

2 Level Control

Space: container, inventory, tank, vessel.

Is the purpose to gather material?

1. Stocks (the necessary quantity);
 - intermediate warehouses between continuous and batch production,
 - min stock (\$, safety) \Leftrightarrow sufficiency stock.
2. Smoothing the disturbances of the continuous processes (averaging);
3. Separation, gathering [oil, mud, air, gas, steam]
 - separator, deaerator

2.1 Liquid Surge Tank

Generally surge tanks are used to "smooth" fluctuations of flow rate in liquid streams flowing between chemical processes [1, 2, 3, 5].

Variables used:

V —volume;

F —volumetric flowrate,

Q —mass flowrate.

Units used: [amount of substance/time units]

$m^3, l, \text{ etc.}$

$m^3/h, l/min, \dots$

$kg/s, kg/h, \dots$

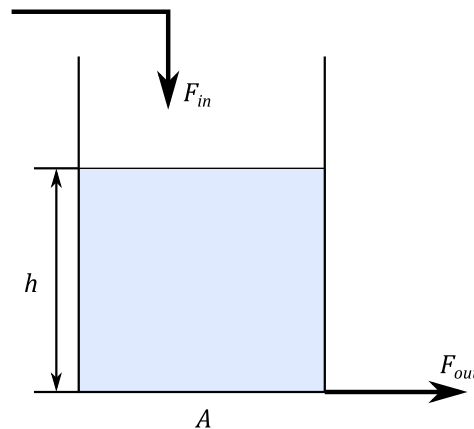


Figure 1.1: Liquid tank

Balance equations based on an rate of change (law of mass conservation):

$$\left[\begin{array}{c} \text{the rate of change of} \\ \text{mass of water in tank} \end{array} \right] = \left[\begin{array}{c} \text{mass flowrate of} \\ \text{water into tank} \end{array} \right] - \left[\begin{array}{c} \text{mass flowrate of} \\ \text{water out of tank} \end{array} \right]$$

The total mass of water in the tank is $V \cdot \rho$.

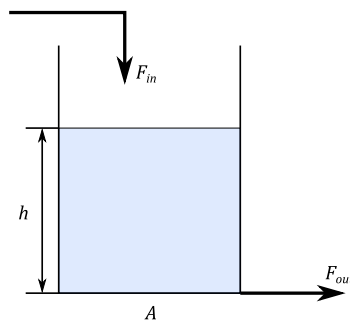
$$\frac{dV \cdot \rho}{dt} = F_{in} \cdot \rho - F_{out} \cdot \rho,$$

if densities of the outflow and the liquid in the tank do not change, then all density terms are equal. Thus, above equation can be simplified to (1.1)

$$\frac{dV}{dt} = F_{in} - F_{out}. \quad (1.1)$$

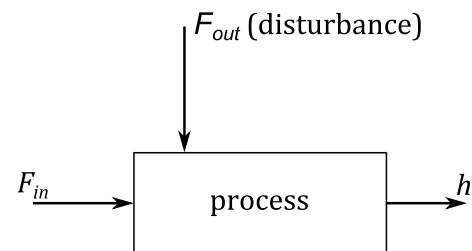
Our system can be considered as shown in the following figure, where outflow F_{out} represents the disturbance.

Physical system:



- Input: inflow F_{in} can be changed by us directly;
- Output: outflow F_{out} depends on pressure, not controllable directly;
- To control: liquid level h .

Controllable system:



- Manipulated value: F_{in} changeable by actuator and final control element;
- System Output: h , what we are measuring and comparing with reference signal ;
- Disturbance: F_{out} , parameter what are changing all the time and influence the system behavior.

$$V = \int (F_{in} - F_{out})dt \quad \text{or} \quad h = \frac{1}{A} \int (F_{in} - F_{out})dt,$$

where A is a cross sectional area of the tank.

1 Open tank with several inputs

The cylindrical open tank in Fig. 1.2 contains water at 18 °C and is being filled through section 1 and 3. Assume incompressible flow.

