4 Mixing processes

Two process streams are mixed to produce one of the feeds [1, 2, 3].

Substance (X) is included in the total quantity of the substance.

$$c = \frac{m_x}{\sum m}[\frac{kg, L, mole}{kg, m^3}] \qquad c = (0\dots1) \text{ or } (0\%\dots100\%)$$

$c \geq 0$ a non-negative!

Mass balance: conservation of mass $m_{in} - m_{out} = m_{accumulation}$

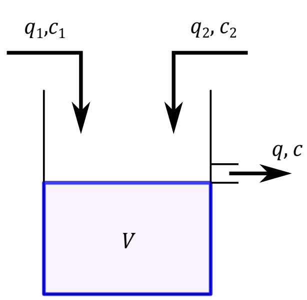
Component balance:

$$\left\{\begin{array}{l}\text{rate of accumulation}\\ \text{of component}\end{array}\right\} = \left\{\begin{array}{l}\text{rate of inflow}\\ \text{of component}\end{array}\right\} - \left\{\begin{array}{l}\text{rate of outflow}\\ \text{of component}\end{array}\right\} + \left\{\begin{array}{l}\text{rate of generation}\\ \text{of component by}\\ \text{chemical reactions}\end{array}\right\} \tag{2.1}$$

$$\frac{\mathrm{d}mc}{\mathrm{d}t} = q_{\text{in}}c_{\text{in}} - q_{\text{out}}c_{\text{out}} - RV, \tag{2.2}$$

where R depends on k_r—kinetic rate constant, c concentrations of the components in chemical reaction.

$m = \rho \cdot V$, if ρdensity is constant and independent of temperature, then $\sum V_{x_{in}} - \sum V_{x_{out}} \approx V_{x_{accum}}$.



Two inflows q_1, q_2

Outflow: Over-flow ($V = const$)

$q = q_1 + q_2$ regardless of the inflows

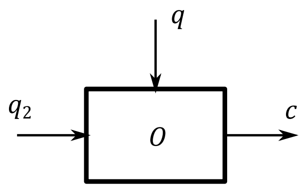
$$\frac{\mathrm{d}(\rho V c)}{\mathrm{d}t} = \text{in} - \text{out} = (\rho F_1 c_1 + \rho F_1 c_1) - \rho(F_1 + F_2)c \tag{2.3}$$

$$V\frac{\mathrm{d}c}{\mathrm{d}t} = F_1 c_1 + F_2 c_2 - Fc,$$

where F_i is volumetric flow.

Steady state: $c = \dots$,

Transfers: exponential time constant $\tau = V/(F_1 + F_2)$



5 Mechanical processes

Mass point

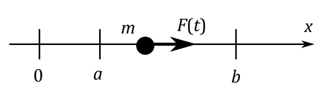
Isaac Newton's formalism. One of the triumphs of Newton's mechanics was the observation that the motion of the planets could be predicted based on the current position and velocities.

The acceleration a of a body is parallel and directly proportional to the net force F acting on the body, is in the direction of the net force, and is inversely proportional to the mass m of the body, i.e., $\sum F = m\ddot{x}$,

where x—is a displacement.

- Inertial system x
- Restrictions to the forces $|F| < F_{\max}$
  - to the movements: space, range $x_1 \ldots x_2$

One dimentional

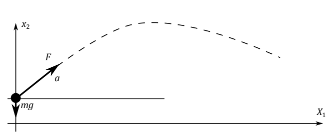


$$m \cdot \ddot{x} = F$$

Example 1 *Mechanical system*

$F = 10\ N,\ \Delta t = 20\ ms,\ m = 0.5\ kg,\ x, v =?$ *What is the state space?*

Two dimensional (in a vertical gravitation field)

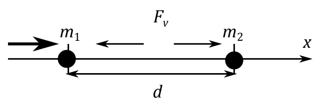


$$m \cdot \ddot{x}_1 = F \cdot \cos\alpha$$
$$m \cdot \ddot{x}_2 = F \cdot \sin\alpha - mg$$

Several (n) points (masses)

