2 Process

To solve control problem it is necessary to understand the process, we need to know what should be automated, no matter how advanced technique we can use (devices used to control are discussed in course ISS0065)!

Process models can be described different ways. We discuss:

- Description of the process (differential equations, transfer functions,...);
- Simple processes (first and second order);
- Process examples (thermal, chemical, level);
- Process models.

The basic steps in control systems are: modeling, controller design and controller validation. The modeling process can be achieved by any combination of the approaches which are described further.

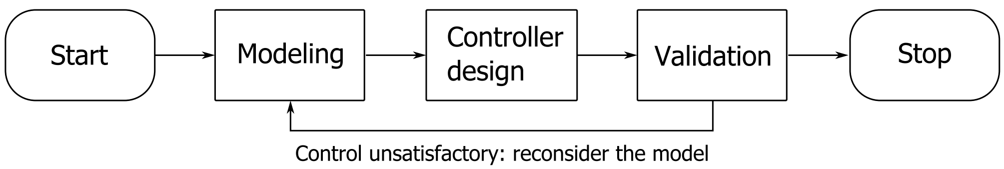


Figure 1.1: Modeling process

Our aim is to obtain a description of system dynamical behavior in terms of some physically significant variables. As nature of the system changes, the system variables change [1].

The purpose of the process description:
Is it possible to control? How to do that? What are the system features?

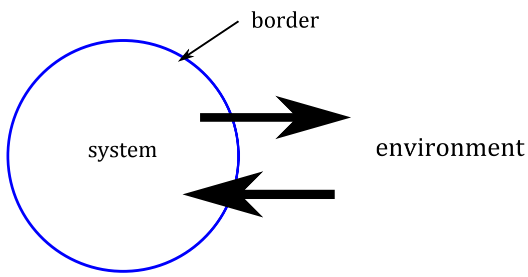
System types:

- Continuous/ pidev/ *непрерывная* or Discrete/ diskreetne/ *дискретная*;
- Deterministic/ stochastic/ chaotic;
- Linear / nonlinear;
- With lumped/distributed parameters;
- SISO/ SIMO/ MIMO/ TITO;
- Stable/ stabiilne/ *устойчивая* or Unstable/ mittestabiilne/ *неустойчивая*;

- Controllable/ juhitav/ *управляемая* or/and Observable/ jälgitav/ *наблюдаемая*,
- Robust/ robustne/ *робастная*.

2.1 System Definition

It is important to remember that process model is noting more than mathematical abstraction of a real process [1]. The more assumptions we make, the simpler the structure of the model will be.



Definitions are used:

- System phases
- System equilibrium

When solving a problem, it is important to specify what is meant by system.

If system is isolated then nothing can enter of leave. Thus, we cannot know anything about an isolated system from the outside.

If heat is allowed to be exchanged, system is called closed.

If both matter and energy can be exchanged, the system is called open.

Table 1.1: Systems types

systems type	Mass	Energy
Isolated	constant	constant
Closed	constant	-
Open	-	-

System: components (parts/units) and relationships between them. There are 3 types of matter/energy changes in the system:

1. Accumulation (gathering, storage)

   Causes the transition processes

   capacity, lag/ mahtuvused/ *емкостные*

2. Flows (movement)

   Flows values are limited ($\neq \infty$)

   conductance/ juhtivus/ *проводимость*

3. Loss

   Energy → Heat

2.2 Conservation Laws

All three types can be gathered using physical value X conservation laws [1]:

- Mass (m),
- Load (q),
- Energy (E),
- Momentum (mv),
- Rotation ($J\omega$), etc.

Changes of the value can be described:

$$X_{\text{accumulation}} = X_{\text{inflow}} - X_{\text{outflow}} \pm X_{\text{leakage/absorbtion}} \tag{1.1}$$

Change rate in the system	Flow through the boarder	Leakage/ absorption

This equation is simple because there may be many "in" and "out" terms.

Dynamic analysis models: obtained by analysis of the physical system at a fundamental level, involving approximations sufficient to simplify the model to a differential equation form in a time domain.

How to obtain a process model from conservation laws?

