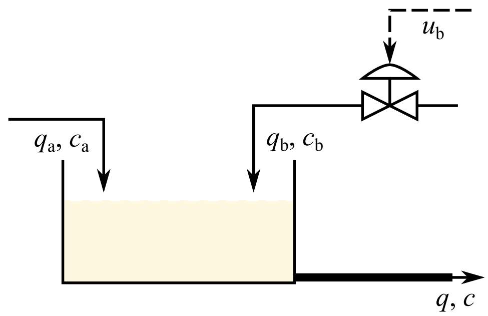
Concentration Control



Two liquids a and b mixed in order to achieve a concentration c of substance X. Outflow concentration is measured (quantity of substance X in total volume %). Inflow concentrations are $c_a = 2\%$ and $c_b = 96\%$.

During mixing process volume does not change. In order to obtain the needed product concentration inflow q_b can be manipulated. Outflow rate is $q = 10 \ldots 100$ $[l/min]$.

Questions

1. Calculate I/O characteristics and the gain (with different consumption rates). Provide result with a graph and equation. With an input rate of $0 \ldots 5\%$ sensor provides output signal $c_x = 0 \ldots 100\%$. The actuator is chosen so, that its 90% control signal u_b provides concentration of 7% with a maximal consumption.
2. Calculate sensor signal $c_x(\%)$, actuator control signal u_b and gain K_{sys} (output c_x dependence on control signal u_b) in order to obtain concentration $c = 4\%$ with minimal and maximal consumption q.
3. What is the transfer function of the system if sensor is placed 100 cm from the tank and pipe $r = 5$ cm.

Parameters used:

- $c_a = 2\%$
- $c_b = 96\%$
- $q_{out} = 10 \ldots 100$ l/min
- q_b— can be manipulated

Notes:

The following assumptions are applied:

1. the mixture volume is constant
2. density is constant and independent of temperature
3. tank is well mixed, outlet concentration is the same as inside the tank

4. concentrations of the substances remain constant
5. the heat of mixing is negligible, as well as rate of chemical reaction

Component balance:

$$\left\{\begin{array}{l}\text{rate of accumulation}\\ \text{of component}\end{array}\right\} = \left\{\begin{array}{l}\text{rate of inflow}\\ \text{of component}\end{array}\right\} - \left\{\begin{array}{l}\text{rate of outflow}\\ \text{of component}\end{array}\right\} + \left\{\begin{array}{l}\text{rate of generation}\\ \text{of component by}\\ \text{chemical reactions}\end{array}\right\}$$

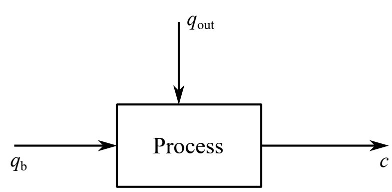
$$\frac{\mathrm{d}mc}{\mathrm{d}t} = q_{\text{in}}c_{\text{in}} - q_{\text{out}}c_{\text{out}} - RV, \tag{1}$$

where R depends on k_r—kinetic rate constants, c concentrations of the components in chemical reaction.

$$\frac{\mathrm{d}mc}{\mathrm{d}t} = \frac{\mathrm{d}(\rho Vc)}{\mathrm{d}t} = \text{in} - \text{out} = (\rho F_a c_a + \rho F_b c_b) - \rho F c$$

We need to control concentration c, so this parameter should be output of the system.

- Only q_b can be manipulated, so evaluate parameter q_a.



product concentration

$$c = \frac{m_x \cdot 100\%}{m} = \frac{q_x \cdot 100\%}{q}$$

- What is the dependency between system output and input?
  - gain $K = \dfrac{\text{d out}}{\text{d in}}$
- System block diagram

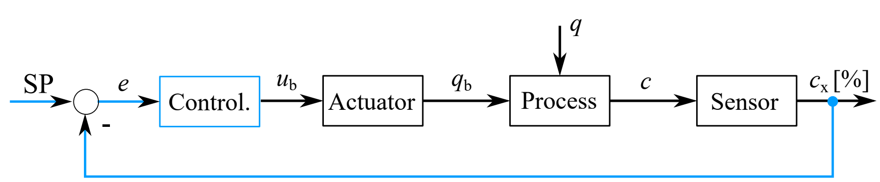


Figure 1: System block diagram