1. Derive an analytic expression for the water-level change $\frac{dh}{dt}$ in terms of arbitrary volumetric flows (F_1, F_2, F_3) and tank cross-sectional area A .
2. At the steady-state determine the exit velocity v_2 for the given data $v_1 = 3 \text{ m/s}$, $d_1 = 5 \text{ cm}$, $d_2 = 7 \text{ cm}$ and $F_3 = 0.01 \text{ m}^3/\text{s}$.

Solution: For the control of liquid level in the tank describe the system



Figure 1.2: Controlled System

Mass Conservation:

$$\frac{d\rho V}{dt} = Q_1 + Q_3 - Q_2,$$

where Q is mass flow.

Assume density ρ is constant.

$$A \frac{dh}{dt} = F_1 + F_3 - F_2 \quad \text{or} \quad \frac{dh}{dt} = \frac{1}{A} (F_1 + F_3 - F_2).$$

Steady-state means there is no change in liquid level, so

$$\frac{dh}{dt} = 0.$$

If h is constant, then

$$F_2 = F_1 + F_3 = v_1 \cdot a_1 + F_3 = v_1 \cdot \frac{\pi d_1^2}{4} + F_3.$$

$$F_2 = 3 \left[\frac{\text{m}}{\text{s}} \right] \cdot \frac{\pi \cdot 0.05^2 [\text{m}^2]}{4} + 0.01 \left[\frac{\text{m}^3}{\text{s}} \right] = 0.0059 \left[\frac{\text{m}^3}{\text{s}} \right] + 0.01 \left[\frac{\text{m}^3}{\text{s}} \right] = 0.0159 \left[\frac{\text{m}^3}{\text{s}} \right].$$

In that case velocity in section 2

$$v_2 = \frac{F_2}{a_2} = \frac{F_2 \cdot 4}{\pi d_2^2}$$

$$v_2 = \frac{0.0159 \left[\frac{m^3}{s} \right] \cdot 4}{\pi \cdot 0.072^2 [m^2]} = 4.13 \left[\frac{m}{s} \right]$$

2.2 Liquids

- Steady state (statics)

- liquid conserves the volume $V = const$
- pressure of liquids $dP/dh = -\rho g$, $P = \rho gh$
- force to the area A is $F = PA$

- Laminar or turbulent flow

The main parameter which correlates the viscous behavior of fluids is the *Reynolds number*:

$$Re = \frac{\rho v d}{\mu} \quad \text{or} \quad \frac{\text{inertial forces}}{\text{viscous forces}},$$

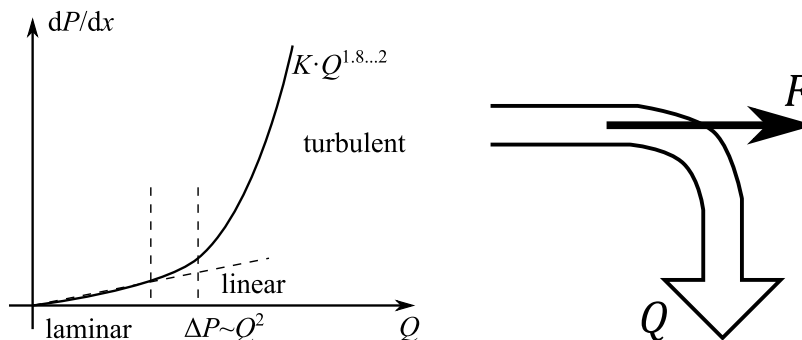
where v is mean velocity, d –diameter if the pipe and μ –viscosity.