Using the definition: flow/rate $F = X/t$

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \sum F_{in} - \sum F_{out} \pm F_l \tag{1.2}$$

This is the value X balance in one dimensional environment.

3 Process description techniques

Different views on reality, their relationships, process characteristics.

Differential equations $\dot{x} = F(x, u)$

Transfer functions $H(s)$, $H(z)$

The model should incorporate all of the important dynamic behavior while being no more complex than is necessary [3].

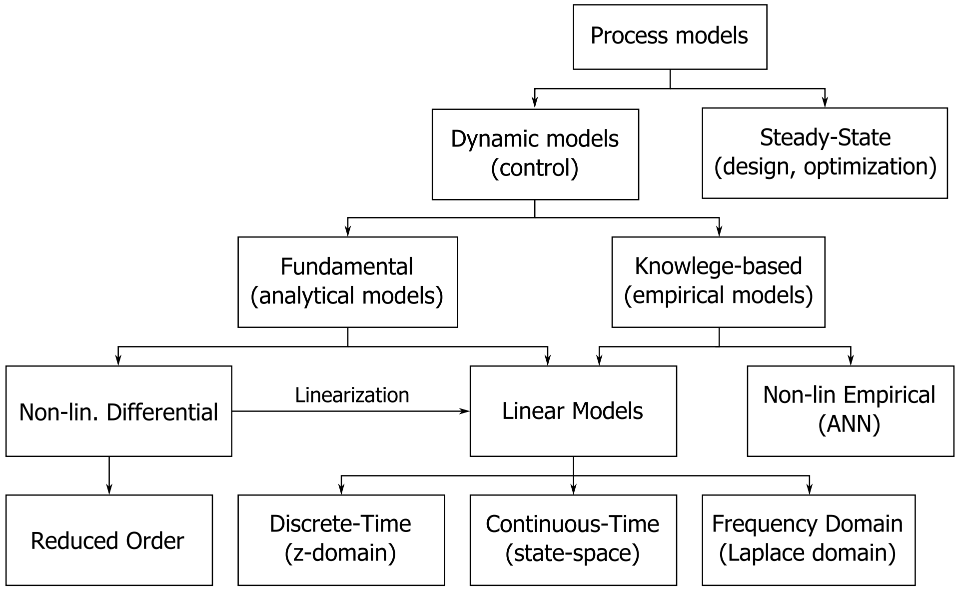


Figure 1.2: The hierarchy of process models [2]

3.1 Differential equations

Time domain

Continuous physical processes

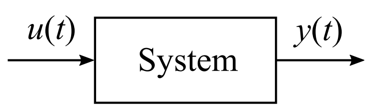


Figure 1.3: Process with inputs and outputs

1. I/O representation $u(t), y(t)$
   - $F(u, \dot{u}, \ldots, y, \dot{y}, \ldots) = 0$
     - u^m, y^n, $n \geq m$, order/ järk/ *порядок*
     - representation: operator $p = \mathrm{dX}/\mathrm{d}t$
2. State-space representation / olekumudel/ *пространство состояний*

- x - state variable

$$\dot{x} = f(x, u, p) \qquad p\text{–parameters}$$
$$y = g(x, u, p) \qquad f(), g()\text{–functions}$$

Linear time invariant equation:

$\dot{x} = Ax + Bu$ $\quad$ A-system matrix
$y = Cx + Du$ $\quad$ B-inputs matrix
C-outputs matrix
D-feedthrough matrix (often = 0)

