Process Dynamics

Aim of the work

Process Dynamics (W.L. Luyben)

Approximate description of the battle with Lanchester's differential equations.

Simulate the processes 1-3, answer the questions, provide graphs.

Report

- Do not provide screenshots of the workspace in MATLAB, and black Scope graphs, use Figures instead!
- Explain ODE parameters, analyze the results using phase plane and time domain representations.

1 Battle of Trafalgar (1805)

27 ships led by Admiral Lord Nelson against 33 ships of the line under French Admiral Pierre-Charles Villeneuve. Ship's destruction speed during the battle is proportional to the number of enemy ships, on the average any ship can be destroyed for a half of the day.

Work flow

1. Write the equation. Provide solution depending on time $N(t), V(t)$ graphically.
2. Who will win? (Victory if number of enemy's vessels = 0).
   - How many ships should be in order to win the battle?
3. Answer the questions.
   - How big are the losses, for how long does the battle last?
   - How can ships' destruction speed change the result?
   - What is the process equilibrium point?
   - Is the process stable or not? Calculate the eigenvalues of the system.
   - If Nelson's ships fought a conventional battle better prepared $k_V = 0.75k_N$, would he win? How many ships would remain in the winning fleet?
4. Provide process trajectory in the state space (N, V) and several more with other initial values (in the same graph). Find eigenvalues of the system.

2 Battle of the Atlantic

Battle between U submarines ($U = 247$) and D destroyers ($D = 132$). Ship's destruction speed during the battle is proportional to the number of enemy ships, on the average one ship destroys 0.25 of the enemy's ship per week. Two U submarines is produced per week.

Provide Solution

1. How many destroyers a week to be produced in order to achieve victory in the battle?
2. Provide the equation to solve $U(t), D(t)$.

3 Battle in space (2200)

Earth (E) has 16 warships Klingon Empire (K) 20 alike (firepower and defense) against a ship. During the modernization of spaceships, half of Earth's ships ($E_1 = \frac{1}{2}E$) increased the firepower in two times compared to K ships, the second half of the warships ($E_2 = \frac{1}{2}E$) has the enhanced defense: damages from K fires decreased by 2 times.

Provide Solution

1. Provide results of the battle without modernization, how many ships are left?
2. Provide results of the battle with modernization, who is the winner?
    - Which part of the warships (E_1, E_2) is more likely to survive?
3. What will happen if E_2 is not able to modernize the ships?

Notes

If you would like to analyze the ordinary differential equations using phase plane and do not have Matlab, the Java version of pplane can be downloaded here pplane the Java version