Laminar flow $Re < 2000$

Transitional flow $2000 < Re < 4000$

Turbulent flow $Re > 4000$

Force is caused by a change in flow the direction



flow energy = $FL = PaL = P \cdot V$

- Hydraulic shot

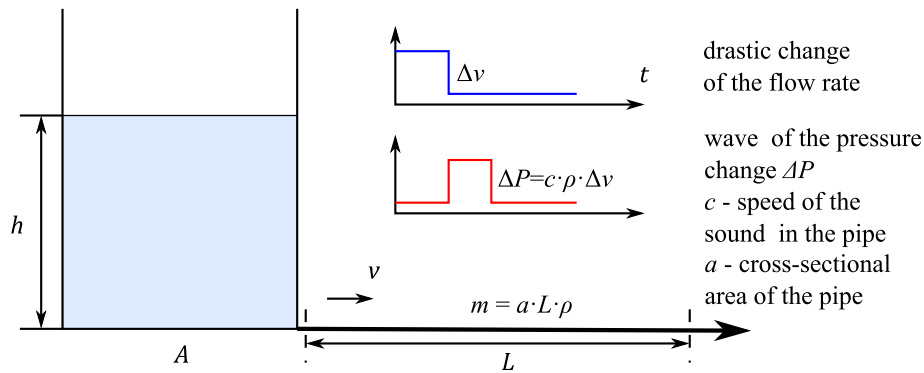



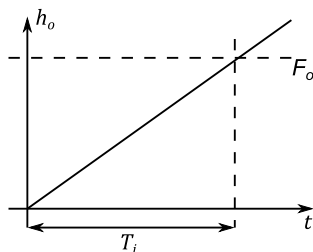
Figure 1.3: Hydraulic shot

On what parameters do the inflow and outflow depend?

1. Pumps

(a)  $F_{in}, F_{out} \neq F(h), F \approx \text{const}$

(b) Function $h(t)$ can be presented as linear piecewise, where we have linear dependency during some intervals with constant inflow and outflow $\Delta h = \frac{F_{in0} - F_{out0}}{A}$.



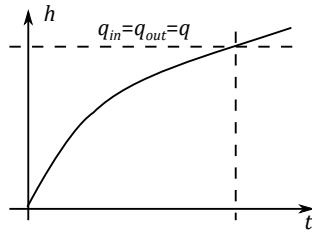
(c)

$$W(s) = \frac{K}{s} \quad (1.2)$$

2. Free outflow. Outflow depends on pressure (level)

(a) $Fv \equiv \sqrt{h}$ - nonlinear!

(b) $A \cdot \frac{dh}{dt} = F_{in} - k\sqrt{h}$, steady state: $F_0 = k\sqrt{h_0}$.



(c) Can be solved numerically, linearized at operating point (F_0, h_0) . See Exercise 2.

$$W(s) = \frac{K_p}{1 + \tau_p \cdot s}. \quad (1.3)$$

Values of the gain K_p and time constant τ_p depend on operation point h_0 !

3 Thermal processes

3.1 Energy balance

Analogous to the mass balance the energy balance for system can be described as

$$\frac{dE}{dt} = (\hat{E}_{m,in} - \hat{E}_{m,out}) + Q + W, \quad (1.4)$$

where \hat{E} is total energy per unit mass [4].

$$E = U + K_E + P_E,$$

where energy balance can be simplified due to the fact that $K_E \approx 0$ (flow velocities are small) and $P_E \approx 0$ (differences in height are small). Simplified representation in that case is $\frac{dE}{dt} \approx \frac{dU}{dt}$.

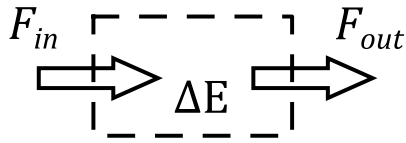
For liquid systems we assume $\frac{dU}{dt} \cong \frac{dH}{dt}$, where H is total **enthalpy** of the liquid in the tank.

The enthalpy is a function of temperature, pressure, phase, and composition. In general the pressure dependency can be neglected.

Thus, we obtain

$$\frac{dH}{dt} = \sum \rho V c \frac{dT}{dt} \quad (1.5)$$

$$Q = Q_{conduction} + Q_{radiation} + Q_{electric}.$$



1. Fix a border of the system (part of the environment);
2. Write down the system's energy equilibrium point.

Variables used:

E —the total energy

Q —heat per unit time

W —amount of work

F —heat flow (power)

U —overall heat transfer coefficient

c_p —heat capacity

T, t, θ —temperature indicator

Processes: heat accumulation, heat transfer.

