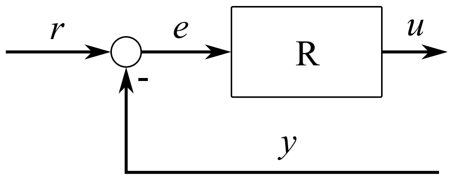
1 Continuous controllers

The output range of the controller is continuous ($u_{\min} \ldots u_{\max}$)

Terms in industry:



SV Set Value (r) or

SP Set Point;

PV Process/ Present Value (y);

CV Control variable (u) or

MV Manipulated Variable.

Set Point / seadesuurus / уставка.
Process Value / hetke (juhitav) värtus / регулируемый (технологический) параметр.
Control variable / juhtsignaal / управляющий сигнал.

The controller's work describe:

Control function: $u(t) = G[E(t)]$, (dynamics!);

Static characteristic: $u = f(e)$;

Transfer function: $W_r(s)$;

Frequency domain characteristic: $W_r(j\omega)(s \to j\omega)$;

Step response: $h(t)$.

Initial data for the controller are:

- Measured values,
- Desired value (set point),
- Information about the object, limitations, etc.

The most common of all continuous industrial process control action is proportional control action. Controllers automatically compare the value of the PV to the SP to determine if an error exists. If there is an error, the controller adjusts its output according to the parameters that have been set in the controller.

PID controller popularity

1. Widespread, available, 85% of regulators are PID

   the industry "working bees".

2. Suitable for the most objects:

   Stable, unstable, etc.

   SISO PID is not suitable for the 5% . . . 10% of the objects:

   - There are some unstable processes what cannot be controlled by PID controller.
   - Some processes is better to control by more advanced controller.

Sad statistics:
Some controllers are not properly used

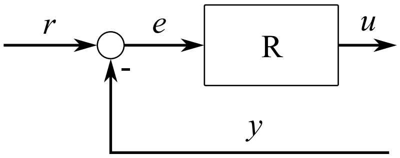
- 80% require re-tuning;
- 30% manually operated;
- 20% use auto-tuned parameters.

PID control law:

- is the sum of three components: the P, I, D;
- 4 standard laws for P, PI, PD, PID.

1.1 Proportional mode

Proportional (gain) control action reproducing changes in input as changes in output.



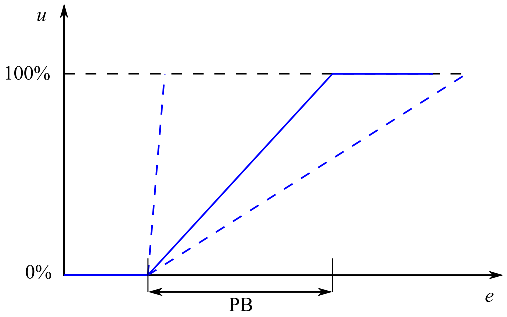
Control law

$$u = K_c \cdot e. \tag{1}$$

Control signal proportional to e,

where K_c is a **gain**/ võimendustegur/ коэффициент усиления.

Controller static characteristic is $u(e)$ with respect to error e. Output value is limited $0 \ldots 100\%$. Proportional (gain) control action reproducing changes in input as changes in output.



Three parts:

$$u = \begin{cases} 0\% & - \ \min \\ K_c \cdot e & - \ linear \\ 100\% & - \ \max \end{cases}$$

Output signals types:
$(4-20)\ mA,\ 0-4000,\ 0\% \ldots 100\%,\ 0-1$.

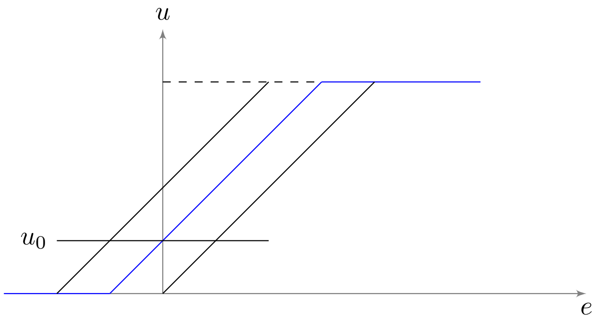
Proportional action responds only to a change in the magnitude of the error.

The controller has an error between PV and SP called **proportional-only offset** / staatiline viga / статическая ошибка, some times called droop.

> Proportional action will not return the PV to set point ($PV \neq SP$). It will, however, return the PV to a value that is within a defined span around the PV [1].

