1 Loop Control

System has a continuous signal (analog) basic notions: open-loop control, close-loop control.

1.1 Open-loop

Open-loop / avatud süsteem / открытая система

Open-loop systems have the advantage of being relatively simple and consequently cheap with generally good reliability. However, they are often inaccurate since there is no correction for errors in the output which might result from extraneous disturbances [1].

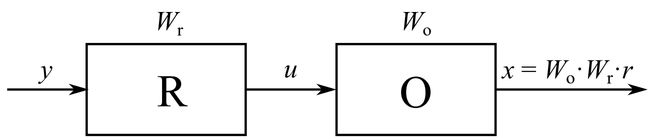


Figure 1: Open-loop Control

From the condition $x = r \quad \Rightarrow \quad W_r = W_o^{-1} \qquad (K_r = 1/K_0, \quad F_r() = F_o^{-1}())$

$u(r) = W_r \cdot r$

Is it possible to implement?

Are any disturbances present?

Feedforward control

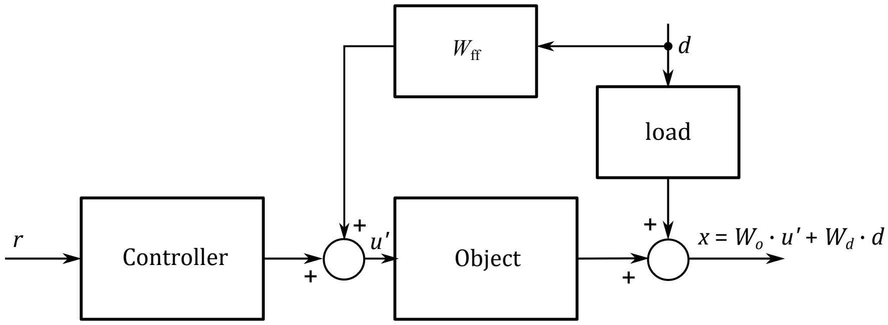


Figure 2: Feedforward Control

To compensate for changes in d a feedforward element W_{ff} may be introduced, see Fig. 2.

From $(x = r)$ follows$x = W_d \cdot d + W_o(W_r \cdot r + W_{ff} \cdot d) = W_d \cdot d + W_o W_r \cdot r + W_o W_{ff} \cdot d = W_o W_r \cdot r + (W_o W_{ff} + W_d) d$, where $W_r = W_o^{-1}$.

If disturbance d is measured it can be compensated.

To satisfy this criterion it is necessary for

$$(W_o W_{ff} + W_d)d = 0.$$

As d cannot be assumed as 0, it follows that

$$W_{ff} = -\frac{W_d}{W_o}.$$

Thus, the feedforward path creates signal which will cancel out the effect of the load operating on disturbance [2].

Control design problems:

1. Assumption of the availability of a perfect model is never true;
2. Computationally difficult or impossible to invert the model W_o^{-1};
3. Disturbances not always can be measured - knowledge of the process is imperfect.

Open-loop one of the alternatives that can be used.

- Low accuracy is needed, fuzzy situation, slow changes (static).
- Does not lead to instability?

Example 1 *Feedforward control*

Heating system of the apartments with adjusting valve.

1.2 Feedback Control

Feedback system / suletud süsteem / замкнутая система

The control mechanism acts using the information fed back from the measurements.

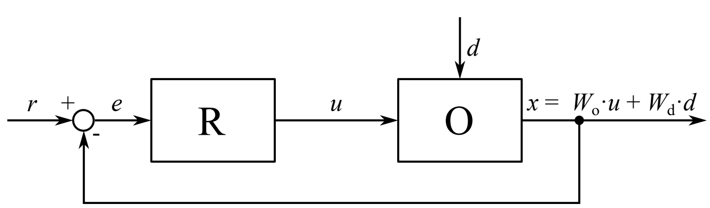


Figure 3: Feedback Control

MIMO: $x = (I - W_o \cdot W_r)^{-1} \cdot W_o \cdot W_r \cdot r + (I - W_o \cdot W_r)^{-1} \cdot W_o \cdot W_d \cdot d$