Initial Parameters

Initial parameters $x(t_0)$ represent system memory of the past $t < t_0$

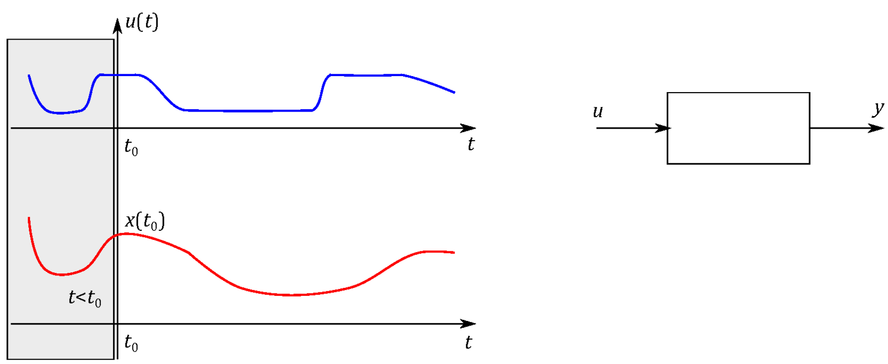
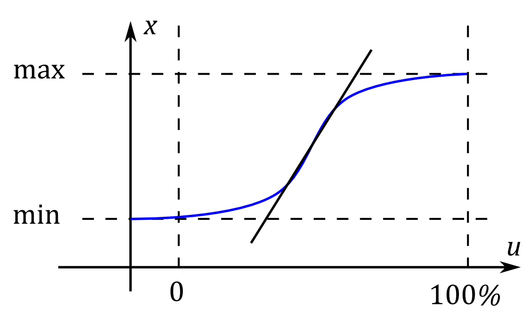


Figure 1.4: Initial States

$x(t_0)$ and $u(t), t > t_0$ define the future behavior of the system $x(t)$.

Steady State

Steady state/ püsiolek/ *статика* is a working point where $\mathrm{d}x/\mathrm{d}t = 0$.



- Limits exist (min, max)
- Deviations from the linear dependencies
  - $\pm 20\%$–imperceptible
  - $\pm 50\%$–appreciable
- Horizontal parts – input does not impact the output
- Vertical parts – $k \gg$, oscillations

$f(x, u) = 0$ associates inputs and outputs, often is nonlinear.
Process steady state is equally important as a process dynamics.

Linearization

What is a linear differential equation? It is one that contains variable only to the first power in any one term of the equation. If square roots, squares, exponentials, products of variables appear in equation, it is non-linear.

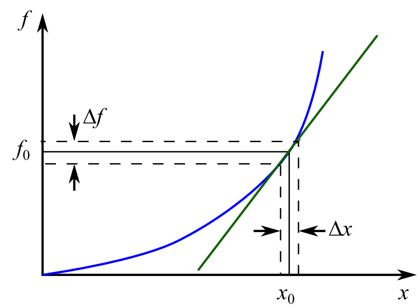


Figure 1.5: Vicinity of working point

The nonlinear function $f(x)$ can be expanded around the reference point x_0 using Taylor series expansion. And if x_0 is a steady state value $\frac{\mathrm{d}x_0}{\mathrm{d}t} = 0 = f(x_0)$, then using only first-order term, we obtain a linear equation (all terms after the first partial derivatives are neglected).

$$\frac{\mathrm{d}x}{\mathrm{d}t} \approx f(x_0) + \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x_0} (x - x_0) = f(x_0) + \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x_0} \cdot \Delta x \tag{1.3}$$

Since steady state is chosen as operating point, so $f(x_0) = 0$

$$\frac{\mathrm{d}x}{\mathrm{d}t} \approx \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x_0} \cdot \Delta x \quad \Rightarrow \quad \frac{\mathrm{d}(x - x_0)}{\mathrm{d}t} \approx \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x_0} \cdot \Delta x$$

or

$$\frac{\mathrm{d}\Delta x}{\mathrm{d}t} \approx \left.\frac{\mathrm{d}f(x)}{\mathrm{d}x}\right|_{x_0} \cdot \Delta x = a \cdot \Delta x, \tag{1.4}$$

where $\Delta x = x - x_0$ – is deviation variable, and a – constant that corresponds to the derivative of the function evaluated at steady state.

We obtained linear equation in terms of deviation variable Δx.

For *one state* and *one input*

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x,u) \approx f(x_0,u_0) + \left.\frac{\partial f(x,u)}{\partial x}\right|_{x_0,u_0} \cdot \Delta x + \left.\frac{\partial f(x,u)}{\partial u}\right|_{x_0,u_0} \cdot \Delta u$$

or

$$\frac{\mathrm{d}\Delta x}{\mathrm{d}t} \approx \left.\frac{\partial f(x,u)}{\partial x}\right|_{x_0,u_0} \cdot \Delta x + \left.\frac{\partial f(x,u)}{\partial u}\right|_{x_0,u_0} \cdot \Delta u = a \cdot \Delta x + b \cdot \Delta u \tag{1.5}$$

We are always interested in tracking the variables of interest as they fluctuate near the equilibrium (steady-state) point. Explaining the dynamic behavior with reference to the desired operating point [4].