To minimize the proportional-only offset we need to increase the controller gain (decreasing its proportional band). That makes controller more "aggressive". However, too much controller gain and control system becomes unstable (oscillations).

Another way: human operator places the controller in manual mode and move the controlled actuator just a little bit more u_0, so $PV = SP$, and then place the controller back into automatic mode. Otherwise, we need more sophisticated control techniques [2].

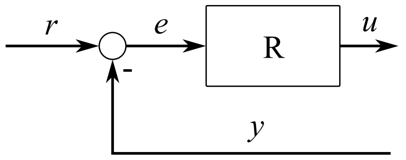


Bias:
$u = K_c \cdot e + u_0$
$u_0 = 25\%, 50\%, 0 \ldots 100\%$

Needed value of the bias depends on:

1. operating point r, process K_p,
2. known disturbance h.

Controller characteristics



In respect to y
$u = K_c \cdot (r - y)$ or
$CV = K_c \cdot (SP - PV)$

I/O characteristic:

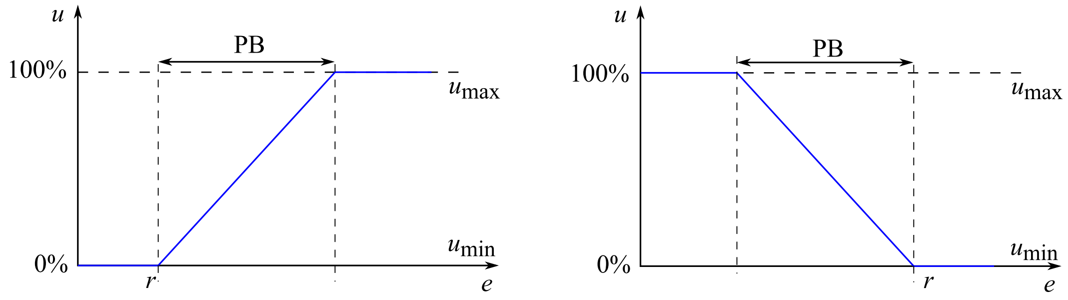


Figure 1: I/O characteristics of P controller: direct action and reverse action

Instead of gain K_c term PB —**proportional band** / proportsionaalsuse tsoon / зона пропорциональности is used.

Proportional Band (PB) answers a question: "What percentage of change of the controller input span will cause a 100% change in controller output?"

$$PB \cdot K_c = 100\% \tag{2}$$

The proportional band (PB) units are PV units.

- Some controllers have several measuring ranges.
- Some controllers do not know what is measured ($4 - 20\ mA$).

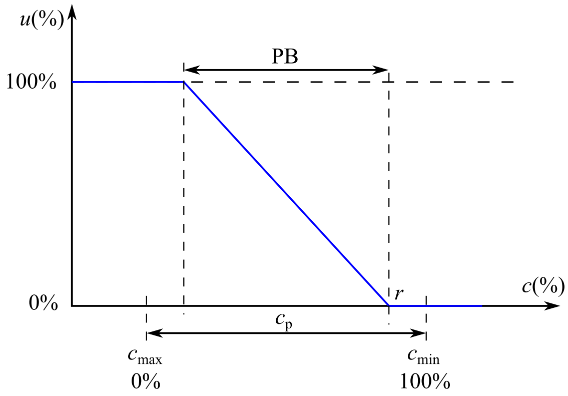


Figure 2: Controller output

"Proportional band" can be also defined as the amount of input change necessary to evoke full-scale (100%) output change in a proportional controller.

K_c	PB
1	100%
2	50%
5	20%
20	5%
0.5	200%

K_c—has no units

1% change of the input signal causes the K_c% of output signal change.

This control action works immediately to match changes in the input signal.

$$PB = \frac{\Delta \text{Input}}{\Delta \text{Output}} = \frac{\Delta e}{\Delta u} \tag{3}$$

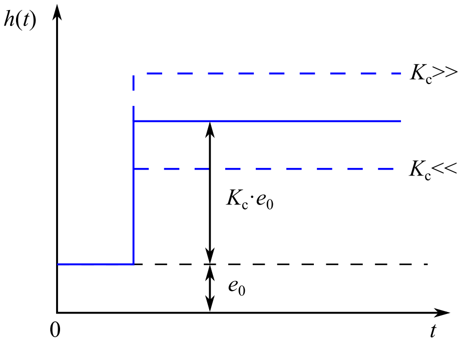


Figure 3: Reaction of the P-controller on input error

Conclusions [1]

1. Proportional Mode Responds only to a change in error;
2. Proportional mode alone will not return the PV to SP.

Advantages —Simple;

Disadvantages —Error;

Settings —PB settings have the following effects:

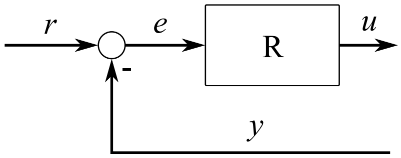
- Small PB—minimize offset,
- High Gain—possible cycling,
- Large PB—large offset,
- Low gain—stable loop.