SISO: $x = \frac{W_o \cdot W_r}{1 + W_o \cdot W_r} \cdot r + \frac{1}{1 + W_o \cdot W_r} \cdot W_d \cdot d$

$$\text{OUT} = \frac{\text{"direct"}}{1 - \text{"} - \text{loop"}} \cdot \text{INP} = \frac{\text{"direct"}}{1 + \text{"loop"}} \cdot \text{INP} \tag{1}$$

open-loop transfer

$W_o \cdot W_r = L$ loop gain

close-loop transfer

$\frac{W_o \cdot W_r}{1 + W_o \cdot W_r} = \frac{L}{1+L} = T$ complimentary sensitivity $T(s)$

$\frac{1}{1 + W_o \cdot W_r} = \frac{1}{1+L} = S$ sensitivity function $S(s)$

$x = T \cdot r + S \cdot W_h \cdot h, \qquad T + S = 1$

Perfect control $(T = 1,\ S = 0)$ is not possible,
Approximate control $(T \approx 1,\ S \ll 1,\ \text{if}\ \ L \gg 1)$ is possible.

Good features of the feedback

1. Reducing the impact of the disturbance d

   - The goal is to compensate, not to eliminate!

     ✓ disturbances in open-loop system: $x = W_d \cdot d$;

     ✓ disturbances in close-loop system: $x = S \cdot W_d \cdot d$

   - $L \gg 1 \Rightarrow S \ll 1$ regulatory problem

     – no disturbance h measurements needed,

     – no process model W_d necessary ,

     – knows the direction of u how to change $x - d$.

2. Tracking the set point as close as possible $x = r$

   - $L \gg 1 \Rightarrow T \approx 1$ servo problem

     – keep within the limits $x = r + \Delta x$.

3. Reducing the impact of the parameters change in loop L.

   - $\frac{dT}{T} = S \cdot \frac{dL}{L}$ the impact of the model errors is reduced;

   - good system $(\frac{dT}{T} <)$ from the bad components $(\frac{dL}{L} >)$;

- linearization of the steady-state (static) model .

4. System dynamics changes

   - faster in general,
   - stabilization of the unstable object.

Drawbacks of the feedback

1. System (stable object + stable controller)

   Can become oscillatory, that can cause stability reduction.

2. Noise impact shows up, for the $e \ll$ it is needed $L <$

   But this is in conflict with the requirement to reduce disturbances,

   Noise acts as input signal.

3. Strong control signals u on the controller output

   Saturation, difficult to realize.

4. Difficulties with the non minimum-phase systems (delays, RHP (Right Half-Plane), RHPZ (Right Half-Plane Zero), ...)

   Imposes *principle constraints* on systems,

   Desired objectives cannot be achieved (stability, performance),

   even if exact model is known and control signals are not saturated.

1.3 The Gang of Four

The feedback loop is influenced by three external signals: the reference r, the load disturbance d and the measurement noise n. The control mechanism acts using the information fed back from the measurements.

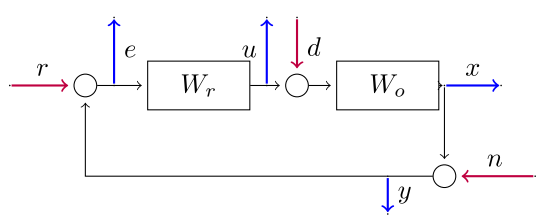


Figure 4: The pure error feedback loop

What are the closed system inputs and outputs?

d y
r → u
n x
e

In that case we have 12 transfers. If all control actions are based on feedback from the error only then the system is completely characterized by four transfer functions called *Gang of Four* [3].

$$S = \frac{1}{1 + W_r \cdot W_o} \qquad \text{sensitivity function } \frac{y(s)}{n(s)}$$

$$W_o S = \frac{W_o}{1 + W_r \cdot W_o} \qquad \text{load sensitivity function } \frac{y(s)}{d(s)}$$

$$T = \frac{W_r \cdot W_o}{1 + W_r \cdot W_o} \qquad \text{complementary sensitivity function } -\frac{u(s)}{d(s)}$$

$$W_r T = \frac{W_r}{1 + W_r \cdot W_o} \qquad \text{noise sensitivity function } -\frac{u(s)}{n(s)}$$

System is stable if Gang of Four is stable.

