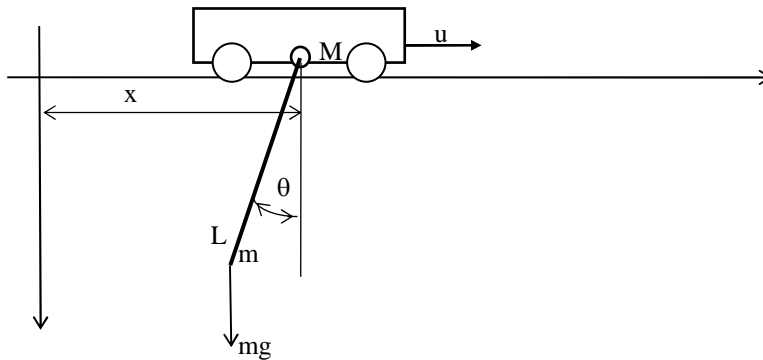


Overhead crane



State-space model ($L = \text{const}$):

θ - angle of rope [rad]

x - position of trolley [m]

m - Mass of load [kg], M - mass of trolley [kg]

L - Length of rope [m]

g - acceleration of gravity [m/s^2]

u - F_x - force [kgm/s^2] (model input)

$$\begin{cases} \dot{X} = A \cdot X + B \cdot U \\ Y = C \cdot X \end{cases} \quad X = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & -\frac{(M+m)g}{ML} & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{ML} \end{bmatrix}$$

$$C = [1 \ 0 \ 0 \ 0]$$

or $C = [1 \ L \ 0 \ 0]$ $x_L = x + L \sin\theta \approx x + L\theta$

Model limitations: from linearization ($\sin\theta \approx \theta$ & $\cos\theta \approx 1$) $\Rightarrow \theta \leq 0.2\text{rad} \Leftrightarrow \text{error} < 1\%$

$L = (0,4+0,05*nr)$ [m] (nt. 1m, $M=2.5$ kg, $m=1$ kg)

$X_{2\text{max}} = 0,2$ rad

$(X_{3\text{max}} = 0,6$ m/s)

$U_{\text{max}} = 10$ kgm/s^2