3.2 Transfer functions

Laplace-transforming the linear ordinary differential equations describing our process in terms of the independent variable t converts them into algebraic equations in the Laplace transform variable s. This provides a very convenient representation of system dynamics.

S domain

Laplace Transform $L[f(t)]$

$f(t) \longleftrightarrow F(s)$ *variable s*

$$F(s) = \int_0^\infty f(t) \cdot e^{-st}\mathrm{d}t$$

1. The transfer function model provides a simpler I/O model than in time domain.
2. It completely describes dynamics of the the output variable.
3. Transfer function is a system property, so independent of the input form.
4. We cannot form the transfer function with nonzero initial conditions. Only defined for deviation parameters.

Differential equation → algebraic equation.

Asymptotic Theorems

If object is stable we can find values of the time domain functions at two extremes $t = 0$ and $t = \infty$, without inverse transform.

- Initial-value theorem

$$\lim_{s\to\infty} [sF(s)] = \lim_{t\to 0} f(t)$$

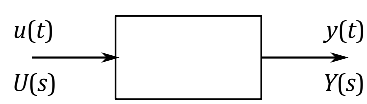
- Final-value Theorem

$$\lim_{s\to 0} [sF(s)] = \lim_{t\to\infty} f(t)$$

In control we use final-value theorem quite often [5].

Our primary use of Laplace transformations in process control involves representing the dynamics of the process in terms of "transfer functions" [6].

Transfer function model $H(s), G(s), W(s), \ldots$



Output depends on the input, zero initial values $Y(s) = H(s) \cdot U(s)$,

$$H(s) = \frac{s^m + \cdots + b_0}{s^n + \cdots + a_0},$$

$$H(s) = k \cdot \frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)},$$

where n and m are orders of poles and zeros.

If poles are real we can use parameter which is called time constant τ:

$$H(s) = K_p \cdot \frac{(\tau_{z_1} \cdot s + 1)\ldots}{(\tau_{p_1} \cdot s + 1)(\tau_{p_2} \cdot s + 1)\ldots}, \tag{1.6}$$

where $\tau_i = -\frac{1}{p_i}$ is a time constant, K_p is the steady-state or static gain.

K_p shows the sensitivity of the process output to a manipulated input or in other words is the ratio of the long-term change in process output to the change in process input.

Mainly time constants are used in industry because those have physical interpretation and can be considered as inertia of the system.

Realizability of the transfer function defines ratio $n : m$

- Strictly proper = realizable if $n > m$

  $\lim_{s\to\infty} H(s) = 0$, in practice $H(\omega \to \infty) = 0$

- Semi-proper, biproper

  $\ldots = /0$

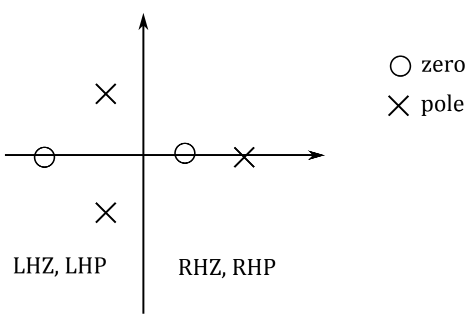
- Improper = not realizable if $n < m$

  $\lim_{s\to\infty} H(s) = \infty$

For physically realistic processes $n \geq m$.

Process is proper if its output depends only on past inputs, and improper if its output depends also on the future inputs.

Designation of the plane $s = \sigma + j\omega$



Stable system: $Re(p) < 0$

Unstable system : $Re(p) > 0$

Integrating system: at the origin.

The poles closer to the origin are dominant!

What about Time Constants?

Increasing the magnitude of imaginary portion makes the response more oscillatory.