System is stable if bounded input signals (d, r, n) generate bounded outputs (y, u, x, e) (internal stability).

- If object W_o is stable then the only requirement is $W_r \cdot S = \dfrac{W_r}{1 + W_r \cdot W_o}$ should be stable;
- If object W_o is unstable then closed system can be made stable, requires all four transfers to be stable;
- It is possible to use unstable controller W_r.

1.4 Controllers

Theory: 1940 - the classics Bode, Nyquist, Evans, Ziegler, etc.
1960 - modern Kalman, Bellman, Zadeh

Usage: ships, milk pasteurization, chemical processes, etc.

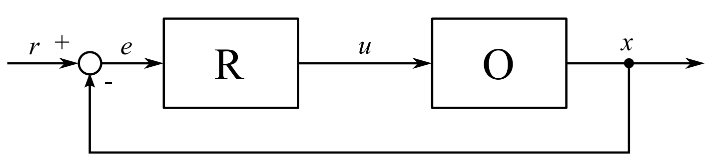


Figure 5: Feedback Control

Given: object O + requirements (statics and dynamics).
Find: a controller (type, tuning)
Main questions:

- Is it possible in general?
- How to obtain?
- What are the best tuning parameters?

There are two distinct control tasks:

1. Precisely track the set point

   $x \approx r, \quad \min e(t).$

2. Minimal sensitivity to disturbances

   $x = T \cdot r + S \cdot W_d \cdot d, \qquad T \approx 1, S \ll.$

Problem: transfer functions $r \to x$ and $h \to x$ can vary significantly,
controller should be designed only for one of them.

Purpose of the control

Stability of the system, what else?

- **Offset** $\to 0$ (staatiline viga / статическая ошбка);
- Dynamics - transfer characteristics (step response);
- Robustness - works well even if object O parameters are changed.

Controller can be
Chosen in different ways;
Connected to the object in different ways;
Tuned in different ways.

We can describe a controller and object (continuous / discrete) using

✓ Differential / difference equations;

✓ **State-space** representation (olekuvõrrand / модель состояний);

✓ **Transfer function** (ülekandefunktsioon} / передаточная функция).

The closed-loop system should be stable, accurate and fast!

Analysis:

1. Is a controller with given accuracy and speed realizable?

   Will face difficulties if:

   ✓ model is inaccurate,

   ✓ object is not linear,

   ✓ signals of the system are limited.

2. Without those difficulties is it still possible (given accuracy and speed)?

   Not always!

   ✓ Restrictions arise when system has: delays, RHPs, RHPZs;

   ✓ There are objects that cannot be controlled faster.

3. Is there an object impossible to control?

   Yes!

   ✓ Unstable object with RHP $\sim$ RHPZ.

   ✓ There are objects which is difficult/impossible to control in principle.

2 Process model

We need to know what we control (object/process).

Model of the process is designed such that

✓ Describes certain characteristics: form, behavior, etc;

✓ Different types: physical, mathematical;

✓ Several models of the same process;

✓ Depends on purpose of the model: understanding the process, predicting behavior, control.

Our aim is control: we are looking for models that provide insight into the feedback control problems.

What kind of models are appropriate for use?

Any model is always approximate, but it must be accurate enough to describe the process and as simple as possible in order to use it.

Model *complexity* $\leftrightarrow$ *accuracy* are related.

If needed model can be simplified or defined more accurately.

We can design more precise model as long as the accuracy of the model will not change it properties at the operating point. If closed-loop system behavior does not differ from the previous one then we deal with satisfactory model - good but not perfect.

How model is associated with an object?

It is known that similar open-loop systems have different characteristics of the closed-loop systems and vice versa.

Often, structure and the parameters of the model describing the process well, does not reflect the physical nature of the process (what are the reasons for that time-constant or delay?)

The process model can be found in a number of ways:

1. Using empirical (experimental) data, so named "black-box";
2. Using analytic techniques, so named functional model or "white-box";

   parameters based on the natural laws
3. Combination of so-called: "gray-box".