1.2 Integral control mode (I)

I—Integral, reset action.

The purpose of I action is to **eliminate offset**. Unlike proportional action, which simply moves the output an amount proportional to any change in PV or SP, integral control action does not stop moving the output until all error has been eliminated.

If proportional action is defined by the error telling the output how far to move, integral action is defined by the error telling the output how fast to move.



$u = K_i \int e \, \mathrm{d}t = \dfrac{1}{T_i} \int e\mathrm{d}t$,

K_i—transfer gain (repeats per minutes);

T_i—integral time constant (minutes per repeat).

The integration symbol tells us the controller will accumulate (“sum”) multiple products of error (e) over tiny slices of time ($\mathrm{d}t$).

Integral is a highly effective mode of process control. In fact, some processes respond so well to integral controller action that it is possible to operate the control loop on integral action alone, without proportional. Typically, though, process controllers are designed to operate as proportional-only (P), proportional plus integral (PI) [2, 3].

Conclusions [1]

Advantages —Eliminates error,

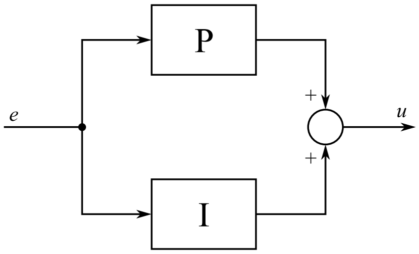
Disadvantages —Reset windup and possible overshoot.

- Fast reset
    - high gain,
    - fast return to set point,
    - possible cycling.
- Slow reset
    - low gain,
    - slow return to set point,
    - stable loop.

PI (Reset) control

$$u = K_c \cdot e + K_i \cdot \int e \ \mathrm{d}t = K_c \left(e + \frac{1}{T_i} \int e \ \mathrm{d}t \right) \tag{4}$$

Two parameters: K_c gain and T_i reset time.



eliminated offset (automatic reset),
faster than I.

The open loop response of a PI controller to a step change in error is depicted Fig. 4.

$$u = K_c \cdot e_0 + \frac{K_c}{T_i} e_0 t$$
$$\text{if } t = T_i$$
$$\Downarrow$$
$$u = 2K_c \cdot e_0 \tag{5}$$

This enables the definition of reset time. T_i is the time taken, in response to a step change in error, for the I action to produce the same change in output as the P action.

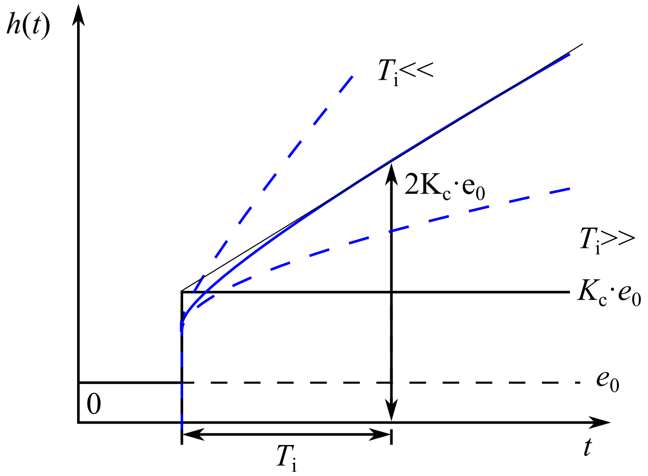


Figure 4: Open loop response of PI controller to a step change in error

Different manufactures cast their control algorithms in slightly different forms. Some use reset rate or transfer gain K_i, instead of reset time. These are simply the inverse of each other:

$$K_i = 1/T_i. \tag{6}$$

It is critical to *know your manufacturer before you start tuning* your controller because parameter values must be matched to your particular algorithm form. Otherwise both PI algorithm representations are equally capable [4].