Units:

$J = W \cdot s, \dots, MW \cdot h$

J/s

J/s

W, kW, MW

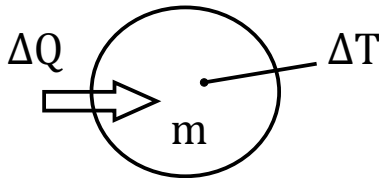
$J/(m^2 \cdot K),$

$kJ/kg \cdot K$

$^{\circ}C, K$

3.2 Heat accumulation

Temperature rise $\Delta Q \rightarrow \Delta T$.



$$\Delta Q = c \cdot m \cdot \Delta T \quad (1.6)$$

or $\Delta Q = (\sum c_i \cdot m_i) \cdot \Delta T$, where c is a heat capacity property of matter.

$$\left[\begin{array}{c} \text{the rate of energy} \\ \text{accumulation} \end{array} \right] = \left[\begin{array}{c} \text{rate of energy} \\ \text{by inflow} \end{array} \right] - \left[\begin{array}{c} \text{rate of energy} \\ \text{by outflow} \end{array} \right] + \left[\begin{array}{c} \text{rate of heat} \\ \text{added} \end{array} \right] + \left[\begin{array}{c} \text{rate of work} \\ \text{done on system} \end{array} \right]$$

3.3 Heat transfer

$$Q = UA\Delta T,$$

where A —surface area.

Mass exchange, heat conduction, convection, radiation.

Table 1.1: Heat capacity

Matter	Heat capacity
water	4.18
iron	0.44
fuel oil	2.5
ice, water vapor	2.0
air	1
AL	0.897

Occurrence / absorption

- heating with power P , burning
calorific value (heat of combustion), MJ/kg
heating
- phase transfers (solid-liquid-gas) , Latent Heat = energy required to change phases

– Solid to Liquid—**fusion**Melting of ice: $L_{ice} = 334 \text{ kJ/kg}$ ($T = 0 \text{ }^\circ\text{C}$),Oxygen: $L_{o, fus} = 13.8 \text{ kJ/kg}$ ($T = -218.8 \text{ }^\circ\text{C}$)Aluminum: $L_{al, fus} = 397 \text{ kJ/kg}$ ($T = 660 \text{ }^\circ\text{C}$)– Liquid to Vapor—**vaporization**boiling water: $L_{steam} = 2256 \text{ kJ/kg}$ ($T = 100^\circ\text{C}$),Oxygen: $L_{o, vap} = 213 \text{ kJ/kg}$ ($T = -182.97 \text{ }^\circ\text{C}$)Aluminum: $L_{al, vap} = 1.14 \times 10^7 \text{ J/kg}$ ($T = 2450 \text{ }^\circ\text{C}$)

where L_{ice} is a Latent Heat of Fusion and L_{steam} - Latent Heat of Vaporization.

$$Q = mL \tag{1.7}$$

Mass exchange: given quantity m of substance, at temperature T

$$Q = c \cdot m \cdot T, \quad F = \frac{Q}{t} = c \cdot q \cdot T \text{ - heat flow}$$

Examples**2 Water Boiler**

m - water mass (50 l, 50 kg)

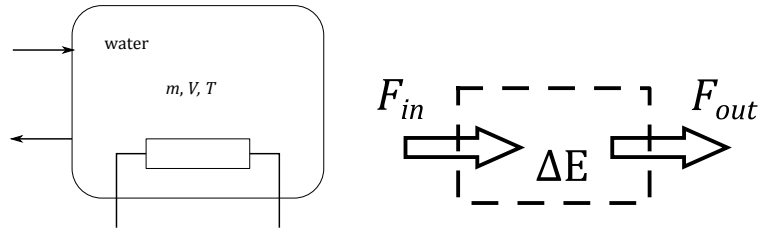


Figure 1.4: Boiler

T - water temperature;

T_e - environment temperature;

Q - heater power (3 kW).

