# ISS0031 Modeling and Identification

Exercises for Lecture 2: Solving Linear Programming Problems

Solve the following problems using the graphical method.

**2.1:**

$$
\begin{aligned}
z = 2x_1 - 2x_2 &\to \max(\min) \\
x_1 - 3x_2 &\leq 0 \\
-x_1 + x_2 &\leq 2 \\
4x_1 + 9x_2 &\leq 36 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

**2.2:**

$$
\begin{aligned}
z = -2x_1 - 3x_2 &\to \max(\min) \\
x_1 - 2x_2 &\leq 0 \\
6x_1 + 9x_2 &\geq 27 \\
-x_1 + x_2 &\leq 4 \\
x_2 &\leq 5 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

**2.3:**

$$
\begin{aligned}
z = x_1^2 + x_2^2 &\to \max \\
2x_1 + 3x_2 &\leq 6 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

**2.4:**

$$
\begin{aligned}
z = (x_1 - 1)^2 + (x_2 - 1)^2 &\to \min \\
2x_1 + 3x_2 &\leq 6 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

**2.5:**

$$
\begin{aligned}
z = 5x_1 - 4x_2 &\to \max \\
x_1 + x_2 &\geq 4 \\
2x_1 + x_2 &\leq 6 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

**2.6:**

$$
\begin{aligned}
z = 5x_1^2 - 8x_2 &\to \max(\min) \\
3x_1 + x_2 &\leq 10 \\
x_1 + x_2 &\leq 6 \\
x_1 \geq 0, x_2 &\geq 0
\end{aligned}
$$

**2.7:**

$$
\begin{aligned}
z = x_1^2 x_2 \to \max(\min) \\
x_1^2 + x_2 \leq 12 \\
x_1^2 + x_2^2 \leq 24 \\
x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

**2.8:**

$$
\begin{aligned}
z = 2x_1 + 4x_2 \to \max(\min) \\
-x_1 + 3x_2 \geq 0 \\
x_1 + 2x_2 \leq 5 \\
x_1 + x_2 \geq 2 \\
x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

**2.9:**

$$
\begin{aligned}
z = 6x_1 + 2x_2 \to \max(\min) \\
x_1 - x_2 \leq 2 \\
3x_1 + x_2 \geq 3 \\
-x_1 + x_2 \leq 5 \\
x_1 < 6, x_2 \leq 6 \\
x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

**2.10:**

$$
\begin{aligned}
z = 14x_1 + 4x_2 \to \max(\min) \\
8x_1 + 9x_2 \leq 72 \\
7x_1 + 2x_2 \geq 14 \\
-x_1 + x_2 \leq 2 \\
4x_1 - 7x_2 \leq 14 \\
x_1 \geq 0, x_2 \geq 0
\end{aligned}
$$

## Answers to problems

1. $z_{\max}(9,0) = 18$ and $z_{\min}(0,0) = 0$.
2. $z_{\max}(0,3) = -9$ and $z_{\min}(0,5) = -35$.
3. $z_{\max}(3,0) = 9$.
4. $z_{\min}(1,1) = 0$.
5. $z_{\max}(2,2) = 2$.
6. $z_{\max}(\frac{10}{3},0) = \frac{500}{9}$ and $z_{\min}(0,6) = -48$.
7. $z_{\max}(\sqrt{8},4) = 8\sqrt{2}$ and $z_{\min}$ does not exist.
8. $z_{\max}(x_1,x_2) = 10$, $x_1 = 3-3\alpha, x_2 = 1+\frac{3}{2}\alpha, 0 \le \alpha \le 1$ and $z_{\min}(\frac{3}{2},\frac{1}{2}) = 8$.
9. $z_{\max}(6,6) = 48$ and $z_{\min}(x_1,x_2) = 6$, $x_1 = \alpha, x_2 = 3-3\alpha, 0 \le \alpha \le 1$.
10. $z_{\max}(\frac{315}{46},\frac{44}{23}) = \frac{2293}{23}$ and $z_{\min}(x_1,x_2) = 28$, $x_1 = 2-\frac{8}{9}\alpha, x_2 = \frac{28}{9}\alpha$.