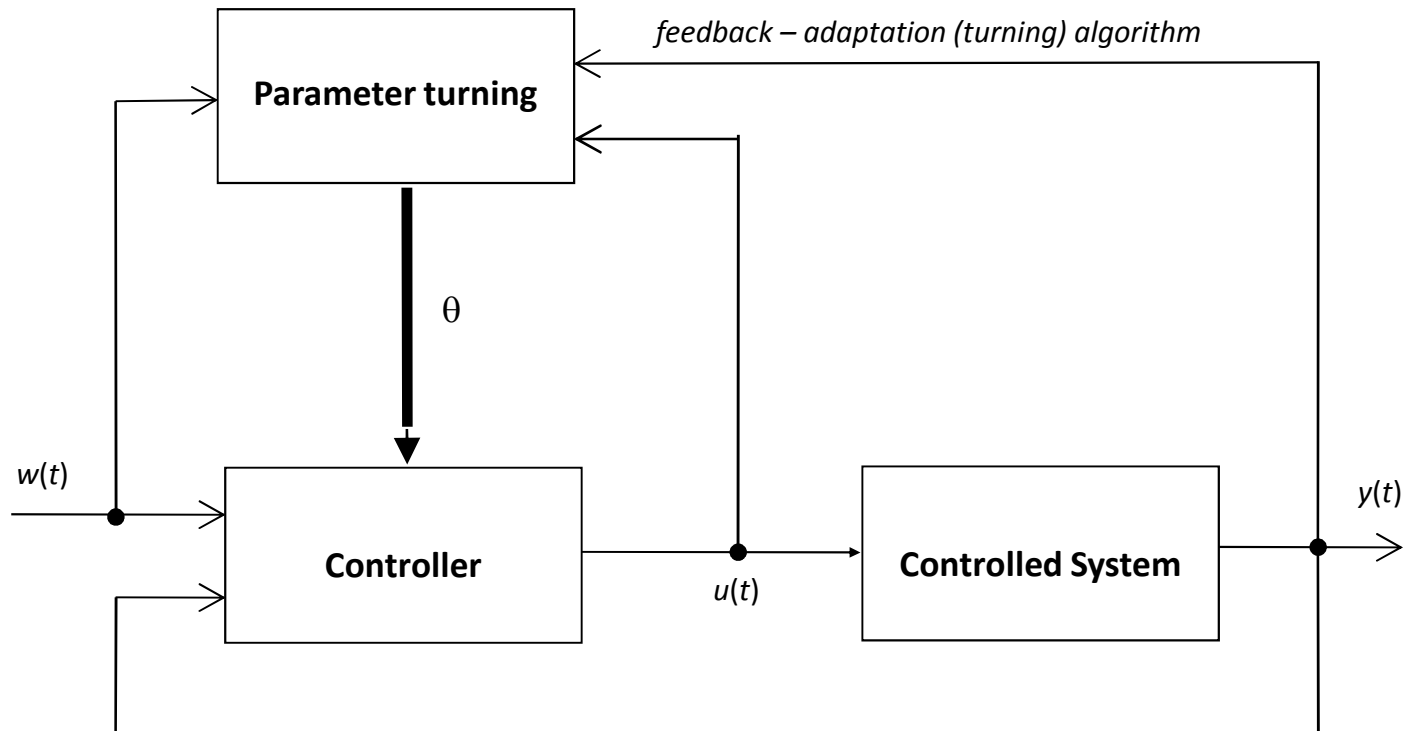


# Part 1

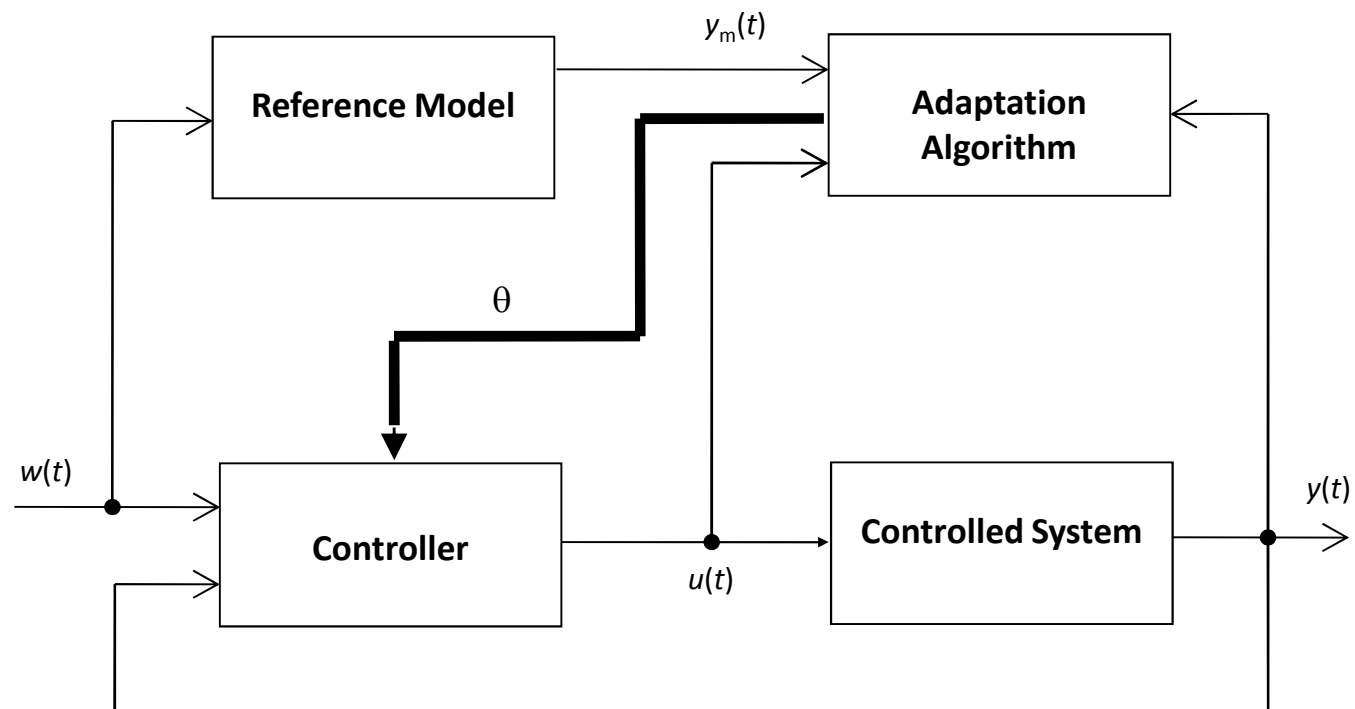
## Reference Model based Adaptive Control

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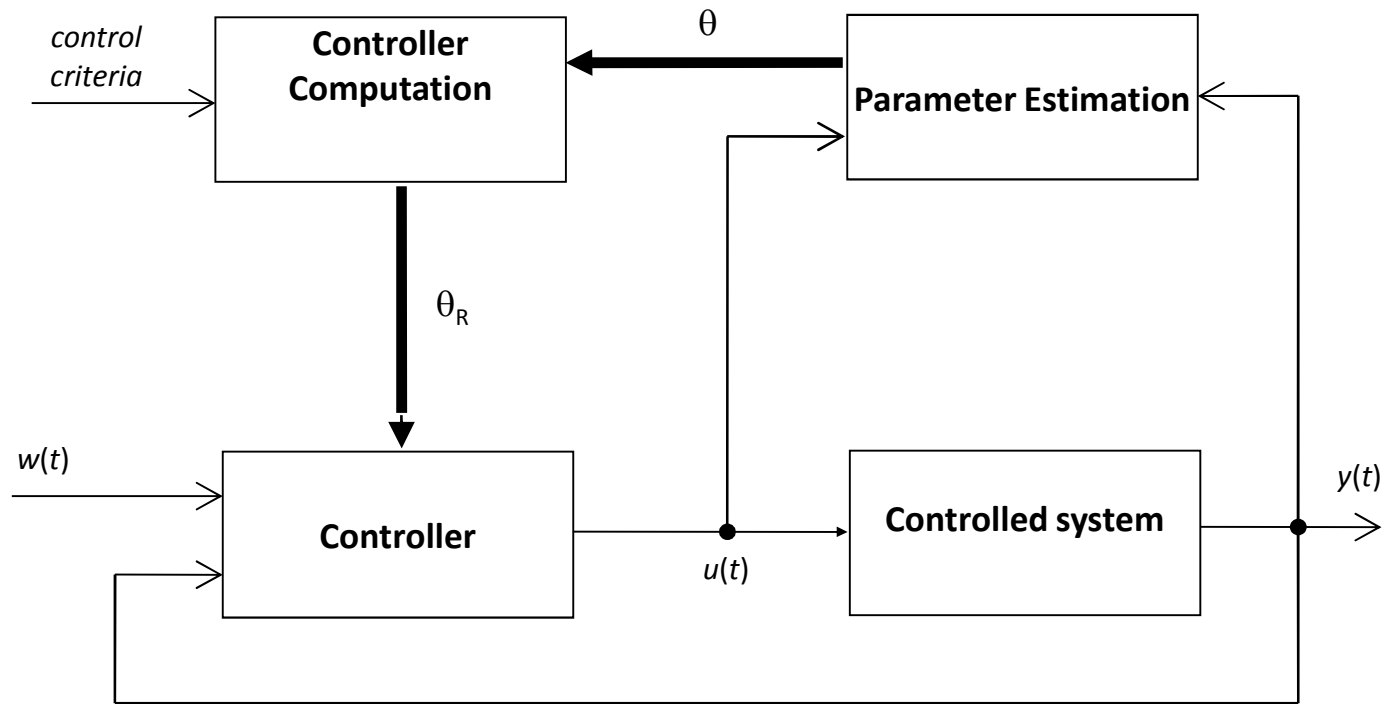
# Structure of an Adaptive System



# Reference Model based Adaptive System



# Identification based Adaptive System



# Reference Model based Control of a Nonlinear System

Controlled nonlinear system:

$$\dot{x}(t) = ax(t) + a_0 f(x) + bu(t)$$

Here  $\mathbf{a}$  ,  $\mathbf{a}_0$  and  $\mathbf{b}$  are unknown constants. State  $\mathbf{x}(t)$  and function  $\mathbf{f}(\mathbf{x})$  are measurable. Nonlinear function  $\mathbf{f}(\mathbf{x})$  is a smooth function of state and  $\mathbf{f}(\mathbf{0})=\mathbf{0}$ .