Integral Windup

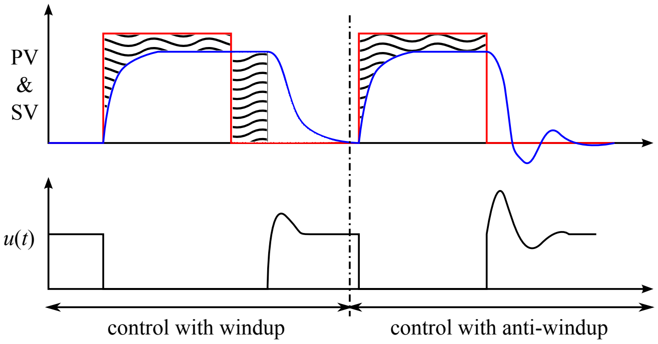


Figure 5: Control with windup and anti-windup protection

Integration is a continual summing. Integration of error means that we continually sum controller error up to the present time.

Reset windup is described as a situation where the controller output is driven from a desired output level because of a large difference between the set point and the process variable. If the controller output saturates whilst an error exists, the I action will continue to integrate the error and, potentially, can become a very large quantity. When eventually the error reduces to zero, the controller output should be able to respond to the new situation (shutdown of the system). However, it will be unable to do so until the error has changed sign and existed long enough for the effect of the integration prior to the change of sign to be canceled out. The output remains saturated throughout this period and the controller is effectively inoperative.

Various techniques exist to manage integral windup. Controllers may be built with limits to restrict how far the integral term can accumulate under adverse conditions. Employing extra "jacketing logic" in the software to halt integration when the $u(t)$ reaches a maximum or minimum value: in some controllers, integral action may be turned off completely if the error exceeds a certain value or human operator can interfere, by placing the controller in manual mode. Another way is recasting the controller into a discrete velocity form that, by its very formulation, naturally avoids windup [2, 3, 4] .

1.3 Derivative mode

D—derivative action.

PD is a control action causing the output signal to be offset by an amount proportional to the rate at which the input is changing. If proportional (P) action tells the output how far to go when an error appears, derivative (D) action tells the output how far to go when the input ramps. If proportional (P) action acts on the present and integral (I) action acts on the past, derivative (D) action acts on the future: it effectively “anticipates” overshoot by tempering the output response according to how fast the process variable is rising or falling [2].

$$u = K_c \cdot e + K_d \frac{\mathrm{d}e}{\mathrm{d}t} = K_c \left(e + T_d \frac{\mathrm{d}e}{\mathrm{d}t} \right) \tag{7}$$

T_d is known as the rate time and characterizes the D action. Adjusting T_d varies the amount of D action, setting it to zero turns off the D action.

Conclusions

Advantages —Rapid output reduces the time that is required to return PV to SP in slow process.

Disadvantages —Dramatically amplifies noisy signals; can cause cycling in fast processes [1].

- Large (Minutes)
  - high gain,
  - large output change,
  - possible cycling.
- Small (Minutes)
  - low gain,
  - small output change,
  - stable loop.

If there is a measuring noise present in $y(t)$ will amplify this noise. Low-pass filter in derivative could insure derivative action only in the frequency band of interest and diminish negative effect of D mode on signal noise. Filter is needed not only because of the effect of noise, but also because it is impossible to build ideal derivative elements since they are noncasual filters. Ideal D action is noncasual dynamics and it cannot be physically realized. Thus, instead of noncasual D mode in control is used casual derivative element (filter):

Realizable controller (zero+pole)—lead-lag

$$W(s) = K_c \left(1 + \frac{T_d \cdot s}{1 + \alpha \cdot T_d \cdot s} \right) = K_c \left(1 + \frac{T_d \cdot s}{1 + \frac{T_d}{t_p} s} \right), \tag{8}$$

where α is a derivative action sharpness or t_p. Majority of the controllers available at market today has t_p value between $3 - 20$, which is satisfying in most situations.

Thus, t_p is used to limit derivative gain on higher frequencies.

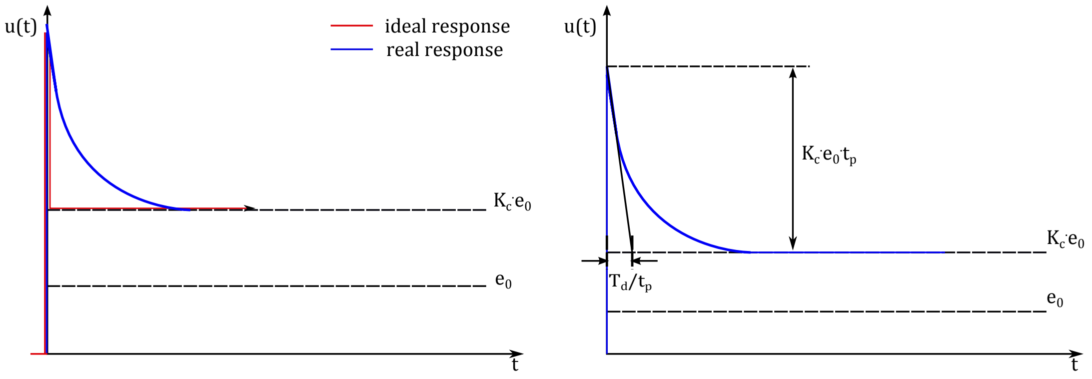


Figure 6: PD controller parameters

1.4 PID control

A combination of all three of the actions described above is more commonly referred to as PID action.