2.1 Functional model

Functional model is based on first principles. In many cases will be overly complex due to the complex nature of many systems and processes. Do we need such a precise (complex) model for the control?

Simplification: reduction of the model order

- State-space representation A, B, C, D - states

  ✓ unimportant states: singular values, etc.
- Transfer function $W(s)$ - poles

  ✓ unimportant poles.

Simplification: important - unimportant

Where to draw a line between?

Time domain: $y(t) = \sum A_i e^{-t/T_i}$

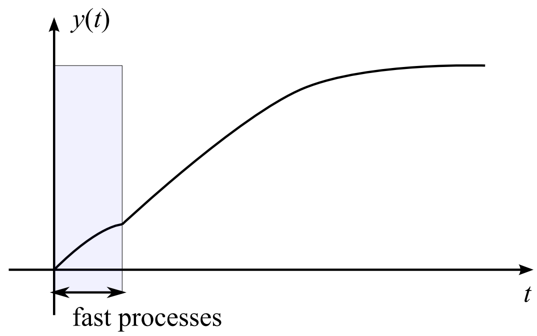


Figure 6: Time domain

1. Consider the dominant pole p_1 ($\min|p| - \max\tau$), ignore others $\Rightarrow$ first order system.

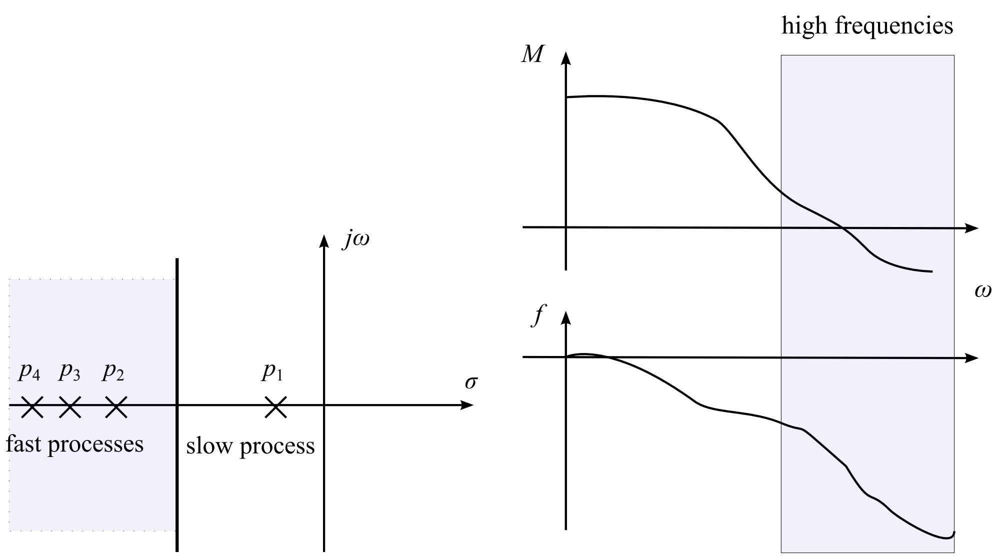


Figure 7: S and Frequancy domains

This kind of model is too simple to describe the whole system with needed accuracy, but can be used to describe system components.

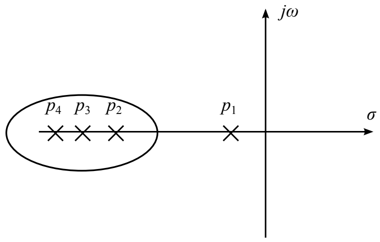
2. Take into account two important poles p_1, p_2, ignore others $\Rightarrow$ second order system.

more complex, more accurate model, but not perfect, there are better solutions

The real world is not purely 1. or 2. order systems, there is always a set of processes which influence must be considered.

Simplification: important - important enough

Approximate Reduced Order Transfer Functions



$|p_1| \ll |p_2, p_3, \ldots|$ or
$\tau_1 \gg \tau_2, \tau_3 \ldots$

How is the distant pole p_i ($|p_i| > |p_1|$) affects the dominant pole p_1 region?