General rule, see Fig. 1.4 :

$$\frac{dE}{dt} = F_{in} - F_{out}, \text{ temperature } T \text{ depends on various parameters } T(t, P, m, T_e, \dots)$$

- heat inflow (heat source) $F_{in} = Q$;
- heat outflow (cooling due to environment) ;

$$F_{out_e} = U \cdot A(T - T_e) = K_l(T - T_e), \quad (1.8)$$

where K_l is cooling coefficient depending on materials (walls, windows, coating. etc).

- heat outflow (running water): mass exchange with quantity $q = \frac{dm}{dt}$, $q = q_{in} - q_{out}$. In that case

$$F_{out_f} = c \cdot q \cdot (T - T_{in}) \quad (1.9)$$

- quantity of heat inside the boiler $Q = c \cdot m \cdot T$

3 General

Heater can be described by mass and energy balance.

1. Differential equation

$$\left\{ \begin{array}{l} \frac{dm}{dt} = Q_{in} - Q_{out} \\ \frac{d(mc_p(T - T_{ref}))}{dt} = q_{in}c_p(T_{in} - T_{ref}) - q_{out}c_p(T - T_{ref}) + Q_{elect} \end{array} \right. ,$$

if $\rho = \text{const}$, $A = \text{const}$, $c_p = \text{const}$ then

$$A \frac{dh}{dt} = F_{in} - F_{out}.$$

Where $dh/dt = f_1(q_{in}, q_{out})$ and $dT/dt = f_2(h, T, q_{in}, q_{out}, P, T_{in})$.

If process is linearised at operating point h_0, T_0 it can be represented as follows

$$\frac{d}{dt} \begin{bmatrix} h \\ T \end{bmatrix} = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \begin{bmatrix} h \\ T \end{bmatrix} + \begin{bmatrix} * & * & * \\ * & * & * \end{bmatrix} \cdot \begin{bmatrix} Q_{elect} \\ q_{in} \\ q_{out} \end{bmatrix}.$$

If we apply the chain rule, then

$$\frac{d(hT)}{dt} = h \frac{dT}{dt} + T \frac{dh}{dt}$$

$$\rho A h c_p \frac{d(T - T_{ref})}{dT} + \rho c_p (T - T_{ref}) (F_{in} - F_{out}) = \rho F_{in} c_p (T_{in} - T_{ref}) - \rho F_{out} c_p (T - T_{ref}) + Q_{elect}$$

$$\begin{aligned} \rho A h c_p \frac{d(T - T_{ref})}{dT} &= \rho F_{in} c_p (T_{in} - T_{ref}) - \rho F_{out} c_p (T - T_{ref}) + Q_{elect} \\ &\quad - \rho F_{in} c_p (T - T_{ref}) + \rho F_{out} c_p (T - T_{ref}). \end{aligned}$$

Canceling common terms gives

$$\rho A h c_p \frac{d(T - T_{ref})}{dT} = \rho F_{in} c_p (T_{in} - T) + Q_{elect}.$$

As T_{ref} is constant, so $d(T - T_{ref})/dt = dT/dt$

Finally we obtain dynamic model, where balances are not coupled.

$$\begin{cases} \frac{dh}{dt} = \frac{1}{A} (F_{in} - F_{out}) \\ \frac{dT}{dt} = \frac{F_{in}}{Ah} (T_{in} - T) + \frac{Q}{Ah\rho c_p} \end{cases} \quad (1.10)$$

2. Steady-state

$$\frac{dh}{dt} = 0, \quad \frac{dT}{dt} = 0$$

From (1.10) it follows

$$\begin{cases} F_{in} = F_{out} \\ \frac{F_s}{Ah} (T_s - T_{in_s}) + \frac{Q_s}{Ah\rho c_p} \end{cases},$$

where s indicates steady-state value.

$$T_s = T_{in_s} + \frac{Q_s}{F_s \rho c_p}$$

It can be seen that volume of the vessel has no effect on the steady-state temperature.