In general transfer function is

$$W(s) = K_c \left(1 + \frac{1}{T_i \cdot s} + T_d \cdot s\right) \tag{9}$$

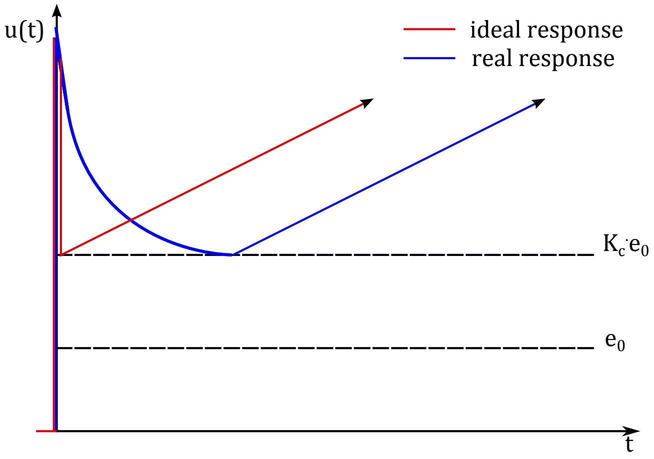


Figure 7: Open loop response of PID controller

There are three major variations how PID equations implemented in modern PID controllers: the **parallel**, **ideal** and **series**. This creates more challenges for controller tuning.

Parallel PID

$$u = K_c \cdot e + \frac{1}{T_i} \int e \ \mathrm{d}t + K_d \frac{\mathrm{d}e}{\mathrm{d}t} \tag{10}$$

In the parallel equation, each action parameter (K_c, T_i, T_d) is independent of the others. Equation can be broken up three parts, each one describing its contribution to the output u.

Ideal/ISA PID

The gain K_c affects all the three actions

$$u = K_c \left(e + \frac{1}{T_i} \int e \ \mathrm{d}t + T_d \frac{\mathrm{d}e}{\mathrm{d}t}\right). \tag{11}$$

The gain constant K_c equally affecting all three control actions. Increasing K_c makes the P, I, D actions equally more aggressive.

Series PID

The gain K_c also affects all three actions. The difference is the fact that both integral and derivative constants have an effect on proportional action as well

$$u = K_c \left(\left(\frac{T_d}{T_r} + 1 \right) e + \frac{1}{T_r} \int e \ \mathrm{d}t + T_d \frac{\mathrm{d}e}{\mathrm{d}t} \right). \tag{12}$$

The proportional term is also affected by the values of the integral and derivative tuning parameters. Therefore, adjusting T_i affects both the I and P actions, adjusting T_d affects both the D and P actions, and adjusting K_c affects all three actions.

Not every process requires a full control strategy.

Table 1: Control loops and Control algorithms [1]

Controlled variable	P control	PI control	PID control
Flow	+	+	-
Level	+	+	Rare
Temperature	+	+	+
Pressure	+	+	Rare
Analytical	+	+	Rare

If small offset has no impact on the process, then only Proportional control can be used.

Usually engineers use full PID controller if we cannot tolerate the offset and dead time is a problem, on the other hand, no noise is present in the system.

Bibliography

[1] PAControl. Instrumentation and control: Process control fundamentals. [Accessed: September, 2017]. [Online]. Available: http://www.pacontrol.com/download/Process%20Control%20Fundamentals.pdf

[2] T. R. Kuphaldt. (2016) Lessons in industrial instrumentation. [Accessed: September, 2017]. [Online]. Available: http://www.pacontrol.com/download/Lessons-in-Industrial-Instrumentation.pdf

[3] J. Love, *Process Automation Handbook: A Guide to Theory and Practice.* Springer-Verlag London Limited, 2007.

[4] ControlGuru. Integral action and PI control. [Accessed: October, 2017]. [Online]. Available: http://controlguru.com/integral-action-and-pi-control/