- Restrictions distance $x_1^2 + x_2^2 + x_3^2 = const$
- $\sum_{j=1}^{N}(m_j \ddot{r}_j - F_j) = \sum \vec{F}_j$

Two masses in one dimensional space

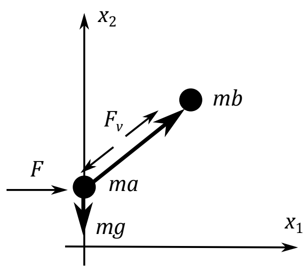


$m_1 \cdot \ddot{x}_1 = F - F_v$

$m_2 \cdot \ddot{x}_2 = F_v$

$x_2 - x_1 = d$

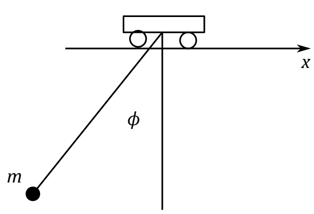
Two mass points in two-dimensional space



Several moving bodies

- Lagrangian formalism
  - $L = E_k - E_p$ difference between kinetic and potential energy

Example 2 *Pendulum*



$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\partial L}{\partial \dot{q}}\Big) = \frac{\partial L}{\partial q}$$ — Euler-Lagrange equation

State: $\begin{pmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{pmatrix}$

$\begin{matrix} x = l\sin\phi & \dot{x} = l\cos(\phi)\dot{\phi} \\ y = l\cos\phi & \dot{y} = -l\sin(\phi)\dot{\phi} \end{matrix}$, where l is the length of the pendulum.

Lagrangian:

$L = E_k - E_p = \frac{1}{2}mv^2 - mgy$, where y is the height of the pendulum.

Velocity is a vector representing the change in position

$$v^2 = \dot{x}^2 + \dot{y}^2 = l^2\cos^2(\phi)\dot{\phi}^2 + l^2\sin^2(\phi)\dot{\phi}^2 =$$
$$= l^2\dot{\phi}^2(\cos^2(\phi) + \sin^2(\phi)) = l^2\dot{\phi}^2$$

Hence Lagrangian is

$$L = \frac{1}{2}ml^2\dot{\phi}^2 - mgl\cos\phi.$$

Lets solve Euler-Lagrange equation for $\ddot{\phi}$

$$\frac{\partial L}{\partial \phi} = 0 + mgl\sin\phi$$

and

$$\frac{\partial L}{\partial \dot{\phi}} = ml^2\dot{\phi} - 0.$$

So,

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\partial L}{\partial \dot{\phi}}\Big) = ml^2\ddot{\phi}.$$

Knowing both sides,we can solve it

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\partial L}{\partial \dot{\phi}}\Big) = \frac{\partial L}{\partial \phi}$$

$$ml^2\ddot{\phi} = mgl\sin\phi \text{ or } \ddot{\phi} = \frac{g}{l}\sin\phi. \tag{2.4}$$

If oscillations are presented in the system

$x = l\sin\phi$ $\quad$ $\dot{x} = l\cos(\phi)\dot{\phi}$

$y = l\cos\phi + A\sin\omega t$ $\quad$ $\dot{y} = -l\sin(\phi)\dot{\phi} + A\omega\cos\omega t$

In this case Euler-Lagrange equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\partial L}{\partial \dot{\phi}}\Big) = \frac{\partial L}{\partial \phi}$$

$$ml^2\ddot{\phi} - mAl\omega\cos\phi\cos(\omega t)\dot{\phi} + mAl\omega^2\sin\phi\sin(\omega t) =$$
$$- mAl\omega\cos\phi\cos(\omega t)\dot{\phi} + mgl\sin\phi$$

$$l\ddot{\phi} + mAl\omega^2\sin\phi\sin(\omega t) = g\sin\phi$$

$$l\ddot{\phi} = g\sin\phi - Al\omega^2 \sin\phi \sin(\omega t) \text{ or}$$

$$\ddot{\phi} = \frac{1}{l}(g - A\omega^2 \sin(\omega t))\sin\phi \tag{2.5}$$

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