In the frequency domain distant pole affects so (we are interested in frequency domain $\omega \approx 1/\tau_1$), that the amplitude does not change and the phase changes linearly $\phi \approx -\omega \cdot \tau_i$.

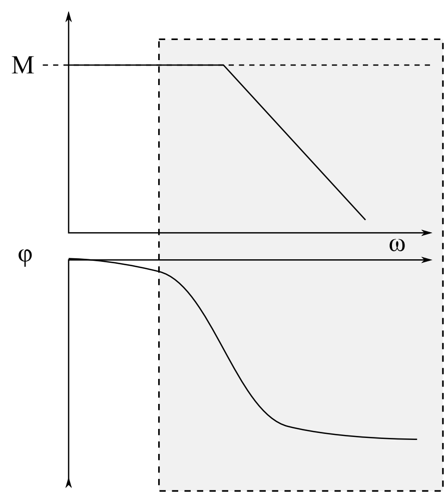


Figure 8: Magnitude anf phase

Delay term $e^{-j\omega\theta}$ with a delay $\theta = \tau_i$ behaves exactly the same way. Thus a distant pole can be considered as delay term.

If we deal with several distant poles $p_1, p_2, \ldots$ with time constants $\tau_1, \tau_2, \ldots$ then time constants are summarized and can be presented as one delay term

$$W(s) = e^{-s \sum \tau_i}. \tag{2}$$

Important dynamic feature of the objects are represented by **Time Constant** τ (ajakonstant / постоянная времени), not so important feature represented by **Delay** θ (hilistumine/ запаздывание).

Models: First-Order-Plus-Dead-Time (FOPDT) and Second-Order-Plus-Dead-Time (SOPDT).

2.2 FOPDT

First-Order-Plus-Dead-Time (FOPDT) models are presented by the next function

$$W(s) = \frac{K_p}{1 + \tau \cdot s} e^{-\theta s}, \tag{3}$$

where

K_p - process gain: the ultimate value of the response (new steady-state) for a unit step change in the input.

τ- time constant: measure of time needed for the process to adjust to a change in the input.

θ- delay: the time at which output of the system begins to change minus the time at which the input step change was made.

✓ 3 parameters: K_p, τ, θ; simple but moderately complex;

✓ Describes dynamics of the system with sufficient accuracy

calculated on this basis controllers work well.

✓ Easily obtainable after simplification of the complex models;

✓ Easy to identify.

Problem: if smaller less important time constant (τ_2) is close to the important time constant (τ_1).

To reduce the order of a transfer function is approximate the small τ with corresponding θ terms.

Example 2 *Approximation with FOPDT*

$$\frac{1}{1 + \tau_p s} \approx e^{-\tau_p s}; \tag{4}$$

$$1 - T_z s \approx e^{-T_z s} \tag{5}$$

If important τ_1 is the largest time constant $\tau_1 > \tau_2 > \tau_3$, the third order system can be approximated as follows:

$$
\begin{aligned}
\frac{K_p}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_3 s)} &\approx \\
&\approx \frac{K_p}{1+\tau_1 s}\cdot\frac{1}{1+\tau_2 s}\cdot\frac{1}{1+\tau_3 s} \\
&\approx \frac{K_p}{(1+\tau_1 s)}\cdot e^{-\tau_2 s}\cdot e^{-\tau_3 s} = \frac{K_p}{(1+\tau_1 s)}e^{-(\tau_2+\tau_3)s}
\end{aligned}
$$

Example 3 *Skogestad method*

The largest neglected time constant should be divided between the smallest retained time constant and the time delay [4]:

For the first order model:

$$\tau_{10} = \tau_1 + \tau_2/2, \quad \theta = \theta_0 + \theta_2/2 + \sum_{i\geq 3}\tau_{p_i} + \sum_j T_{z_j} \tag{6}$$

2.3 SOPDT

Second-Order-Plus-Dead-Time have one additional time constant

$$W(s) = \frac{K_p}{(1+\tau_1 s)(1+\tau_2 s)}e^{-\theta s} \tag{7}$$

- Gain, 2 time constants, delay.
- Used then $\tau_2 > \theta$.
- Setting the parameters in not an easy task!