Reference model:

$$\dot{\mathbf{x}}_m(t) = \mathbf{a}_m \mathbf{x}_m(t) + \mathbf{b}_m \mathbf{w}(t), \quad \mathbf{a}_m < \mathbf{0}$$

# Reference Model based Control of a Nonlinear System

Turnable controller:

$$\mathbf{u}(t) = -\mathbf{k}_1(t)\mathbf{x}(t) - \mathbf{k}_2(t)\mathbf{f}(\mathbf{x}) + \mathbf{k}_0(t)\mathbf{w}(t)$$

Turning algorithm

$$\dot{\mathbf{k}}_1(t) = \mathbf{sign}(b)\mathbf{e}(t)\mathbf{x}(t)$$

$$\dot{\mathbf{k}}_2(t) = \mathbf{sign}(b)\mathbf{e}(t)\mathbf{f}(\mathbf{x})$$

$$\dot{\mathbf{k}}_0(t) = -\mathbf{sign}(b)\mathbf{e}(t)\mathbf{w}(t)$$

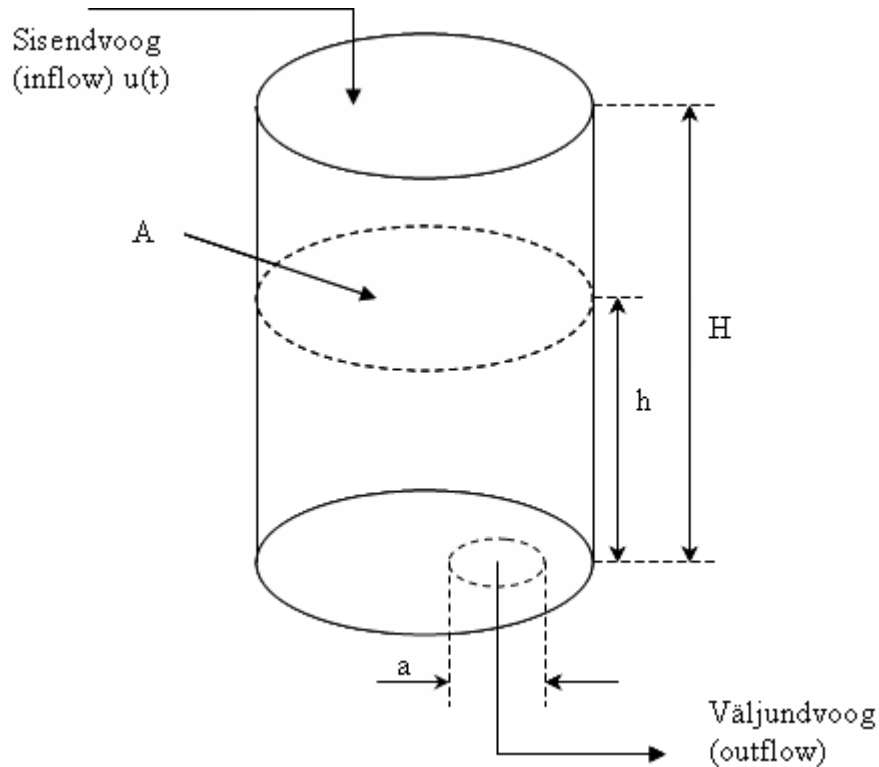
Let the error be

$$e(t) = x(t) - x_m(t) \rightarrow 0$$

Equilibrium state  $\mathbf{e}(t)=0$ ,  $\mathbf{k}_1(t)=\mathbf{k}_1$ ,  $\mathbf{k}_2(t)=\mathbf{k}_2$  and  $\mathbf{k}_0(t)=\mathbf{k}_0$  is stable, which means that the designed adaptive system precisely follows the reference model



## Nonlinear system – tank (liquid level control)



$$\frac{dV}{dt} = u(t) - \text{outflow}(t)$$

$$\rho gh = \frac{\rho v^2}{2}$$

$$\text{outflow}(t) = Sv = S\sqrt{2gh} \quad \left( S = \frac{\pi a^2}{4} \right)$$

$$\frac{dy}{dt} = \frac{1}{100AH} (u(t) - 100S\sqrt{2gHy(t)})$$

$$[u(t)] = \left[ \frac{dal}{s} \right]$$

$$\left. \begin{array}{l} H = 0.5m \\ A = 0.2m^2 (d \approx 0.16m) \\ a \approx 0.04m \end{array} \right\} \Rightarrow \frac{dy}{dt} = u(t) - 0.44\sqrt{y(t)}$$