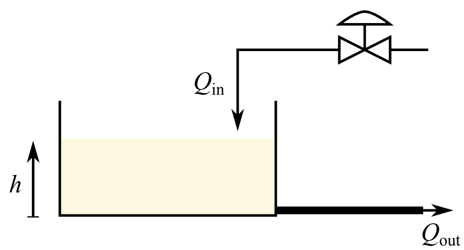
1 Process description: differential equation, linearization



- Aim: liquid level control.
  - How does liquid level depend on volumetric flows?
  - What technique to use?

Quantity that provides information about the state of the tank is the total mass of the liquid in the tank.

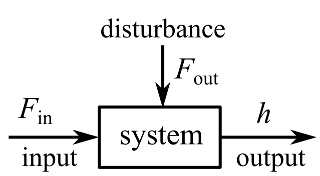
1. Use conservation law of mass

$$\frac{[\text{Accum. of total mass }]}{\text{time}} = \frac{[\text{Input of total mass}]}{\text{time}} - \frac{[\text{Output of total mass}]}{\text{time}} \tag{1}$$

2. Express total mass in terms of state variable

$$\text{total mass } m = \rho V = \rho A h,$$

where ρ is a density, V is a volume of liquid, A–cross sectional area of the tank and h–level of the liquid.



$$\frac{\mathrm{d}(\rho A h)}{\mathrm{d}t} = \rho F_{in} - \rho F_{out}, \tag{2}$$

where F are volumetric flow rates.

3. Make some assumptions if needed.

We assume that the density ρ is constant

$$A\frac{\mathrm{d}h}{\mathrm{d}t} = F_{in} - F_{out}$$

We have equation that describes the change of level in the tank as a function of inflow and outflow.

1.1 Differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{A}(F_{in} - F_{out}) \tag{3}$$

What is the F_{out}?

Questions:

1. Provide differential equation (process model) where outflow depends on the liquid level.
2. What is the steady-state of the given process?
3. Linearize system near some operating point.
4. Laplace domain representation: what are the time constant τ and gain K_p values? Provide system transfer function $W(s) = \dfrac{K_p}{1 + \tau_p \cdot s}$.

2 Approximation with FOPDT

FOPDT model: $W(s) = \dfrac{K}{1 + \tau_o \cdot s} e^{-\theta s}$ where $s = j\omega$

SOPDT model: $W(s) = \dfrac{K}{(1 + \tau_1 \cdot s)(1 + \tau_2 \cdot s)} e^{-\theta s}$,

Simplify the next transfer functions:

1. $W_1 = \dfrac{13.7}{(1 + 120s)(1 + 36s)}$
2. $W_2 = \dfrac{K}{(1 + 10s)(1 + 36s)(1 + 120s)(1 + 14s)}$
3. $W_3 = \dfrac{13.7}{(1 + 120s)(1 + 36s)} e^{-18s}$
4. $W_4 = \dfrac{13.7(1 + 12s)}{(1 + 10s)(1 + 6s)} e^{-s}$
5. $W_5 = \dfrac{13.7(1 - 12s)}{(1 + 120s)(1 + 36s)} e^{-18s}$
6. $W_6 = \dfrac{K(1 + 12s)(1 - 4s)}{(1 + 120s)(1 + 36s)(1 + 14s)(1 + 10s)} e^{-9s}$
7. $W_7 = \dfrac{1 + 10s}{(1 + 12s)(1 + 23s)} e^{-3s}$

Use both General and Skogestad method.

For more information see,
course ISS0065 Control Instrumentation,
lecture Loop Control and Process Models,
topic FOPDT Lecture 4