Process gain is defined by the derivative of output with respect to the input, evaluated at steady state.

$$K_p = \frac{\partial T_s}{\partial Q_s} = \frac{1}{F_s \rho c_p}$$

4 No losses

There is no flow and cooling into environment $F_{out_e} = F_{out_f} = 0$.

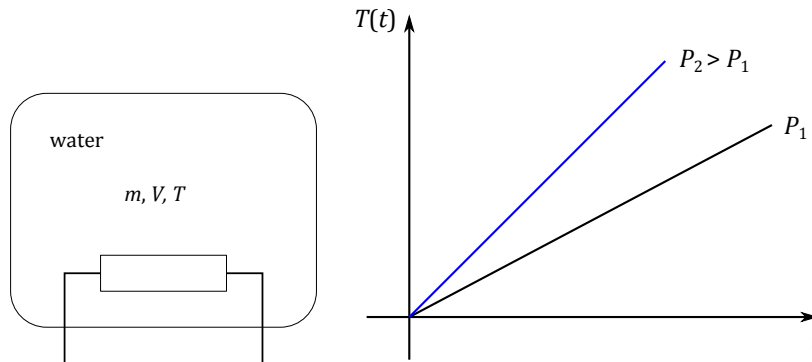


Figure 1.5: Closed isolated boiler

Water is heated.

$$\frac{d(c \cdot m \cdot T)}{dt} = Q \quad cm \frac{dT}{dt} = Q \text{ - integrator}$$

$P \rightarrow T$ transfer

Questions:

- Steady state $T(P)$?
- Step response (heating)
- What is the initial temperature $T_o(0)$?
- What does happen if heating is stopped $P = 0$?

5 No flow

There is no flow, the cooling into environment is present $F_{out_f} = 0$.

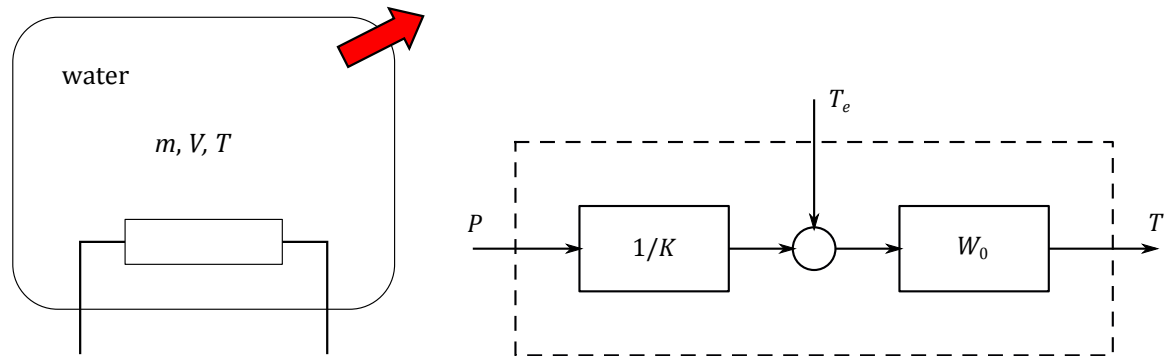


Figure 1.6: Looses into environment

$K \neq 0$. Differential equation:

$$\frac{d(c \cdot m \cdot T)}{dt} = P - K \cdot (T - T_e)$$

First order system representation is

$$\tau \frac{dy}{dt} + y = K_p u,$$

so lets rewrite differential equation according to that format

$$cm \frac{dT}{dt} + K \cdot T = P + K \cdot T_e \quad \Rightarrow \quad \frac{cm}{K} \frac{dT}{dt} + T = \frac{P}{K} + T_e.$$

Thus, transfer function $P \rightarrow T$ is

$$W(s) = \frac{\frac{1}{K}}{1 + \frac{cm}{K} \cdot s},$$

where τ is a time constant, K_p - gain of the process.