Example 4 *Skogestad SOPDT method*

For the second order model [4]:

$$\tau_{10} = \tau_1, \quad \tau_{20} = \tau_2 + \tau_3/2, \quad \theta = \theta_0 + \theta_3/2 + \sum_{i\geq 4}\tau_{p_i} + \sum_j T_{z_j} \tag{8}$$

$$\frac{K_p}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_3 s)(1+\tau_4 s)} \approx \frac{K_p\cdot e^{-(\tau_3/2+\tau_4)s}}{(1+\tau_1 s)(1+[\tau_2+\tau_3/2]s)}$$

If we have positive numerator against neighboring denominator

$$\frac{1+T_z s}{1+\tau_p s} \approx \begin{cases} T_z/\tau_p & \text{for } T_z \geq \tau_p \geq \theta \\ T_z/\theta & \text{for } T_z \geq \theta \geq \tau_p \\ 1 & \text{for } \theta \geq T_z \geq \tau_p \\ T_z/T_p & \text{for } \tau_p \geq T_z \geq 5\theta \\ \frac{\tilde{\tau_p}/\tau_p}{(\tilde{\tau_p}-T_z)s+1} & \text{where } \tilde{\tau_p} = \min(\tau_p, 5\theta) \geq T_z \end{cases} \tag{9}$$

2.4 Identification

Run a test to monitor I/O signals

- Planning
  - A priori info: stable, static;
  - Operating point, input (step, value), measurand;
  - Time and safety.
- Test
  - The existence of nonlinearity;
  - Stability of another inputs.
- Structure of the model
  - What is known?
- Parameter estimation
  - Graphically, statistically (regression analysis).
- Model evaluation
  - How precise the model is?
- Verification
  - Model check with an additional data.

Step response

- Often used, easy to understand;
- Does not contain information about the high-frequency behavior;
- Does not have theoretical advantages;
- Works well with a noise; if signal/noise ratio <5 it is hard to found the derivative;
- Not perfect step change in the input causes the error
  - rise time $T_s = 0.1 \cdot \theta$ of the input signal causes the measurement error of a delay θ up to 20%.

Simple identification tests provide a simple process models, which can be used to design the control system with limited features. Sometimes that is enough.

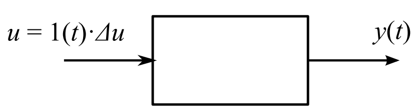
Table 1: Test types

Model structure	Test type				Model parameters
	step	impulse	2 impulses	PRBS	
1st order	✓	✓			K_p, τ
2nd order	✓				$K_p, \tau_2, \tau_1/\omega, \xi$
FOPDT	✓				K_p, τ, θ
SOPDT			✓		$K_p, \tau_1, \tau_2, \theta$
ARX				✓	$y(t+1) = f(y, x)$

Other methods

- Two impulses
- Pseudo-Random Binary Sequence (PRBS)

2.5 Experimental estimation of the FOPDT model parameters



step response of a static object
input step change Δu at time instance $t(0)$.

Register a reaction curve in response to a step change in the input from one steady-state value to another $y_1 \to y_2$.

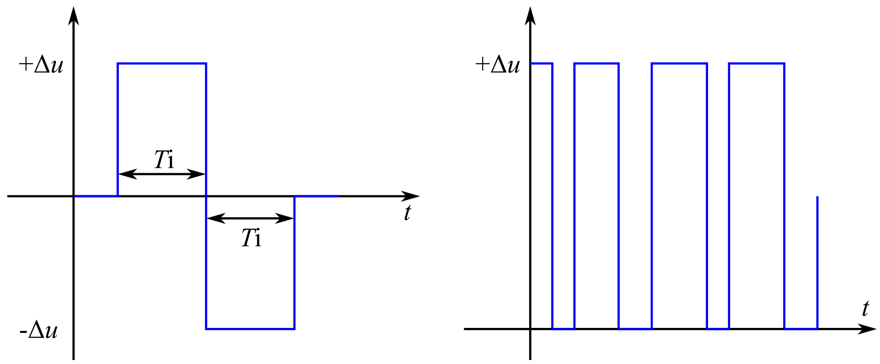


Figure 9: Test signals

The process gain K_p

$$K_p = \frac{\Delta y}{\Delta u} = \frac{y(t \to \infty)}{u(t \to \infty)}. \tag{10}$$

