ISS0031 Modeling and Identification

Exercises for Laboratory Work #2

Linear Programming

The MATLAB function

```
x = linprog(c, A, b, Aeq, beq, lb, ub)
```

solves the linear programming problem specified by

$$z = c^{\mathrm{T}} x \to \min$$

subject to constraints

$$A \cdot x \leqslant b$$
$$A_{eq} \cdot x = b_{eq}$$
$$lb \leqslant x \leqslant ub.$$

Example 1

Consider the following linear programming problem from Lecture 2:

$$z = x_1 + 2x_2 \to \min$$
$$x_1 + x_2 \geq 1$$
$$x_1 \geqslant 0, x_2 \geqslant 0.$$

First convert $x_1 + x_2 \geqslant 1$ to equivalent $-x_1 - x_2 \leqslant -1$ and type in MATLAB

```
c  = [1; 2];
A  = [-1 -1];
b  = -1;
lb = zeros(2,1);
```

Thereafter, run the `linprog` function specifying nonexistent matrices as empty:

```
>>  [x, fval] = linprog(c, A, b, [], [], lb)
    Optimization terminated.

    x =
        1.0000
        0.0000

    fval =
        1.0000
```

Thus, the minimum $z_{min}(1, 0) = 1$ is obtained.

Example 2

$$\begin{aligned} z = 3x_1 + 4x_2 &\to \max \\ 2x_1 + 4x_2 &\leqslant 120 \\ 2x_1 + 2x_2 &\leqslant 80 \\ x_1 \geqslant 0, x_2 &\geqslant 0. \end{aligned}$$

This is a problem of maximization. To use MATLAB to solve it we need to convert the objective function as follows:

$$z' = -3x_1 - 4x_2 \to \min,$$

where $z' = -z$. Now, construct the matrices c, A, b, and lb and solve this problem.

Individual Exercises

1. Solve the problem from today's lecture:

$$\begin{aligned} z = 4x_1 + 3x_2 &\to \max \\ 3x_1 + x_2 &\leqslant 9 \\ -x_1 + x_2 &\leqslant 1 \\ x_1 + x_2 &\leqslant 6 \\ x_1 \geqslant 0, x_2 &\geqslant 0. \end{aligned}$$

2. Learn to use the `optimtool` optimization tool and use it to solve the above problems.

Nelder-Mead Simplex Method

Use the supplied `optimize` function to locate the minima of the following functions.

- **Rosenbrock function**

$$f_R(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2.$$

- **Himmelblau's function**

$$f_H(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2.$$

  This function has several minima. Locate them by means of

  - Choosing different initial points in the search space;
  - By using search variable bounds;
  - By using constraints.

Hint: use MATLAB's anonymous function mechanism to construct the cost function that is passed to `optimize`.

Individual Exercise

Write a MATLAB script for automating the task of finding all roots of Himmelblau's function using Nelder-Mead simplex algorithm.