Poles of the system p_i are state matrix A eigenvalues

- $[A, B, C, D] \rightarrow H = C \cdot (sI - A)^{-1} \cdot B + D$

  during the transfer function calculation zeros and poles can withdraw

- backward $[A\ B\ C\ D] \leftarrow$ is not uniquely defined, depend on the state choice

  H can contain less information than $[A\ B\ C\ D]$

First Order Systems

Many systems can be represented as single-state models, where the output is the state. In that case in can be represented by the first-order differential equation

$$\tau_p \frac{\mathrm{d}y}{\mathrm{d}t} + y = K_p u, \tag{1.7}$$

where y is the output variable and u–the input variable.

$$L\left[\tau_p \frac{\mathrm{d}y}{\mathrm{d}t}\right] = \tau_p L\left[\frac{\mathrm{d}y}{\mathrm{d}t}\right] = \tau_p\left[sy(s) - y(0)\right] = \tau_p \cdot sy(s)$$
$$L\left[y\right] = y(s)$$
$$L\left[K_p u(t)\right] = K_p \cdot L\left[u(t)\right] = K_p u(s)$$

So Laplace transform is $(\tau_p s + 1)y(s) = K_p u(s)$, and the first order transfer function

$$y(s) = \frac{K_p}{\tau_p s + 1} u(s). \quad (1.8)$$

Many processes have a delayed response to a process input. Such processes can be described as First-Order-Plus-Dead-Time (FOPDT) transfer function

$$H(s) = \frac{K_p}{\tau_p s + 1} e^{-\theta s}, \quad (1.9)$$

where θ is a time delay. It is also called dead time or transport lag.

$$\tau_p \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = K_p u(t - \theta), \quad (1.10)$$

Time delays have a strong influence on the control system and may drastically degrade the performance [4].

Characteristics

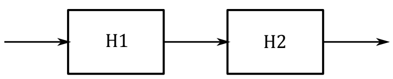
1. Step $g(t)$
   - $u(t) = 1(t)$
   - $U(s) = 1 \cdot 1/s$
2. Pulse response $h(t)$
   - $u(t) = \delta(t)$
   - $U(s) = 1$
3. Ramp r(t)
   - $u(t) = \sigma(t) = \begin{cases} 0, & \text{t<0;} \\ t, & \text{t>0.} \end{cases}$
   - $U(s) = 1/s^2$

Those co-called "singular functions" d/d$t = \infty$ are suitable for system tests. Extreme modes: if works here, then works well with other signals as well.

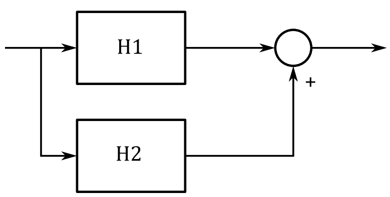
Block-diagram reduction

Block-diagrams illustrate a cause-and-effect relationship.

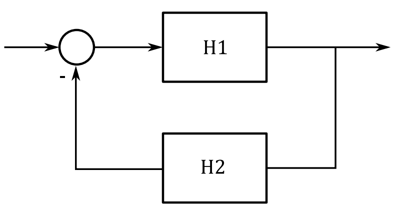
Figure 1.6: Step, pulse, ramp signals



$$H = H_1 \cdot H_2$$



$$H = H_1 + H_2$$



$$H = \frac{H_1}{1 + H_1 \cdot H_2} = \frac{\text{"direct"}}{1 - -\text{"loop"}}$$

Blocks are used to represent transfer functions and lines for indirect information transmission.

Using transfer functions

✓ Get results without resolving differential equations

stability,

final-values ($t = \infty$).

✓ Impacts are clearly shown

differences and similarities.

✓ Suitable for linear SISO systems.

3.3 Frequency Response

Description of the frequency domain.

If sinusoidal input is imposed and frequency response is measured the dynamic behavior of the system can be studied. Bode and Nyquist plots are graphical representations of functional dependence of magnitude and phase on frequency.

$H(j\omega) = y/u = M \cdot e^{j\phi}$

Magnitude and phase frequency characteristic

1. Phase frequency characteristic $\phi(\omega)$
2. Complex frequency characteristic $M(j\omega)$

For more information see

Course ISS0031 Modeling and Identification,

Lecture: Linear systems. Control design.

Topic: Frequency Domain Analysis of SISO Systems L05_Linear_systems.pdf.

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