Steady state:

$$\frac{dT}{dt} = 0,$$

so final temperature depends on power and environmental temperature $T(P, T_e)$.

$$T - T_e = \frac{P}{K} \quad \text{or} \quad \Delta T = \frac{P}{K}$$

Step response, see Fig. 1.7:

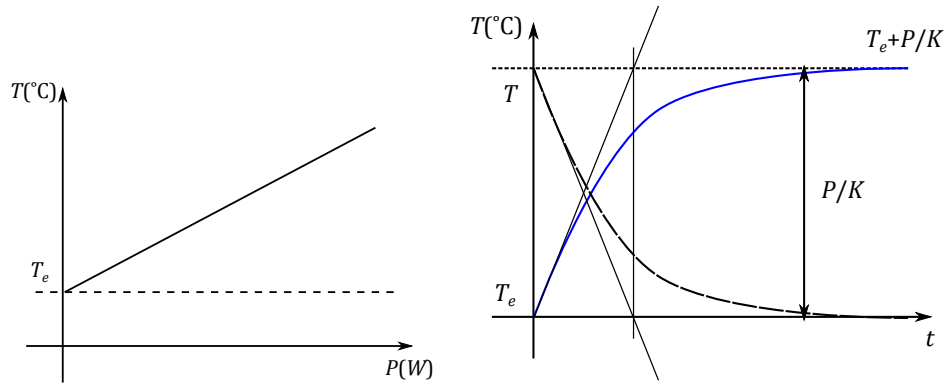


Figure 1.7: Step response

Equation is valid if T is lower than the boiling temperature!

Cooling: $P = 0, T_0 \neq T_e$

6 Good isolation

Running water is heated, $K = 0$.

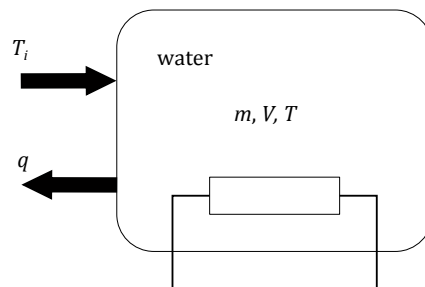


Figure 1.8: Running water with heating

$$\frac{d(c \cdot m \cdot T)}{dt} = P - c \cdot q(T - T_{inp})$$

First order system representation is

$$\tau \frac{dy}{dt} + y = K_p u,$$

so lets rewrite differential equation according to that format

$$cm \frac{dT}{dt} + cq \cdot T = P + cq \cdot T_{inp} \quad \Rightarrow \quad \frac{cm}{cq} \frac{dT}{dt} + T = \frac{P}{cq} + T_{inp}.$$

Thus, transfer function $P \rightarrow T$ is

$$W(s) = \frac{\frac{1}{cq}}{1 + \frac{m}{q} \cdot s},$$

where time constant is $\tau = \frac{m}{q}$ and process gain $K_p = \frac{1}{cq}$.

Special cases:

1. Water is running through the tank, no heating $P = 0$, $K = 0$. Such systems are used as a filter for the inflow water temperature T_{inp} , see Fig. 1.9,

$$W(s) = \frac{1}{1 + \frac{m}{q} \cdot s}.$$

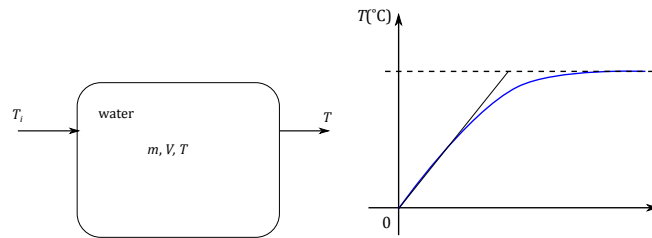


Figure 1.9: Running water

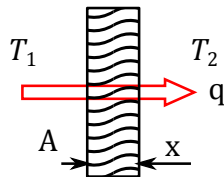
2. If several tanks are set in series, then $W = W_1 \cdot W_2 \cdot \dots$.
3. If several inflows, then $(q_1, T_1) + (q_2, T_2) \rightarrow (q_1 + q_2, T)$.

Question:

What will happen if $V = 0$?