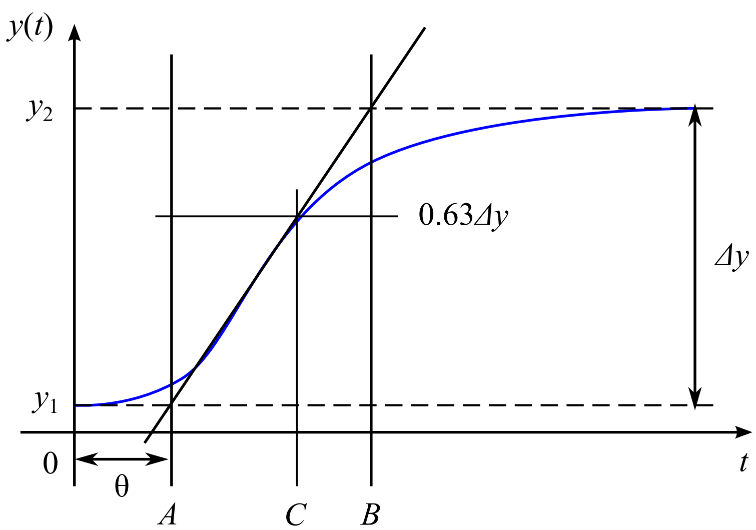


Figure 10: Process reaction curve

Maximum Slope Method

Locate the inflection point of the process reaction curve and draw a **tangent** (puutuja / касательная) along it. The intersection of the tangent line and the time axis y_1 corresponds to the estimate of time delay $\theta = [0\ A]$. The intersection with the final steady-state line helps to calculate the time constant $T = [A\ B]$.

Numerical Application of 63% Method

Time delay can be found like in previous method: $\theta = [0\ A]$. Time constant τ can be obtained by looking at the 63% response time. Calculate the $0.63 \cdot \Delta y$ of the output signal. Mark the time instance then output value is equal to it, so $\tau = [A\ C]$. More accurate technique.

Two-Point Method

Here the time required for the process output to make 28.3% as $t_{28.3\%}$ and 63.2% as $t_{63.2\%}$.

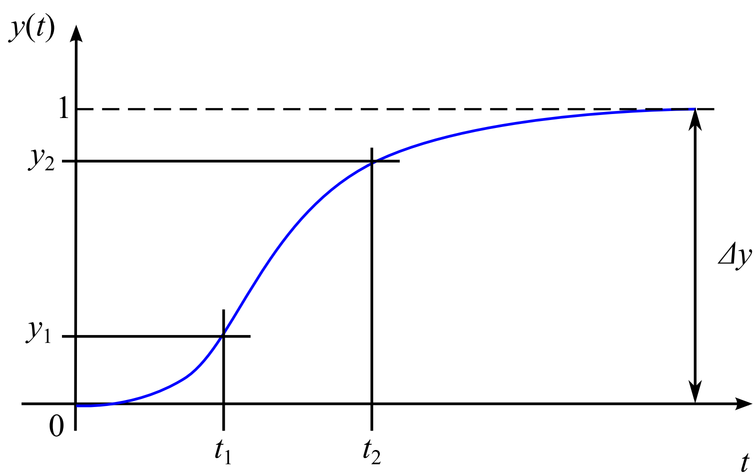


Figure 11: Two-Points Methods

The time constant and time delay can be estimated the following way

$$\tau = 1.5(t_{63.2\%} - t_{28.3\%})$$
$$\theta = t_{63.2\%} - \tau \quad (11)$$

In case of the 20% and 80% levels

$$\tau = 0.721(t_{80\%} - t_{20\%})$$
$$\theta = 1.161t_{20\%} - 0.161t_{80\%} \tag{12}$$

The primary limitation to using step responses to identify FOPDT transfer functions is the amount of time required to assure that the process is approaching a new steady state. That is, the major limit is the time required to determine the gain of the process. For large time constant processes it is often desirable to use a simpler model that does not require a long step test time [5].

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