ISS0031 Modeling and Identification

Practical Work #4: Nonlinear Systems

These exercises are meant to be completed in the MATLAB environment. The first task is to get familiar with the `pplane8` application. In order to launch it, type

```
>> pplane8
```

in the directory which contains the corresponding m-function. Try the default example or load some examples using the *Gallery* menu option. Notice that you can specify parametrized systems. Parameters may be numerical (e.g.: "$A = 1.5$") or represent expressions (e.g.: "$h = x_1 + x_2$"). Once you click *Proceed* you will see the main working window of the program. Here, learn how to

- ☐ draw the trajectories,
- ☐ zoom in and out (you may also use the *pplane8 Setup* window),
- ☐ locate and plot equilibrium points,
- ☐ plot the state evolution in time (i.e., obtain the "state vs. time" graph).

Once you are familiar with the graphical user interface, enter into `pplane8` the description of the pendulum

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\dfrac{g}{l}\sin x_1 - \dfrac{k}{m}x_2, \end{cases}$$

where x_1 is the angle of the pendulum, x_2 is the angular velocity, $g \approx 9.8\mathrm{m/s}^2$ is the gravitational acceleration, l is the length of the pendulum rod, k is the friction, and m is the mass of the payload. Initially choose $l = k = m = 1$. Study the dynamical characteristics of the pendulum. Now, set $k = 0$. Observe the changes in the motion of the system trajectories.

Phase Plane Analysis

For the following systems, locate all equilibrium points and determine the type of each isolated equilibrium. Discuss the qualitative behavior of each system.

1. $\dot{x}_1 = x_2, \quad \dot{x}_2 = x_1 - 2\arctan(x_1 + x_2)$.
2. $\dot{x}_1 = (1 - x_1 - 2h(x))\, x_1, \quad \dot{x}_2 = (2 - h(x))\, x_2$, where $h(x) = x_2/(1 + x_1)$.

To find the equilibrium points, solve the equations for zero dynamics (i.e., $\dot{x}_1 = \dot{x}_2 = 0$) using any applicable method (e.g., the graphical method using the built-in MATLAB function `ezplot`).

Feedback Linearization

Construct the simplified pendulum model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a\sin x_1 - bx_2 + cu \end{aligned}$$

in Simulink. Take $a = b = c = 1$ as parameters of the model. Now, design the feedback linearizing control law

$$u = \frac{a}{c}\sin x_1 + \frac{v}{c}$$

and apply it to the system. Then, choose appropriate feedback coefficients k_1 and k_2 for

$$v = -k_1 x_1 - k_2 x_2$$

and discuss the characteristics of the resulting state feedback control system.