Thermal conduction

Conduction takes place within stagnant gas or liquid layers (layers, which are sufficiently thin that no convection as a result of temperature gradients can occur), solids or on the boundary between solids [4].



$$\frac{q_{\text{conduction}}}{A} = \lambda \frac{\Delta T}{\Delta x}$$

heat flow q [W/m²] per surface $A = 1 \text{ m}^2$ is
 \sim temp. gradient

With a fixed thickness x (area $A = 1 \text{ m}^2$) parameters:

$$q = \alpha \cdot \Delta T = \Delta T / R,$$

where α - heat transfer coefficient $\alpha = \lambda/x$ W/(m² · K),

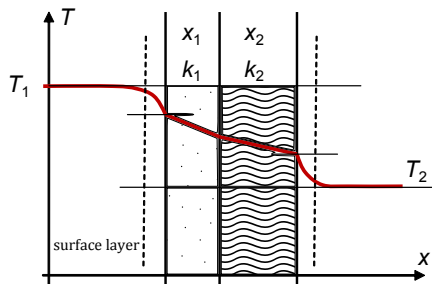
R - thermal resistance $R = x/\lambda$ m² · K/W.

Thermal conductivity λ - property of the material, units are W/(m · K)

- Cu 390,
- Al 210,
- Fe 50,
- Steel 16 ,
- Concrete 1.55,
- Water 0.61 ,
- Ice 2.2,
- Wood 0.11,
- Silicate cotton 0.04,
- Leather 0.017,
- Fat 0.21,
- Air 0.024.
- Glass, window 0.96

Convection

At an interface between gas and liquid or solid or between liquid and solid, convective heat transfer can take place when those media have a temperature difference. It can be in the form of free convection, such as in the case of a central heating radiator, or it can be forced convection, for example, an air flow from a blower[4].



$$q_{\text{convection}} = hA\Delta T,$$

where h is the convection heat transfer coefficient

- gas, natural convection 3-25
- motionless air 2.7
- fluid, natural convection from 30 to 60
- boiling water 4000 to 15000
- vapor condensation from 10000 to 20000

Thermal radiation

Heat transport can also take place from one object to another object through radiation. An object radiates energy proportional to the fourth power of its absolute temperature

$$\frac{q_{\text{rad,max}}}{A} = \sigma T^4,$$

where $\sigma = 56.7 \times 10^{-9} \text{ [W/m}^2\text{K}^4]$ is the Stefan-Boltzmann constant.

Radiation heat transfer can be described by reference to the “black body”. A black body is a hypothetical body that completely absorbs all wavelengths of thermal radiation incident on it. Such bodies do not reflect light, and therefore appear black if their temperatures are low enough so as not to be self-luminous. All black bodies heated to a given temperature emit thermal radiation.

If the temperature difference between T_1 and T_2 is small, this relationship is often written as a linear relationship

$$q_{\text{rad}} = \epsilon \sigma A (T_1^4 - T_2^4),$$

where ϵ is emissivity of the surface $A \left[\frac{\text{W}}{\text{m}^2} \right]$.

- Aluminum foil 0.07,
- Polished copper 0.03,
- Polished silver 0.02,
- Polished stainless steel 0.17
- Black paint 0.98
- White paint 0.9
- Red brick 0.93-0.96
- Human skin 0.95
- Water 0.96
- Wood 0.82-0.92

Bibliography

- [1] B. Wayne Bequette. *Process Control: Modeling, Design and Simulation*. Prentice Hall Professional, 2003.
- [2] Pao C. Chau. *Process Control: A First Course with MATLAB*. Cambridge University Press, 2002.
- [3] William L Luyben. *Process modeling, simulation and control for chemical engineers*. McGraw-Hill Higher Education, 1989.
- [4] Brian Roffel and Ben Betlem. *Process Dynamics and Control: Modeling for Control and Predictions*. Wiley and Sons, 2006.
- [5] F Greg Shinskey. *Process control systems: application, design and tuning*. McGraw-Hill, Inc., 1990.