) = 4 \cdot \mathbf{1}(t)$$

Leida $y(t)$, $y(0)$, $y(\infty)$?

Lahendus:

$$Y(s) = H u_1 y(s) \cdot U_1(s) + H u_2 y(s) \cdot U_2(s)$$

$$Hu_1 y(s) = \frac{H_1(s)H_2(s)}{1 - H_1(s)H_2(s)H_3(s)} = \frac{3}{s^2 + 4s}$$

$$Hu_2 y(s) = \frac{H_2(s)}{1 - H_1(s)H_2(s)H_3(s)} = \frac{s+3}{s^2 + 4s}$$

$$Y(s) = \frac{3}{s^2 + 4s} \cdot \underbrace{\frac{3}{s+1}}_{L[3e^{-t}]} + \frac{s+3}{s^2 + 4s} \cdot \underbrace{\frac{4}{s}}_{L[4\mathbf{1}(t)]} = \frac{4s^2 + 25s + 12}{s^2(s+1)(s+4)}$$

Lahutame osamurdudeks

$$Y(s) = \frac{4s^2 + 25s + 12}{s^2(s+1)(s+4)} = \frac{\overset{5/2}{K_1}}{s} + \frac{\overset{3}{K_2}}{s^2} + \frac{\overset{-3}{K_3}}{s+1} + \frac{\overset{1/2}{K_4}}{s+4}$$

$$\begin{aligned} 4s^2 + 25s + 12 &= K_1 s(s+1)(s+4) + K_2(s+1)(s+4) + \\ &\quad + K_3 s^2(s+4) + K_4 s^2(s+1) \end{aligned}$$

$$y(t) = \frac{5}{2}\mathbf{1}(t) + 3t - 3e^{-t} + \frac{1}{2}e^{-4t}; \quad y(0) = \frac{5}{2} - 3 + \frac{1}{2} = 0; \quad y(\infty) = \infty$$